Chap. 1 Introduction (序論)

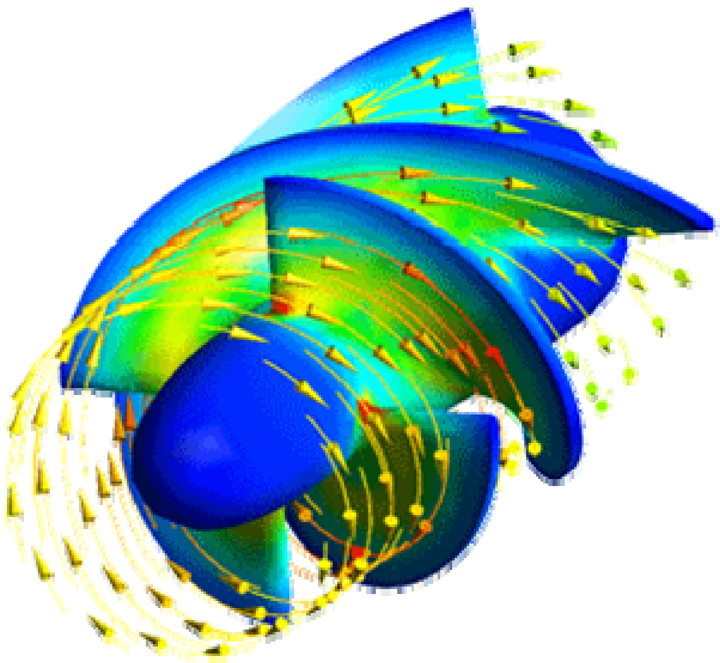
1-1 Definition of a Fluid

1-2 Dimensions and Units

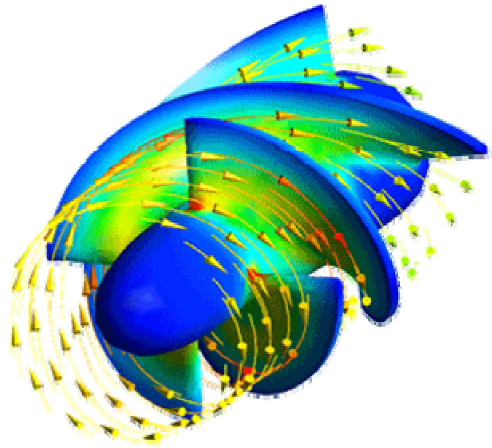
1-3 Properties of Fluid

1-4 Fluid Statics

1-5 Method of Analysis

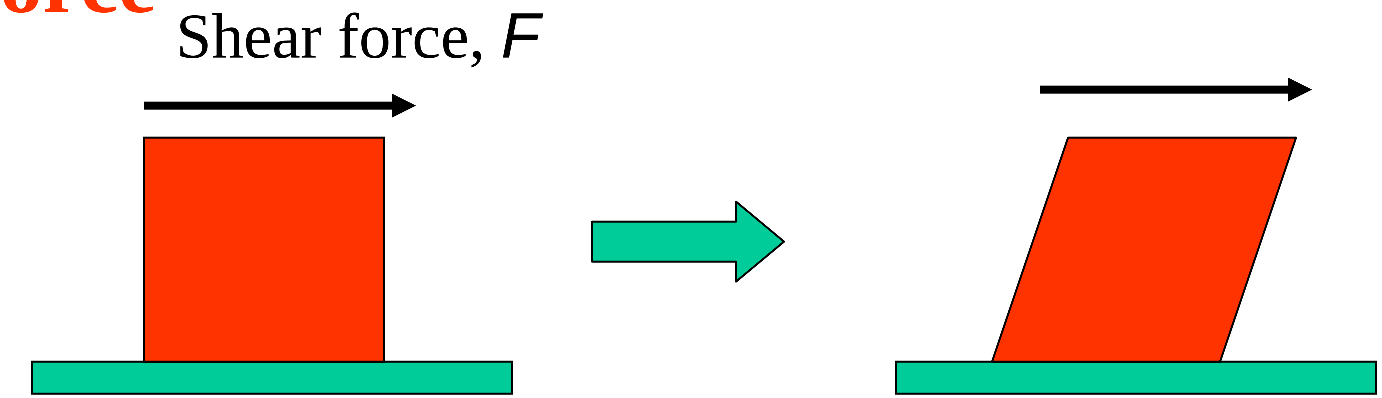


1-1 Definition of a Fluid



- A substance that **deforms continuously** under the application of a **shear (tangential) stress no matter how small** the stress may be.
- There is **no slip at the boundary**.
  - i.e.the fluid in direct contact with the solid boundary has **the same velocity** as the boundary itself;

- **Solid** under the action of a **constant shear force**



Within **elastic limit** of the solid, the deformation is proportional to the applied shear stress.

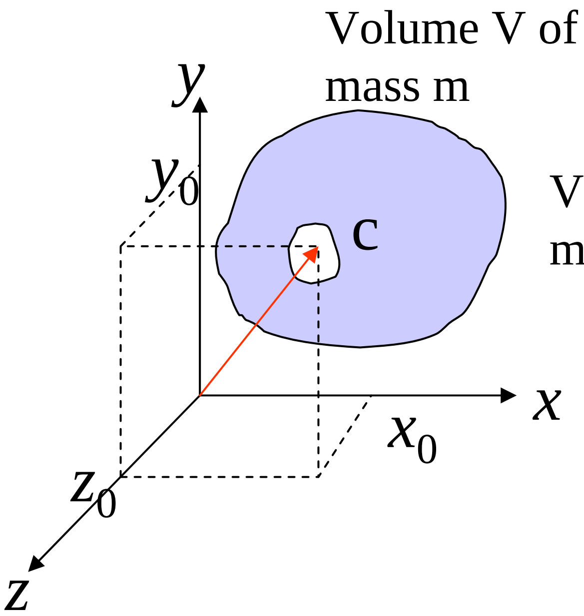
$$\frac{\delta V}{V} \propto \frac{F}{A}$$

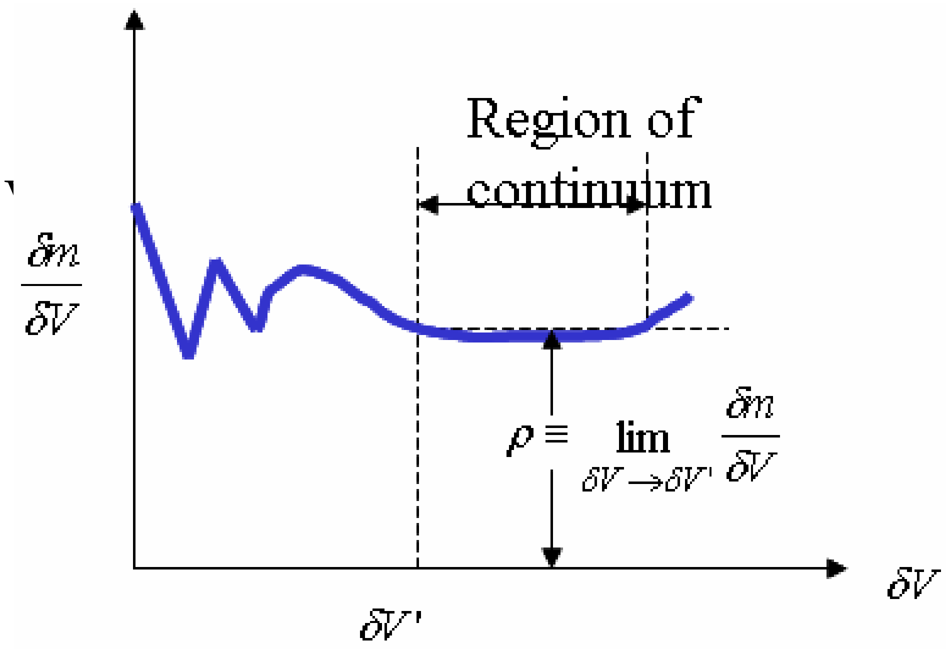
Fluid As a **Continuum**

- Fluid is treated as an **infinitely divisible substance**.
- Each fluid property (T, P, V, ρ, μ, U, H, S, etc.) is assumed to **have a definite value** at every **point** in space.
- The fluid properties are **continuous** function of position and time $\Phi = \Phi(x,y,z,t)$.

e.g. density at a "point"

$$\boxed{\rho \equiv \lim_{\delta V \to \delta V'} \frac{\delta m}{\delta V} = \rho(x,y,z,t)}$$





1-2 Dimensions and Units

Systems of Dimensions(定性)

a. Mass [M], length [L], time [t], temperature [T].

