教材

A. Introduction & Review

※ Notation

1. Vectors e.g. Velocity

$$\vec{V} = (u, v, w) = (u_1, u_2, u_3)$$

或 $(\underline{V}) = u_i, \quad i = 1, 2, 3$

$$\vec{X} = (X_1, X_2, X_3) = (X, y, z) = X_i, \quad i = 1, 2, 3$$

2. Tensors 張量

τ_{ij} = shear stress tensor

$\underline{\underline{\tau}}$ or $\underline{\underline{\underline{\tau}}}$ $\quad i = 1, 2, 3 \; ; \; j = 1, 2, 3$

3. Internal energy

u or $\hat{u}$ for internal energy

e = total energy $= \hat{u} + \frac{1}{2}V^2 + gz$
(or e_t)

※熱力學一般忽略動能、位能

$$h = e + \frac{P}{\rho}$$

(enthalpy)

4. Dot & Cross Product

Dot $\vec{A} \cdot \vec{B}$ (or $\underline{A} \cdot \underline{B}$)

Cross $\vec{A} \times \vec{B}$ ($\underline{A} \times \underline{B}$)

$\vec{A} \wedge \vec{B}$ ($\underline{A} \wedge \underline{B}$)

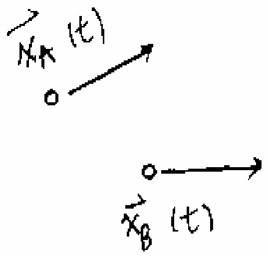
B. Review

1. Description of Fluid Flow

(a) Lagrangian Description 追蹤（同樣粒子）

→ follow or track individual particles.

→ apply laws of motion directly to each particle.

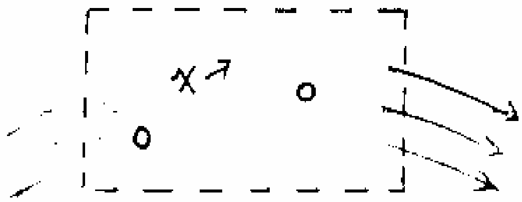


(b) Eulerian Description 取空間（不同的粒子）

→ Look at a fixed field or control volume.

→ Watch the particles pass through this field.

$$\vec{V} = \vec{V}(x, y, z, t)$$



→ To go from laws in Lagrangian description to laws in Eulerian description.

use Substantial Derivative

total :

material :

particle :

Let Q = some quantity in Eulerian Description

$Q = Q(x, y, z, t)$

Total derivative of Q

$$\rightarrow \; dQ = \frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial y}dy + \frac{\partial Q}{\partial z}dz + \frac{\partial Q}{\partial t}dt$$

use $dx = u\,dt$, $dy = v\,dt$, $dz = w\,dt$

$\div dt$

$$\underbrace{\frac{DQ}{Dt} = \frac{dQ}{dt}}_{\text{實質改變量}} = \underbrace{u\frac{\partial Q}{\partial x} + v\frac{\partial Q}{\partial y} + w\frac{\partial Q}{\partial z}}_{\text{convective}} + \underbrace{\frac{\partial Q}{\partial t}}_{\text{local}}$$

Substantial derivative (or unsteady)

* Even for steady flow $\rightarrow \frac{\partial Q}{\partial t} = 0$

you still have $\frac{dQ}{dt} \neq 0$

because convective term $\left(u\frac{\partial Q}{\partial x} + v\frac{\partial Q}{\partial y} + w\frac{\partial Q}{\partial z}\right)$

is not necessary zero.

C. In vector notation.

$\nabla \rightarrow$ gradient operator

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

So

$$\frac{dQ}{dt} = \frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + (\vec{V}\cdot\nabla)Q$$

If $Q = \vec{V}$ itself

$$\frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}$$

$\hookrightarrow \vec{a}$ (加速度)

So Newton law

$$\frac{\vec{F}}{\forall} = \frac{m\vec{a}}{\forall} = \rho \vec{a}$$

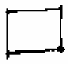
$$\underbrace{\rho\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}\right)}_{\hookrightarrow \vec{a}} = \text{force terms}$$

$$= \frac{\vec{F}}{\forall} \text{ (per unit volume)}$$

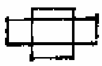
Navier - stokes eq.

