Order of expression

→ no# of free index

zero'th order expression (no free indices)

3^{order} = no. of expression

$3^0 = 1$ expression

scalar

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (\nabla \cdot \vec{v} = 0)$$

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0$$

1st order expression (1 free index)

$$u_j \frac{\partial u_i}{\partial x_j} = A_i \quad , i = 1, 2, 3$$

dummy index

$$u_1 \frac{\partial u_i}{\partial x_1} + u_2 \frac{\partial u_i}{\partial x_2} + u_3 \frac{\partial u_i}{\partial x_3} = A_i$$

$3^1 = 3$ expression

→ a vector expression

2nd order expression (2 free index)

e.g. → strain $\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$

$3^2 = 9$ expression $\quad i = 1, 2, 3$

$\quad j = 1, 2, 3$

2nd order tensor

b) derivatives

1) Comma notation

eg. $\frac{\partial u_i}{\partial x_j} = u_{i,j}$

$$\frac{\partial^2 u_i}{\partial x_j \partial x_j} = u_{i,jj} \quad (\text{Laplacian } \nabla^2 \vec{V})$$

$$\frac{\partial^2 Q}{\partial x_i \partial x_j} = Q_{,ij}$$

2) Subscripted del notation

$$\frac{\partial u_i}{\partial x_j} = \partial_j u_i$$

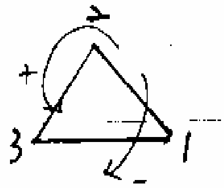
$$\frac{\partial^2 u_i}{\partial x_j \partial x_j} = \partial_j \partial_j u_i \quad (\text{Laplacian})$$

3) Time derivative

$$\frac{\partial u_i}{\partial t} = \dot{u}_i$$

Permutation Sympol

ε_{ijk}



$$\varepsilon_{ijk} = \begin{cases} 1 & \text{if } ijk = 1,2,3 \text{ , } 2,3,1 \text{ , or } 3,1,2 \text{ 逆} \\ -1 & \text{if } \quad = 3,2,1 \text{ , } 1,3,2 \text{ , or } 2,1,3 \text{ 順} \\ 0 & \text{if } i=j \text{ , or } j=k \text{ 兩兩相等} \end{cases}$$

Application

Cross Product

e.g. $\vec{C} = \vec{b} \times \vec{a}$, $C_i = \varepsilon_{ijk}\, b_j\, a_k$

e.g. Vorticity $\vec{\omega} = \nabla \times \vec{V}$

$$\omega_i = \varepsilon_{ijk} \frac{\partial}{\partial x_j} u_k$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ u_1 & u_2 & u_3 \end{vmatrix} \qquad \left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3} \right) \vec{i}$$