- Force [F] is a secondary dimension

b. Force [F]], length [L], time [t], temperature [T].

- Mass [M] is a secondary dimension

c. Force [F]], Mass [M], length [L], time [t], temperature [T].

- The constant of proportionality, g_c, in Newton's second law in not dimensionless.

Systems of Units(定量)

- SI(Systéme International d'Unités) system of units(MLtT)
  - [M]=kg, [L]=m, [t]=s, [T]=K(kelvin)
  - **[F]=N(Newton)=kg · m/s^2**
- British Gravitational system of units(FLtT)
  - [F]=lbf, [L]=ft, [t]=s, [T]=^{o}R(degree Rankine)
  - **[M]=slug=lbf · s^2/ft**
- English Engineering system of units(FMLtT)

English Engineering system of units(FMLtT)

- [F]=lbf, [M]=lbm, [L]=ft, [t]=s, [T]= ^{o}R

$$\vec{F} = \frac{1}{g_c} m\vec{a} \qquad\qquad 1\,\text{lbf} \equiv \frac{1\,\text{lbm} \times 32.2\ \text{ft/s}^2}{g_c}$$

$$g_c \equiv 32.2\ \text{ft} \cdot \text{lbm/(lbf} \cdot \text{s}^2)$$

$$\boxed{1\,\text{slug} \equiv 1\,\text{lbf} \cdot \text{s}^2\,/\,\text{ft} \equiv 1\,\text{lbm} \times 32.2\ \text{ft/s}^2 \cdot \text{s}^2\,/\,\text{ft} \equiv 32.2\,\text{lbm}}$$

1-3 Properties of Fluid

- **Density---** ρ **=M/V**

$$\rho_{\mathrm{Hg}} = (13.55)(1.94\ \mathrm{slugs/ft^3}) = 26.3\ \mathrm{slugs/ft^3}$$

$$\rho_{\mathrm{Hg}} = (13.55)(1000\ \mathrm{kg/m^3}) = 13.6 \times 10^3\ \mathrm{kg/m^3}$$

- **Specific Volume** $$v = \frac{1}{\rho}$$
- **Specific Weight** $$\gamma = \rho g$$
- **Specific Gravity** $$SG = \frac{\rho}{\rho_{\mathrm{H_2O@4°C}}}$$

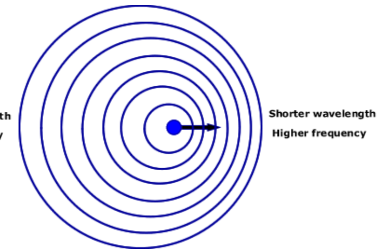
- **Bulk Modulus--Compressibility of Fluids**

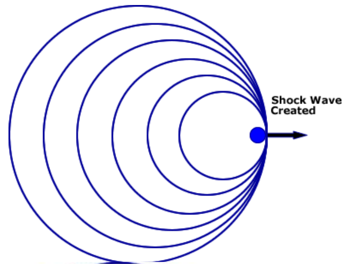
(*bulk modulus of elasticity*)

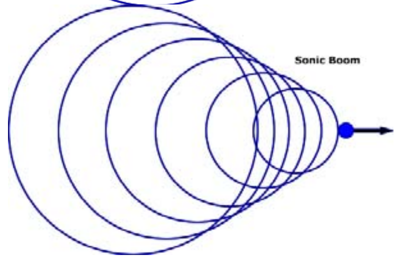
$$E_v = -\frac{dp}{d\forall/\forall} \quad E_v = \frac{dp}{d\rho/\rho}$$

N/m^2 (Pa)

lb/in.^{2} (psi)







Speed of Sound

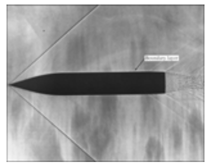
$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{kp}{\rho}} = \sqrt{kRT}$$

Incompressible fluid:

$d\rho/\rho \ll 1$ or **V<<100m/sec or Mach number<<0.3**

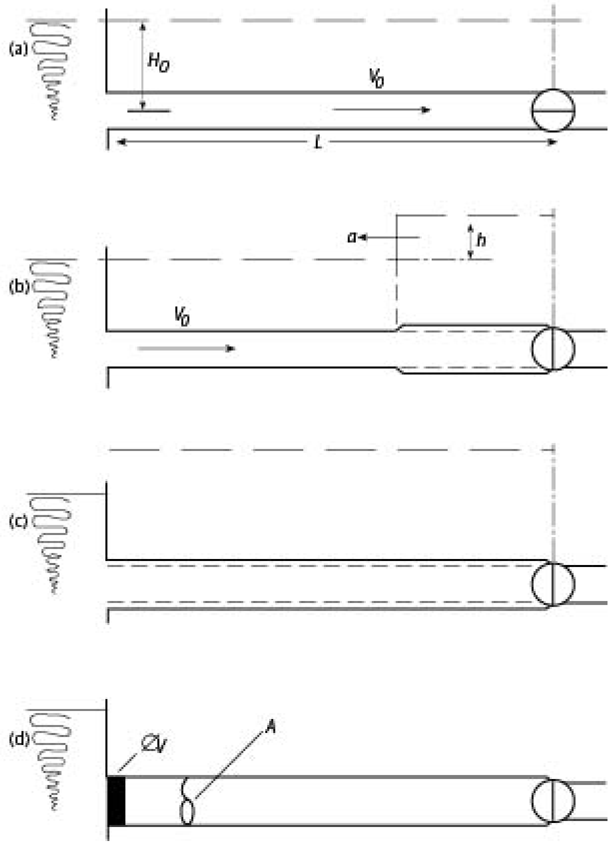
壓縮機

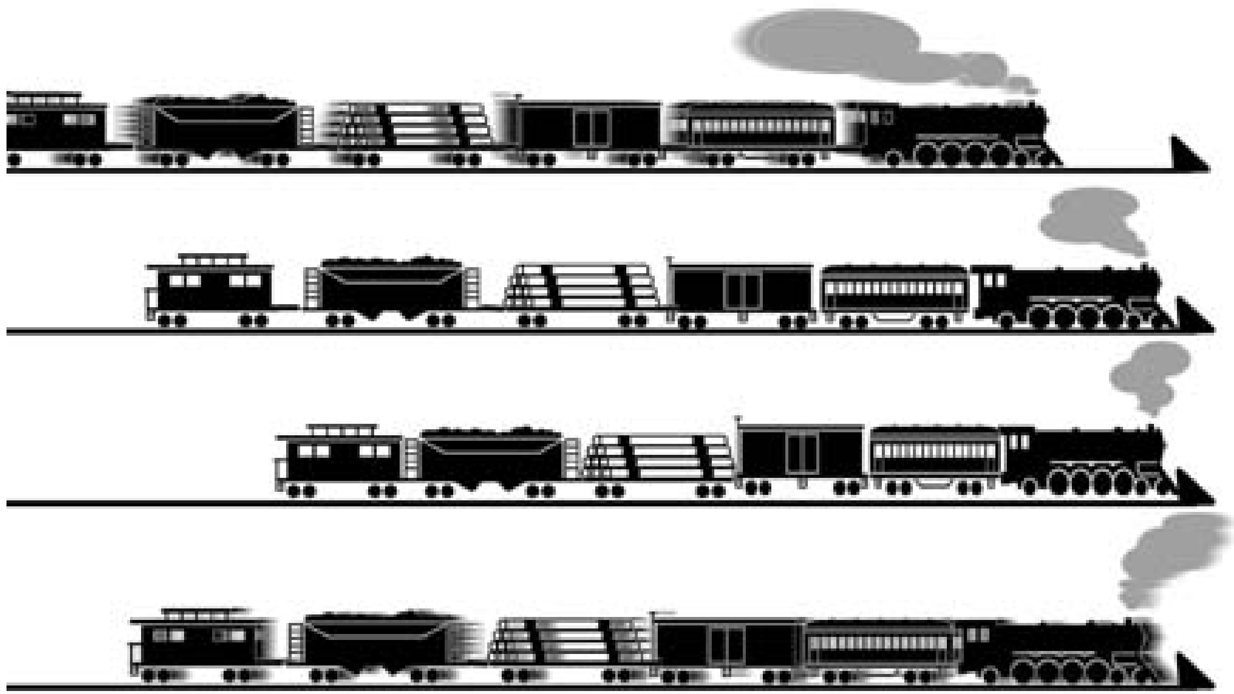
Water Hammer



十七級颱風 =62.4m/s

Water Hammer





※ Viscosity（黏滯力）

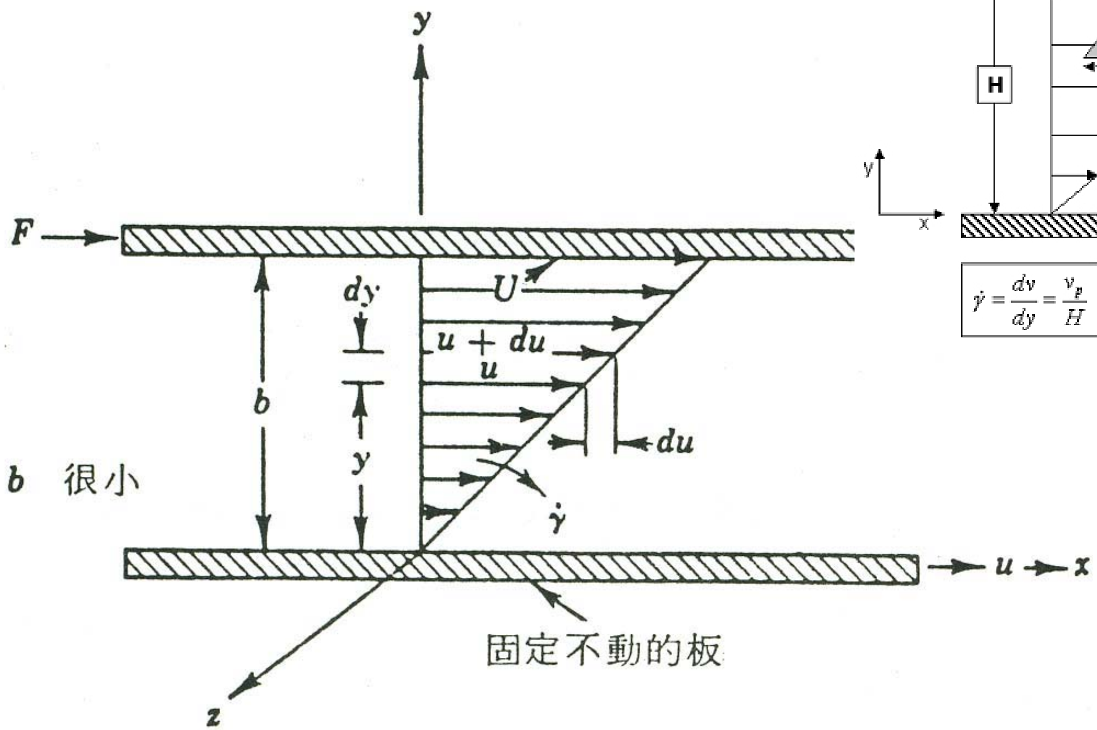
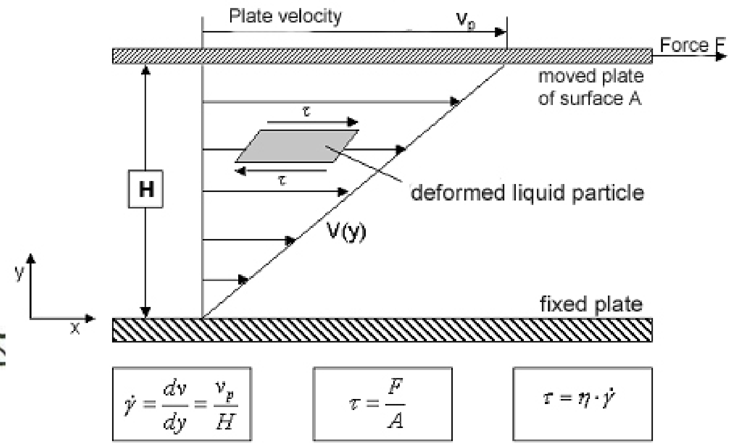
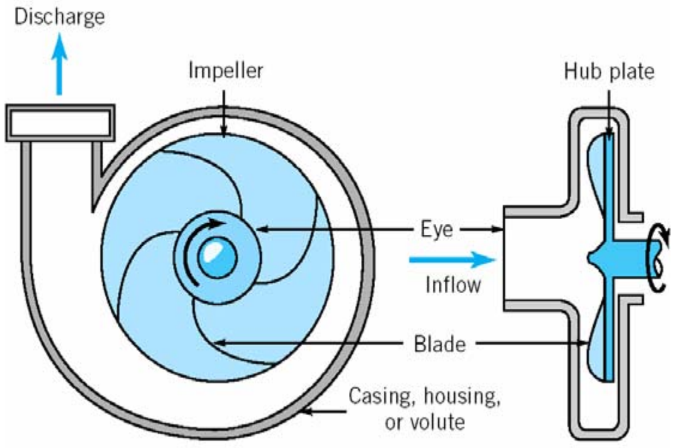