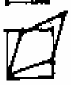
※ Types of fluid motion

4 Types.

(a) translation 

(b) rotation 

(c) dilatation  (stretching & shrinking) 拉伸 & 压缩

(d) shear strain 

In Fluid mech, we discuss <u>rate</u> of motion

In Solid mech, = <u>motion</u>

(a) rate of translation $= \frac{d\vec{x}}{dt} = \vec{V}$

(b) rate of rotation $= \frac{d\vec{\Omega}}{dt}$ (angular velocity)

$\Omega \to$ angle of rotation

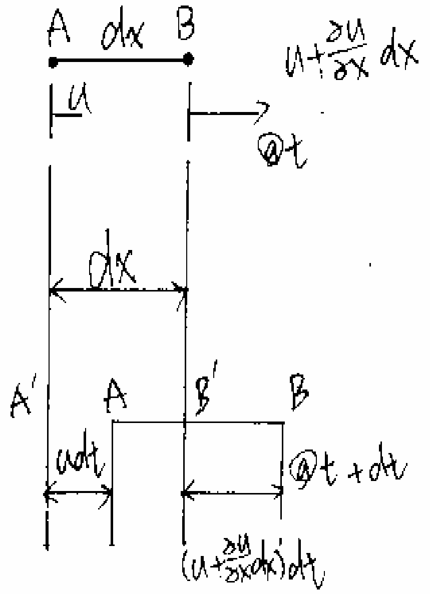
$\vec{\omega}$ = Vorticity vector

$\vec{\omega} = 2\frac{d\vec{\Omega}}{dt}$, $\vec{\omega} = \nabla \times \vec{V} = \text{curl}(\vec{V})$

※ rate of dilatation

dilation → fractional increase in length

eg. in x-direction



% dilation =

$$\frac{[(dx+\overline{BB'})-\overline{AA'}]-dx}{dx}$$

$$\frac{dx-udt+(u+\frac{\partial u}{\partial x}dx)dt-dx}{dx}=\frac{\partial u}{\partial x}dt$$

rate of dilatation (in x-direction)

$$=\frac{\frac{\partial u}{\partial x}dt}{dt}=\frac{\partial u}{\partial x}$$

$$=\varepsilon_{xx}\quad \text{strain rate}$$

Volume dilatation

$$=\frac{1}{\forall}\frac{d\forall}{dt}$$

$$=\frac{1}{\cancel{dx\,dy\,dz}}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\cancel{dx\,dy\,dz}\,\frac{\cancel{dt}}{\cancel{dt}}$$

$$=\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=\nabla\cdot\vec{V}$$

$$\nabla\cdot\vec{V}=0$$

Rate of dilatation in x-dir $=\varepsilon_{xx}=\frac{\partial u}{\partial x}$

〃 y 〃 $=\varepsilon_{yy}=\frac{\partial v}{\partial y}$

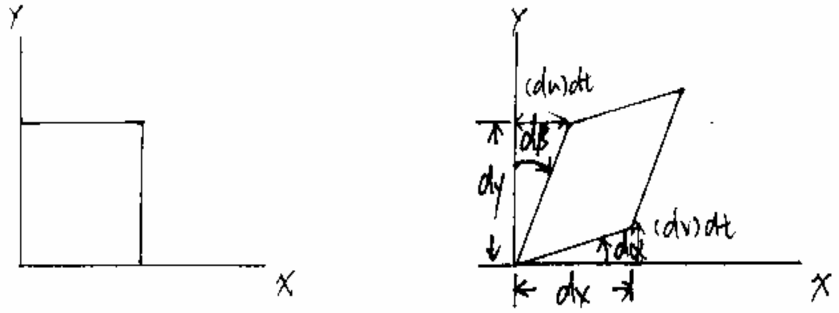
〃 z 〃 $=\varepsilon_{zz}=\frac{\partial w}{\partial z}$

The total rate of dilation

$$\Rightarrow\ \varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}=\frac{1}{\forall}\frac{d\forall}{dt}$$

Rate of shear strain

shear strain = the average decrease in ∢ between two lines which initially ⊥



$$\left(\begin{matrix}\text{shear strain} \\ \varepsilon_{xy}\end{matrix}\right) = \frac{1}{2}(d\alpha + d\beta) \Rightarrow \frac{1}{2}\left(\frac{dv}{dx}dt + \frac{du}{dy}dt\right)$$

So Rate of shear strain $= \frac{d\varepsilon_{xy}}{dt}$

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \quad \varepsilon_{ij} = \frac{1}{2}\left(\frac{u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Strean rate tensor

Tensor Notation 張量符号

Definitions : (1) at free - index - any index that appear only once in an additive term.

. Rules of Tensor Notation

Rule 1 : The words "for i = 1,2,3" are implied by free index

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad i, j = \text{free index}$$

Ralae index (2) at dummy index - any index that appear twice in the same additive term.

dummy index $u_i \frac{\partial Q_i}{\partial x_i} = \sum_{i=1}^{3} u_i \frac{\partial Q_i}{\partial x_i}$

Rule 2 : Summation from i = 1 to 3 is implied by a dummy index.

Rule 3. free index must balance for each additive term in an eq.

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}$$ vaild

$$u_i \frac{\partial u_k}{\partial x_k} = \varepsilon_{ij} A_i$$ no vaild

$$u_i \frac{\partial u_k}{\partial x_k} = 0$$ vaild