1,2,3   1,3,2

+   -

§ Transport Properties

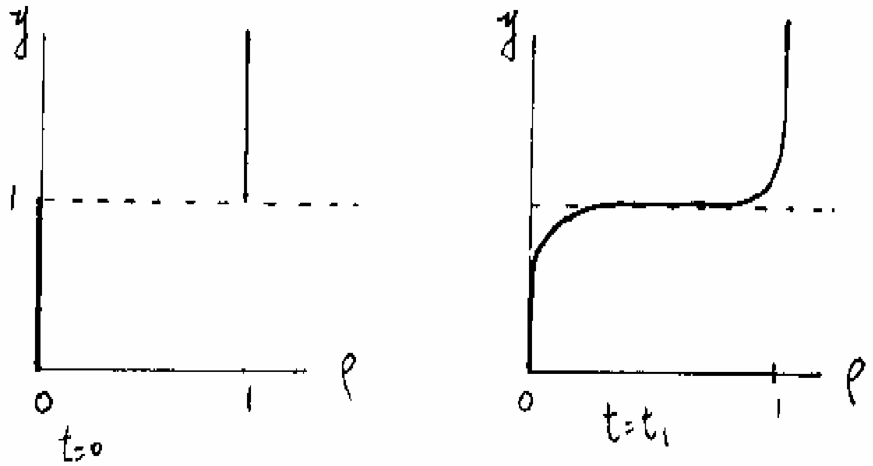
Mass

Momentum

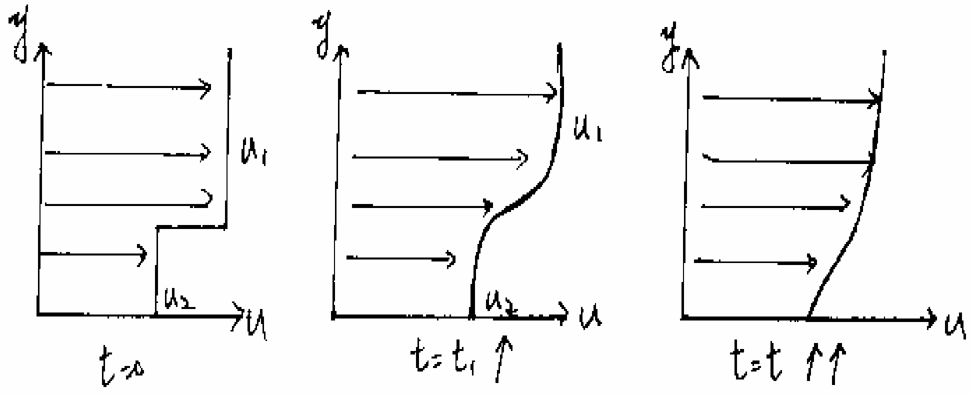
Energy

① Mass Diffusion

D_{12} - coefficient of mass diffusivity



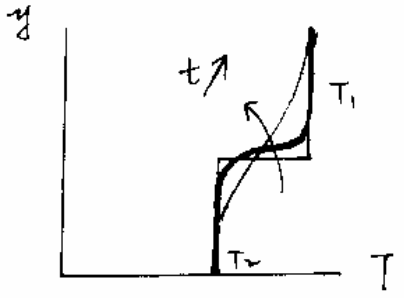
② Momentum diffusion



μ = Viscosity = a transport property

ν = (kinematic viscosity) = $\frac{\mu}{\rho}$

③ Heat (Energy) diffusion



k = coefficient of thermal conductiving
 = a transport property

$$\alpha = \frac{k}{\rho C_p} = \text{Thermal diffusivity}$$

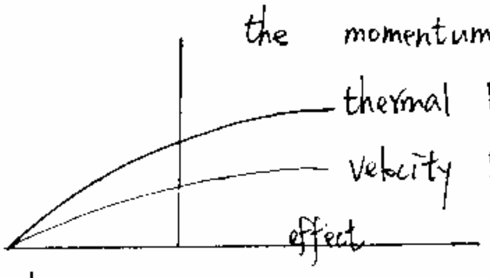
注意, D, ν, α all have dimensions of L^2/t
m^2/s

$$\text{Schmidt}\# \equiv \frac{\nu}{D}$$

$$\text{Prandtl}\# \equiv \frac{\nu}{\alpha}$$ viscosity / thermal diffusivity

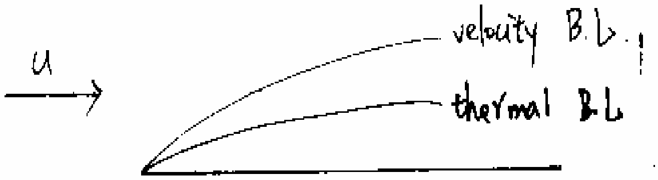
$$\text{Lewis}\# \equiv \frac{D}{\alpha} = \frac{\rho C_p D}{k}$$

Prandtl number : in Air $P_r \doteq 0.7$
the momentum diffusivity is 0.7 time of heat diffusivity



thermal B.L is stronger than momentum B.L effect

Water , $P_r \approx 7$



- in Air $S_c \approx 1$

In Water $S_c \approx 1000$

you can use dye to visualize the water flowfield

- Summary

D - relates diffusion of mass to a concentration gradient

ν - ———————— momentum ——— velocity ———

α - ———————— heat ——— temperture ———

- The higher the coefficient is the faster is the diffusion.