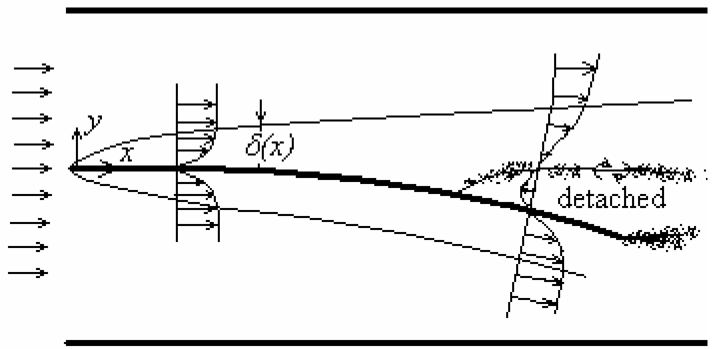
圖 1.1

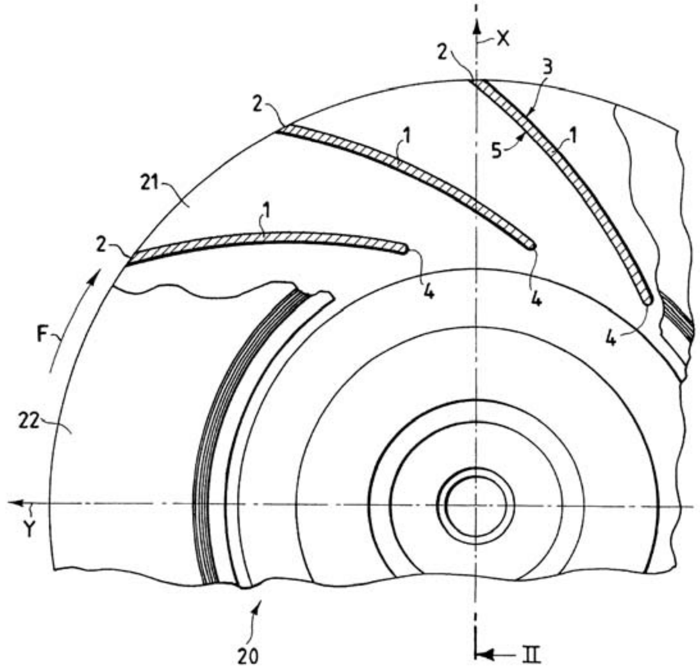


Air -- Boundary layer + Separation

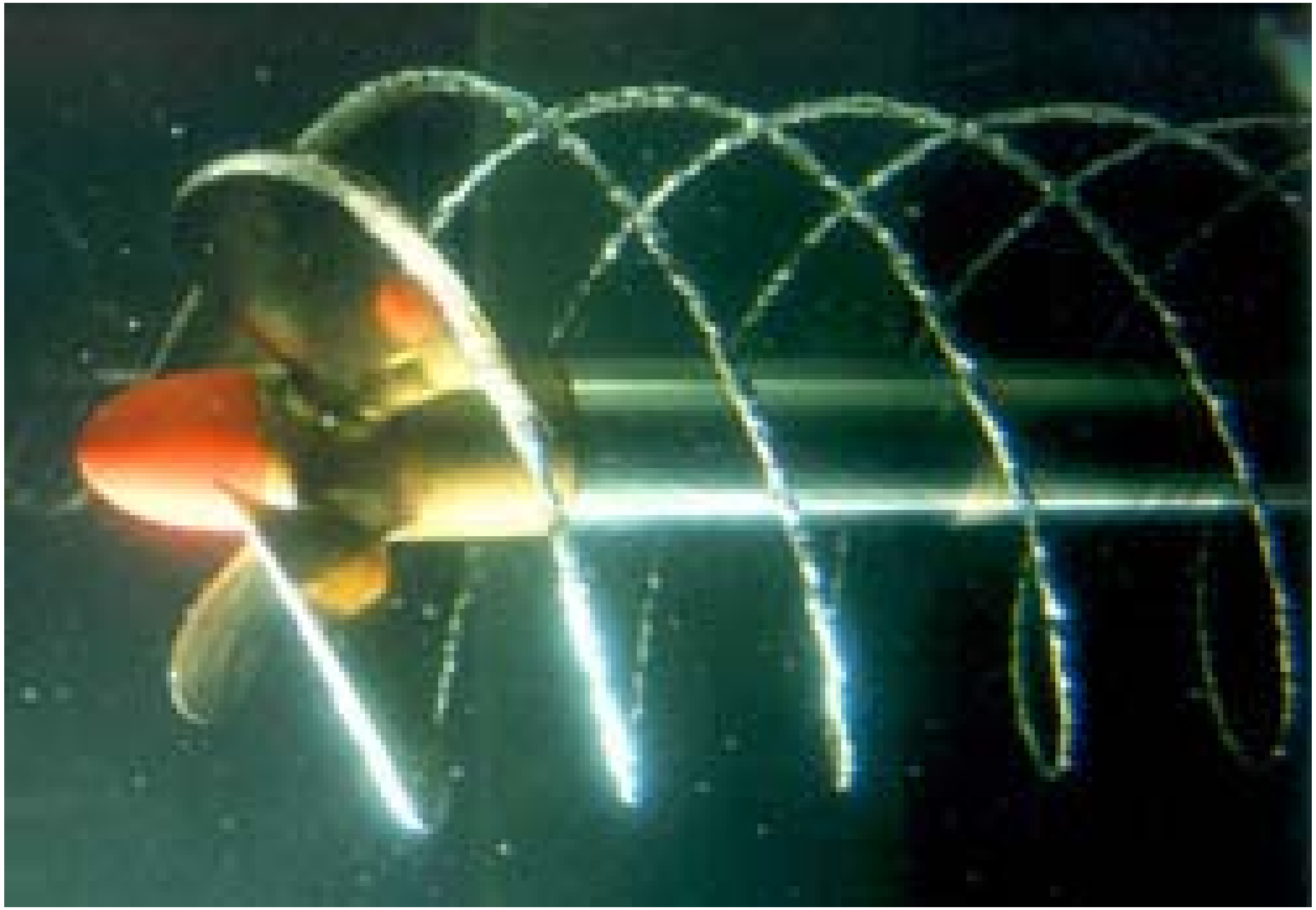


(a) (b)





Water -- Boundary layer + Separation



Dimension and Unit of Viscosity

- Viscosity has dimensions $[F][t]/[L]^2$

SI unit

$$\boxed{\mu = \frac{\tau}{du/dy} = \frac{\mathrm{N/m^2}}{\mathrm{m/s/m}} = \frac{\mathrm{N}}{\mathrm{m^2}} \cdot \mathrm{s} = \mathrm{Pa} \cdot \mathrm{s} = \frac{\mathrm{kg}}{\mathrm{m} \cdot \mathrm{s}}}$$

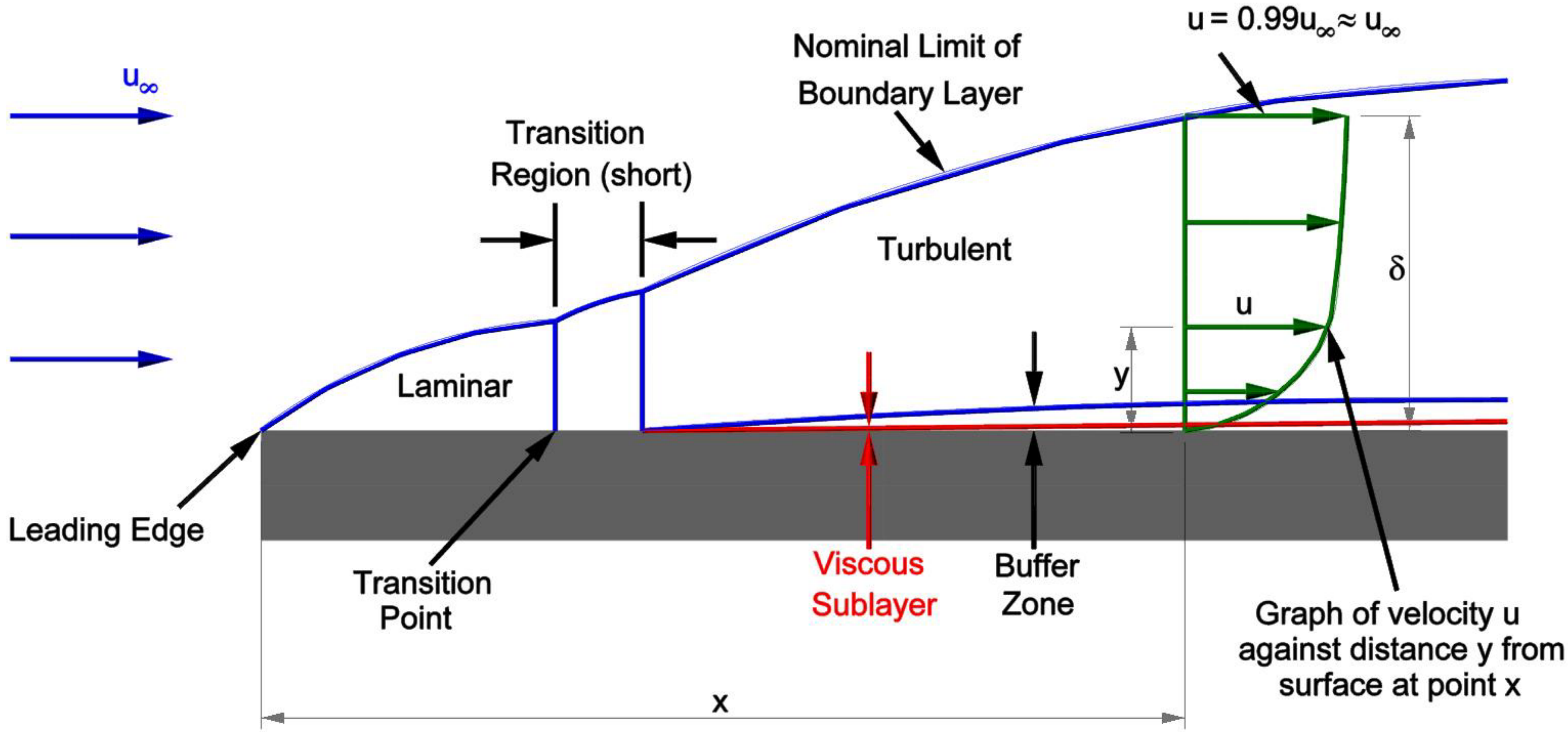
1 poise = 1 g/(cm· s)

Kinematic Viscosity

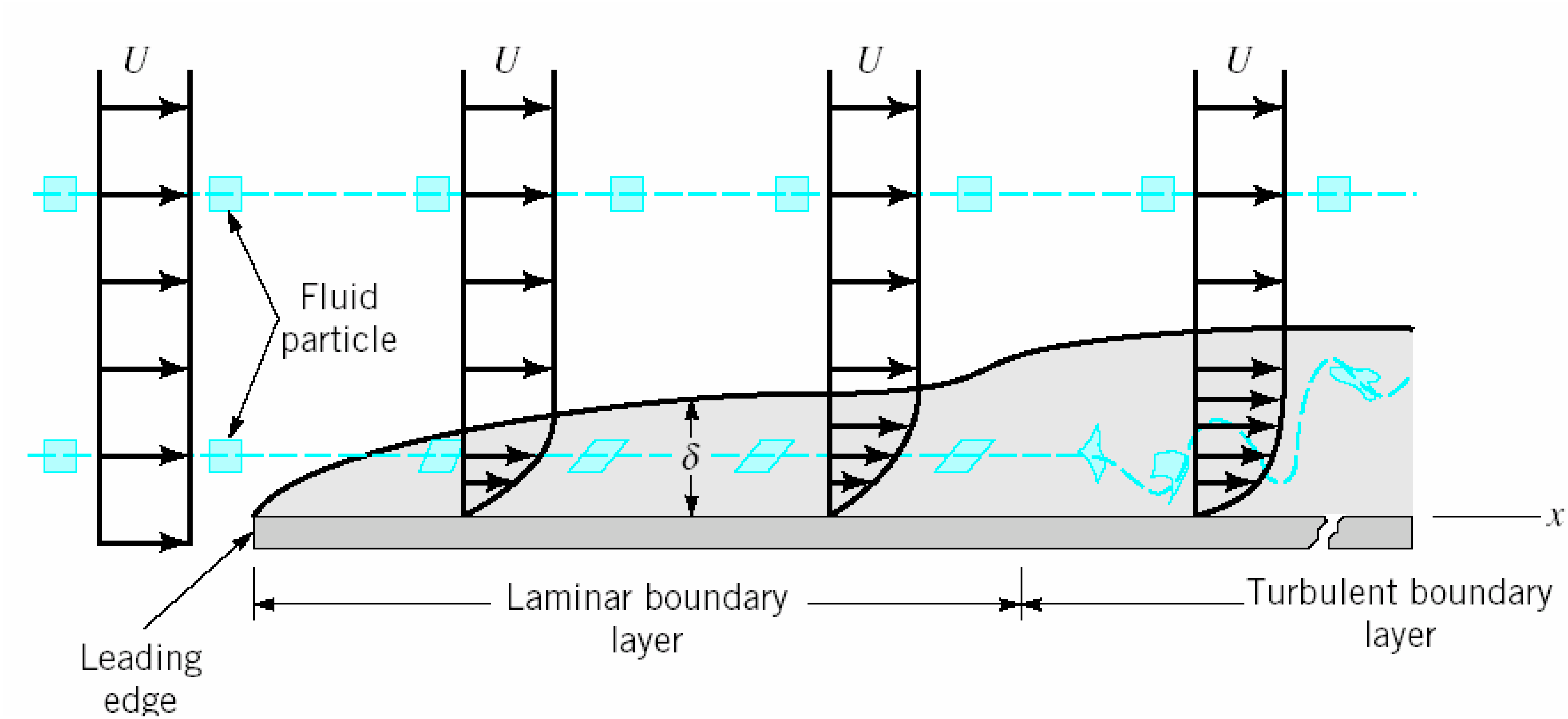
$$\boxed{\nu \equiv \frac{\mu}{\rho}}$$

Dimensions of kinematic viscosity are $[L^2/t]$

1 stoke = 1 cm^2/s

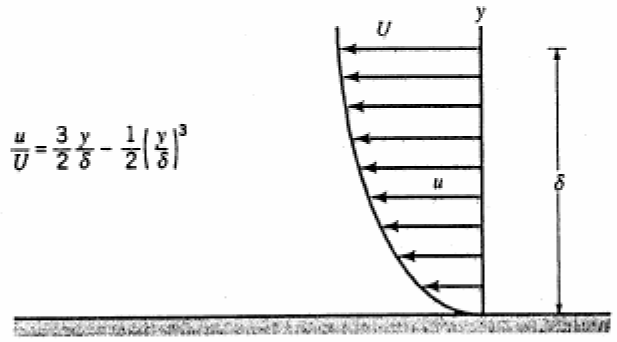


u_∞
u = 0.99u∞ ≈ u∞
Nominal Limit of Boundary Layer
Transition Region (short)
Turbulent
δ
u
y
Laminar
Leading Edge
Transition Point
Viscous Sublayer
Buffer Zone
Graph of velocity u against distance y from surface at point x
x

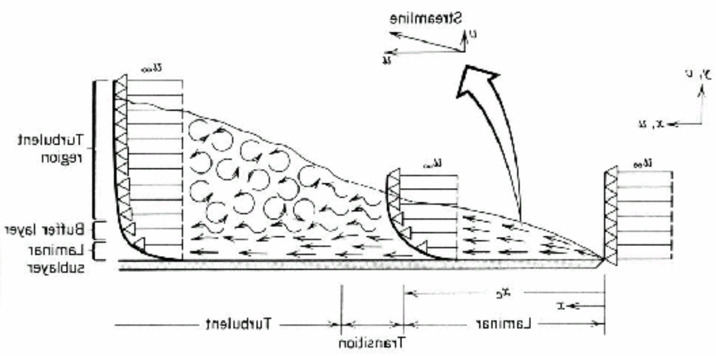


U
U
U
U
Fluid
particle
δ
x
Leading
edge
Laminar boundary
layer
Turbulent boundary
layer

1.58 A Newtonian fluid having a specific gravity of 0.92 and a kinematic viscosity of $4 \times 10^{-4}\ m^2/s$ flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero (as shown in Video V1.2), and the velocity profile near the surface is shown in Fig. P1.58. Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of U and δ, with U and δ expressed in units of meters per second and meters, respectively.



■ FIGURE P1.58



$$\tau_{\substack{surface \\ (y=0)}} = \mu \left(\frac{du}{dy}\right)_{y=0}$$

$$\frac{du}{dy} = U\left(\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right)$$

$$@\ y=0, \quad \frac{du}{dy} = \frac{3}{2}\frac{U}{\delta}$$

$$\text{Since, } \mu = \nu\rho$$

$$\tau_{surface} = \nu\rho\left(\frac{3}{2}\frac{U}{\delta}\right)$$

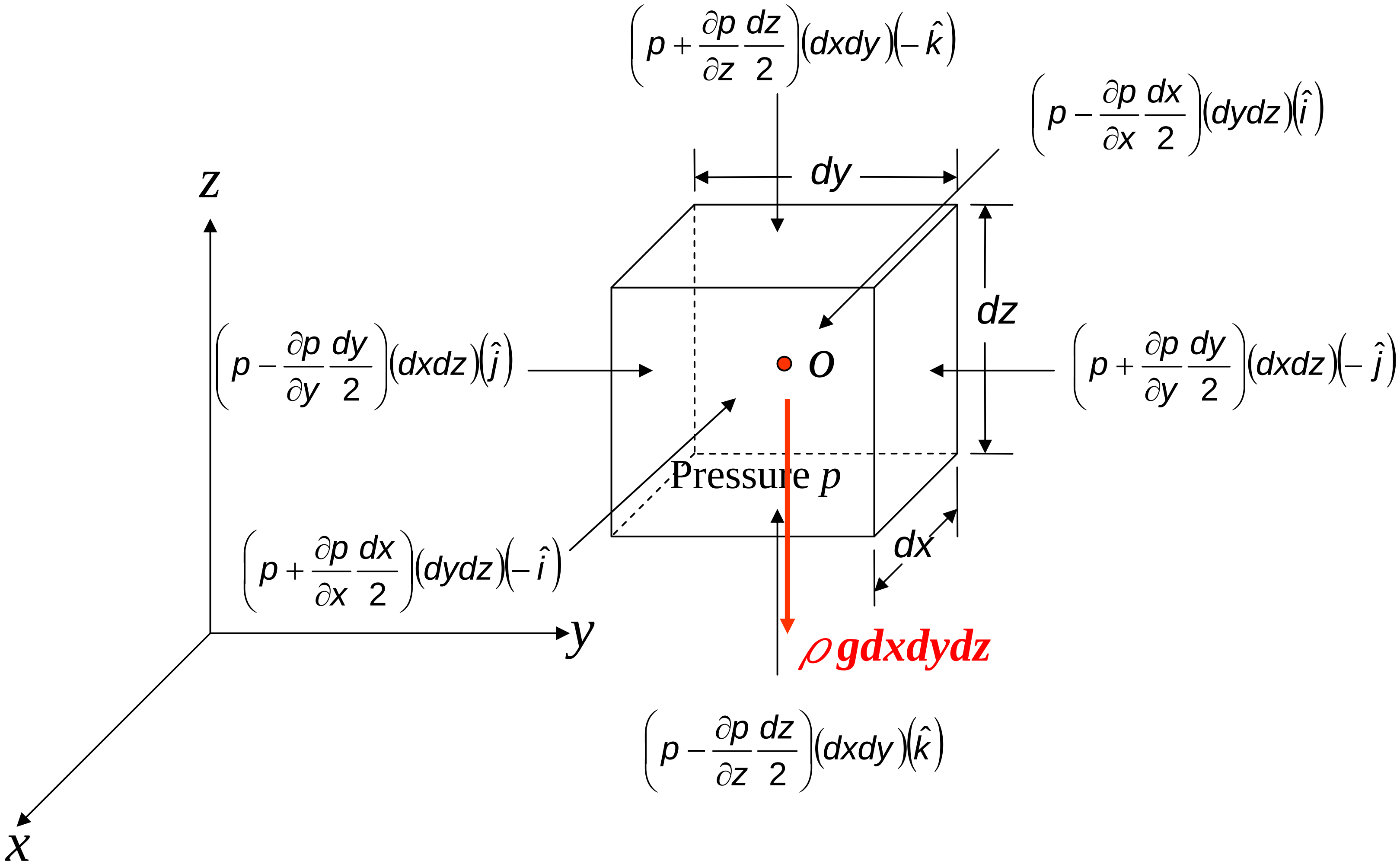
$$= \left(4 \times 10^{-4}\frac{m^2}{s}\right)\left(0.92 \times 10^3 \frac{kg}{m^3}\right)\left(\frac{3}{2}\right)\frac{U}{\delta}$$

$$= \underline{\underline{0.552\frac{U}{\delta}\ N/m^2 \text{ acting to left on plate}}}$$

1-4 Fluid Statics

- The Basic Equation of Fluid Statics
- Pressure Variation in a Static Fluid
- Measurement of Pressure
- Fluids in Rigid Body Motion

Pressure Forces on a **Static** Fluid Element



Body Force on a Fluid Element

- The gravitational body force acing on an element of volume, dV, is $d\vec{F} = \rho\vec{g}dV = \gamma dV$

ρ is the density (mass per unit volume)

$\vec{g}$ is the local gravitational acceleration.

$$\frac{d\vec{F}}{dV} = \rho\vec{g} = \gamma$$ gravitational body force per unit volume

Sum of the Pressure Forces

$$d\vec{F}_S = dF_x\hat{i} + dF_y\hat{j} + dF_z\hat{k}$$

$$\boxed{dF_x = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)(dydz) - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right)(dydz) = -\left(\frac{\partial p}{\partial x}\right)(dxdydz)}$$

$$\boxed{dF_y = \left(p - \frac{\partial p}{\partial y}\frac{dy}{2}\right)(dxdz) - \left(p + \frac{\partial p}{\partial y}\frac{dy}{2}\right)(dxdz) = -\left(\frac{\partial p}{\partial y}\right)(dxdydz)}$$

$$\boxed{dF_z = \left(p - \frac{\partial p}{\partial z}\frac{dz}{2}\right)(dxdy) - \left(p + \frac{\partial p}{\partial z}\frac{dz}{2}\right)(dxdy) = -\left(\frac{\partial p}{\partial z}\right)(dxdydz)}$$

$$d\vec{F}_S = -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)(dxdydz)$$

Pressure Gradient

$$d\vec{F}_S = -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)(dxdydz)$$

$$grad\ p \equiv \nabla p \equiv \left(\hat{i}\frac{\partial p}{\partial x} + \hat{j}\frac{\partial p}{\partial y} + \hat{k}\frac{\partial p}{\partial z}\right) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)p$$

$$\nabla \equiv \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

$$d\vec{F}_S = -\nabla p(dxdydz)$$

$$\nabla \equiv \left(\hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{e}_z\frac{\partial}{\partial z}\right)$$

Total Force

$$d\vec{F} = d\vec{F}_S + d\vec{F}_B = \left(-\nabla p + \rho\vec{g}\right)\left(dxdydz\right) = \left(-\nabla p + \rho\vec{g}\right)d\forall$$

$$\frac{d\vec{F}}{d\forall} = -\nabla p + \rho\vec{g}$$

For a static fluid

$$\boxed{\frac{d\vec{F}}{d\forall} = -\nabla p + \rho\vec{g} = 0}$$

Physical Meaning of Force Balance Equation

$$\underbrace{\left\{\begin{array}{c}\text{net pressure force}\\ \text{per unit volume}\\ \text{at a point}\end{array}\right\}}_{}^{-\nabla p} + \overbrace{\left\{\begin{array}{c}\text{body force per}\\ \text{unit volume}\\ \text{at a point}\end{array}\right\}}^{\rho\vec{g}} = 0$$

$$\boxed{\begin{aligned} -\frac{\partial p}{\partial x} + \rho g_x &= 0 \\ -\frac{\partial p}{\partial y} + \rho g_y &= 0 \\ -\frac{\partial p}{\partial z} + \rho g_z &= 0 \end{aligned}}$$

If z axis is directed vertically upward,

$$-\frac{\partial p}{\partial x} = 0$$

$$-\frac{\partial p}{\partial y} = 0$$

$$\boxed{\frac{\partial p}{\partial z} = -\rho g_z}$$

Pressure Variation in a Static Fluid

- Incompressible Liquids (ρ=constant)

For g=constant

$$\frac{\partial p}{\partial z} = -\rho g = \text{constant}$$

$$\Downarrow$$

$$\int_{p_0}^{p} dp = -\int_{z_0}^{z} \rho g dz$$

$$p - p_0 = -\rho g (z - z_0) = \rho g (z_0 - z) \Rightarrow \boxed{p = \gamma h + p_0}$$

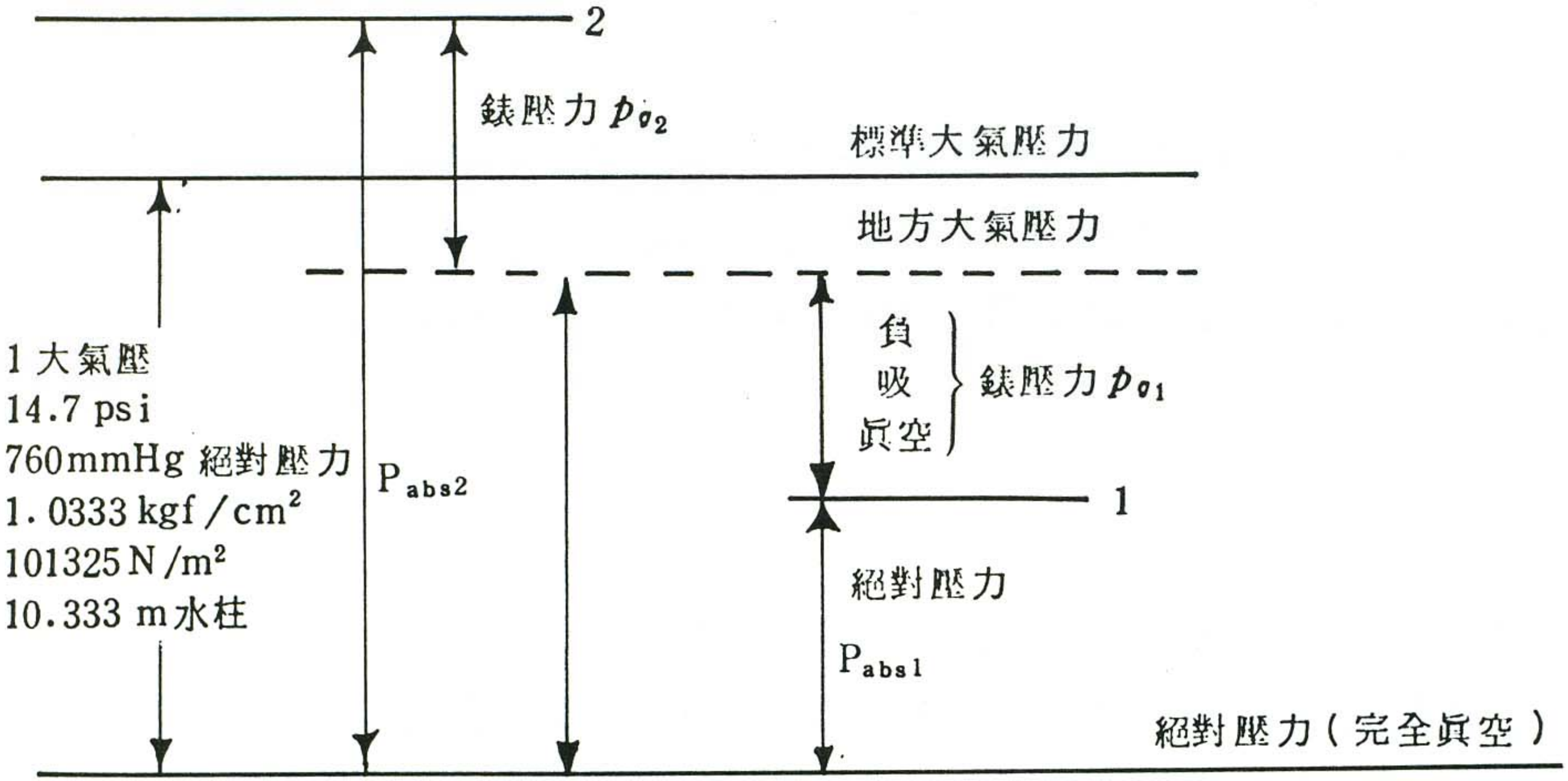
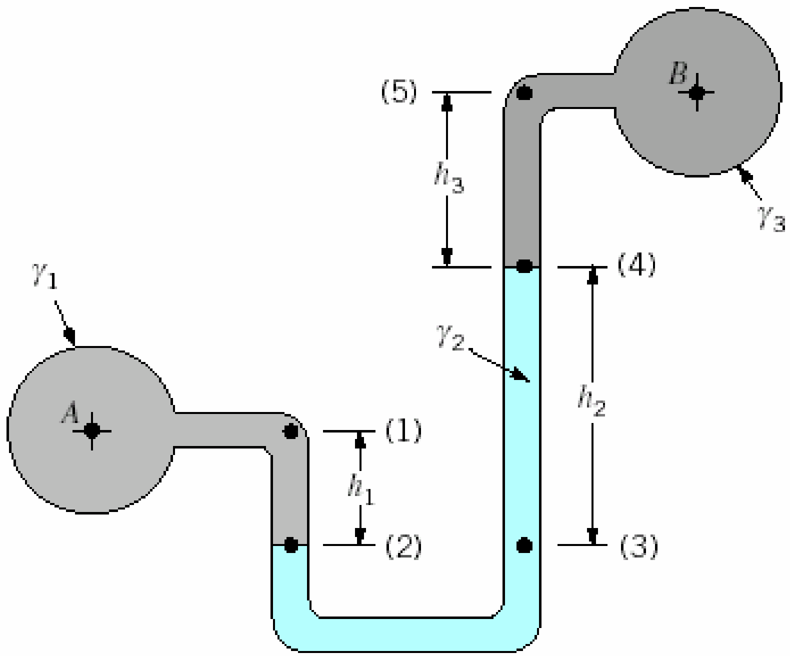

2
錶壓力 p_{o2}
標準大氣壓力
地方大氣壓力
1 大氣壓
14.7 psi
760mmHg 絕對壓力
1.0333 kgf/cm²
101325 N/m²
10.333 m水柱
P_{abs2}
負
吸
眞空
錶壓力 p_{o1}
1
絕對壓力
P_{abs1}
絕對壓力(完全眞空)

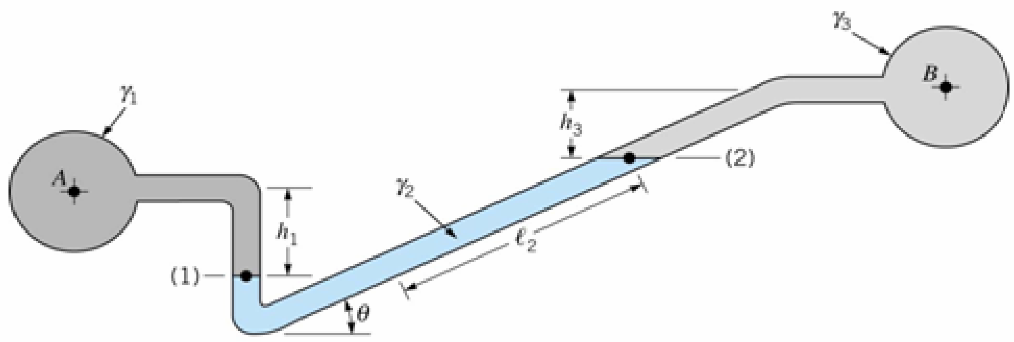
圖 1.3



$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

Inclined-tube manometer.



$$p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 \ell_2 \sin\theta$$

$$p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

$$\ell_2 = \frac{p_A - p_B}{\gamma_2 \sin\theta}$$

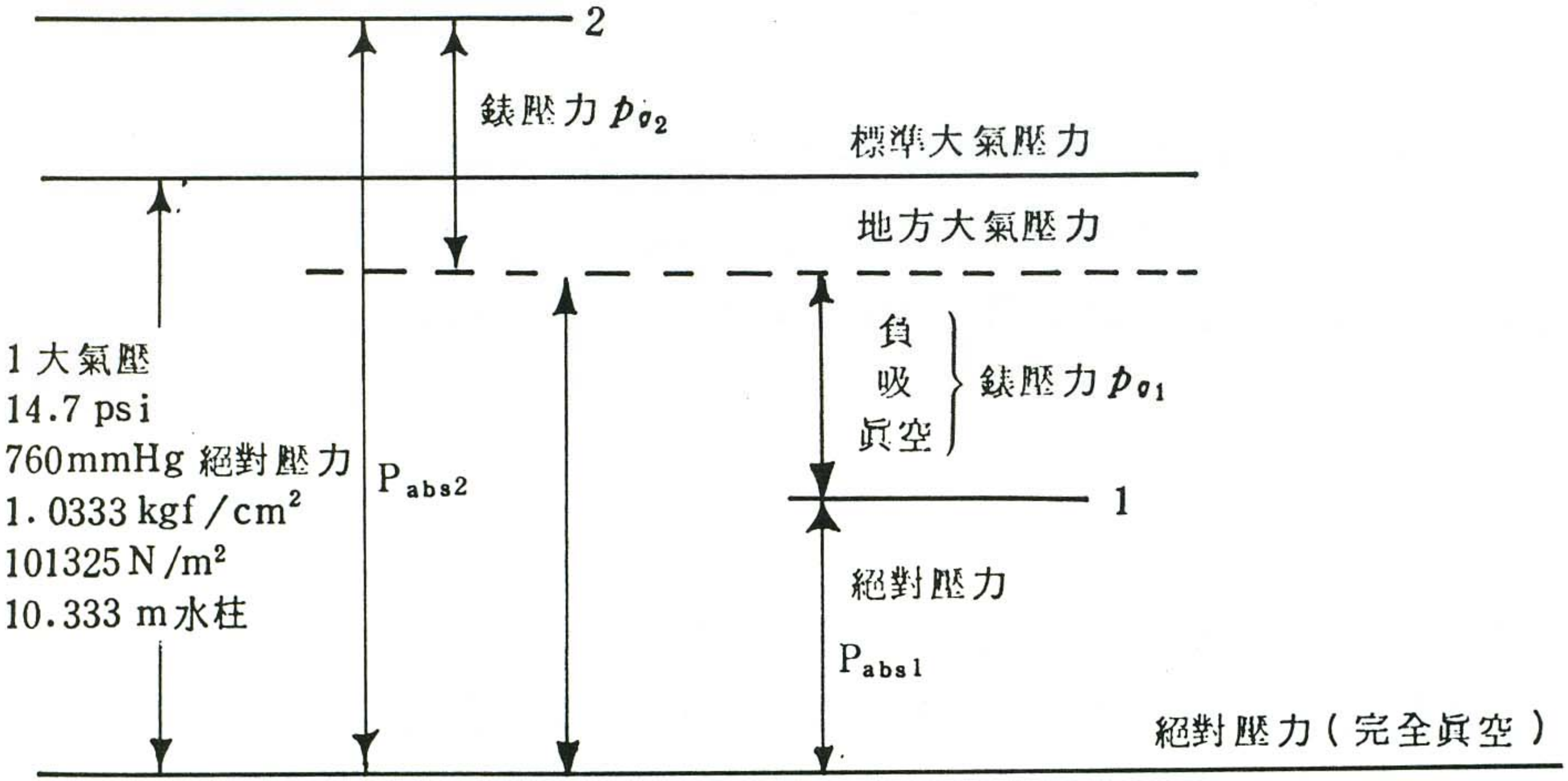
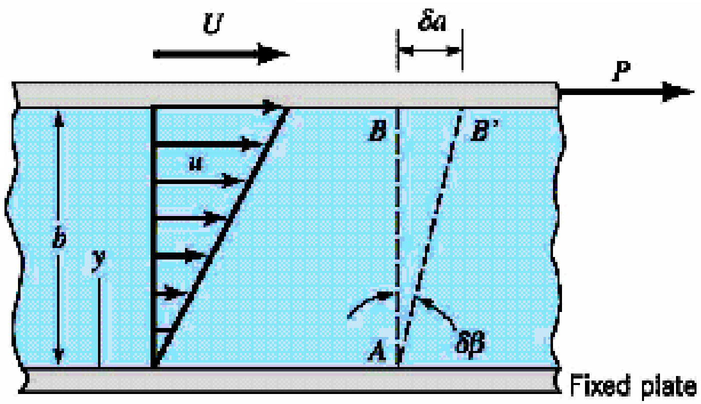

2
錶壓力 p_{o2}
標準大氣壓力
地方大氣壓力
1 大氣壓
14.7 psi
760mmHg 絕對壓力
1.0333 kgf / cm²
101325 N / m²
10.333 m水柱
P_{abs2}
負
吸
眞空
錶壓力 p_{o1}
1
絕對壓力
P_{abs1}
絕對壓力(完全眞空)

圖 1.3

1.53 For a parallel plate arrangement of the type shown in Fig. 1.3 it is found that when the distance between plates is 2 mm, a shearing stress of 150 Pa develops at the upper plate when it is pulled at a velocity of 1 m/s. Determine the viscosity of the fluid between the plates. Express your answer in SI units.



$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = \frac{U}{b}$$

$$\mu = \frac{\tau}{\left(\frac{U}{b}\right)} = \frac{150 \frac{N}{m^2}}{\left(\frac{1 \frac{m}{s}}{0.002\,m}\right)} = \underline{\underline{0.300 \frac{N \cdot s}{m^2}}}$$

hat

$$\rho_f = \left[\frac{70 \text{ psi (abs)}}{14.7 \text{ psi (abs)}}\right]^{\frac{1}{1.31}} \left(1.29 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}\right) = \underline{\underline{4.25 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}}}$$

$$T_f = \frac{p_f}{\rho_f R} = \frac{\left(70 \frac{\text{lb}}{\text{in.}^2}\right)\left(144 \frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(4.25 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}\right)\left(3.099 \times 10^3 \frac{\text{ft} \cdot \text{lb}}{\text{slug} \cdot {}^\circ\text{R}}\right)}$$

$$= 765\ {}^\circ\text{R}$$

$$T_f = 765\ {}^\circ\text{R} - 460 = \underline{\underline{305\ {}^\circ\text{F}}}$$

2.34 Small differences in gas pressures are commonly measured with a *micromanometer* of the type illustrated in Fig. P2.34. This device consists of two large reservoirs each having a cross-sectional area, A_r, which are filled with a liquid having a specific weight, γ_1, and connected by a U-tube of cross-sectional area, A_t, containing a liquid of specific weight, γ_2. When a differential gas pressure, $p_1 - p_2$, is applied a differential reading, h, develops. It is desired to have this reading sufficiently large (so that it can be easily read) for small pressure differentials. Determine the relationship between h and $p_1 - p_2$ when the area ratio A_t/A_r is small, and show that the differential reading, h, can be magnified by making the difference in specific weights, $\gamma_2 - \gamma_1$, small. Assume that initially (with $p_1 = p_2$) the fluid levels in the two reservoirs are equal.

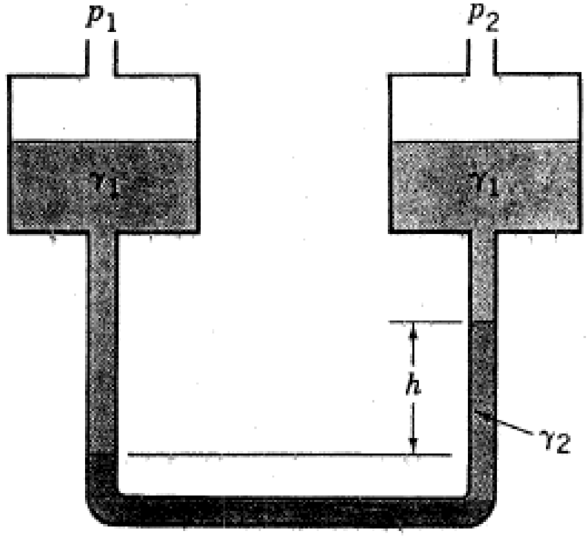
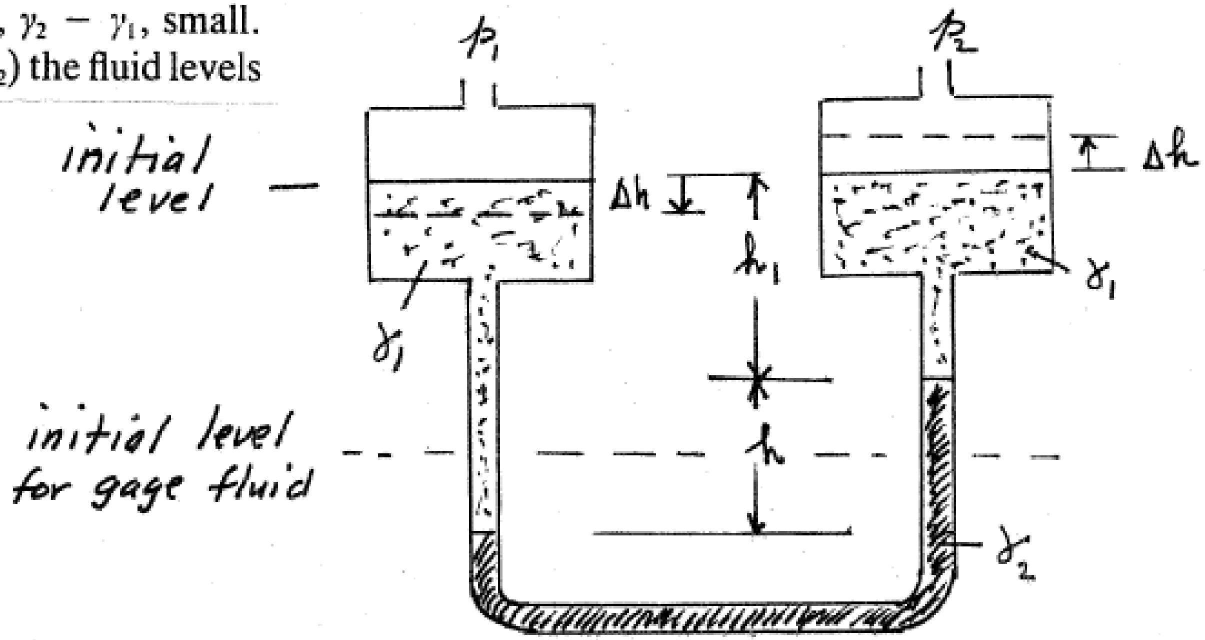


FIGURE P2.34



When a differential pressure, $p_1 - p_2$, is applied we assume that level in left reservoir drops by a distance, Δh, and right level rises by Δh. Thus, the manometer equation becomes

$$p_1 + \gamma_1 (h_1 + h - \Delta h) - \gamma_2 h - \gamma_1 (h_1 + \Delta h) = p_2$$

or

$$p_1 - p_2 = \gamma_2 h - \gamma_1 h + \gamma_1 (2 \Delta h) \qquad (1)$$

Since the liquids in the manometer are incompressible,

$$\Delta h \, A_r = \frac{h}{2} A_t \qquad \text{or} \qquad \frac{2 \Delta h}{h} = \frac{A_t}{A_r}$$

and if $\frac{A_t}{A_r}$ is small then $2 \Delta h << h$ and last term in Eq.(1) can be neglected. Thus,

$$p_1 - p_2 = (\gamma_2 - \gamma_1) h$$

or

$$\underline{\underline{h = \frac{p_1 - p_2}{\gamma_2 - \gamma_1}}}$$

and large values of h can be obtained for small pressure differentials if $\gamma_2 - \gamma_1$ is small.

2.45 Determine the new differential reading along the inclined leg of the mercury manometer of Fig. P2.45, if the pressure in pipe A is decreased 10 kPa and the pressure in pipe B remains unchanged. The fluid in A has a specific gravity of 0.9 and the fluid in B is water.

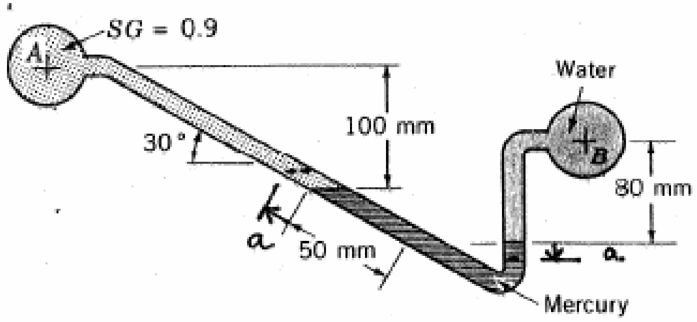


FIGURE P2.45

For the initial configuration:

$$p_A + \gamma_A (0.1) + \gamma_{Hg} (0.05 \sin 30°) - \gamma_{H_2O} (0.08) = p_B \qquad (1)$$

where all lengths are in m. When p_A decreases left column moves up a distance, a, and right column moves down a distance, a, as shown in figure. For the final configuration:

$$p_A' + \gamma_A (0.1 - a \sin 30°) + \gamma_{Hg} (a \sin 30° + 0.05 \sin 30° + a) -$$

$$\gamma_{H_2O} (0.08 + a) = p_B \qquad (2)$$

where p_A' is the new pressure in pipe A.

Subtract Eq. (2) from Eq. (1) to obtain

$$p_A - p_A' + \gamma_A (a \sin 30°) - \gamma_{Hg}\, a (\sin 30° + 1) + \gamma_{H_2O}(a) = 0$$

Thus,

$$a = \frac{-(p_A - p_A')}{\gamma_A \sin 30° - \gamma_{Hg}(\sin 30° + 1) + \gamma_{H_2O}}$$

For $p_A - p_A' = 10\ kPa$

$$a = \frac{-10\ \frac{kN}{m^2}}{(0.9)(9.81\ \frac{kN}{m^3})(0.5) - (133\ \frac{kN}{m^3})(0.5+1) + 9.80\ \frac{kN}{m^3}}$$

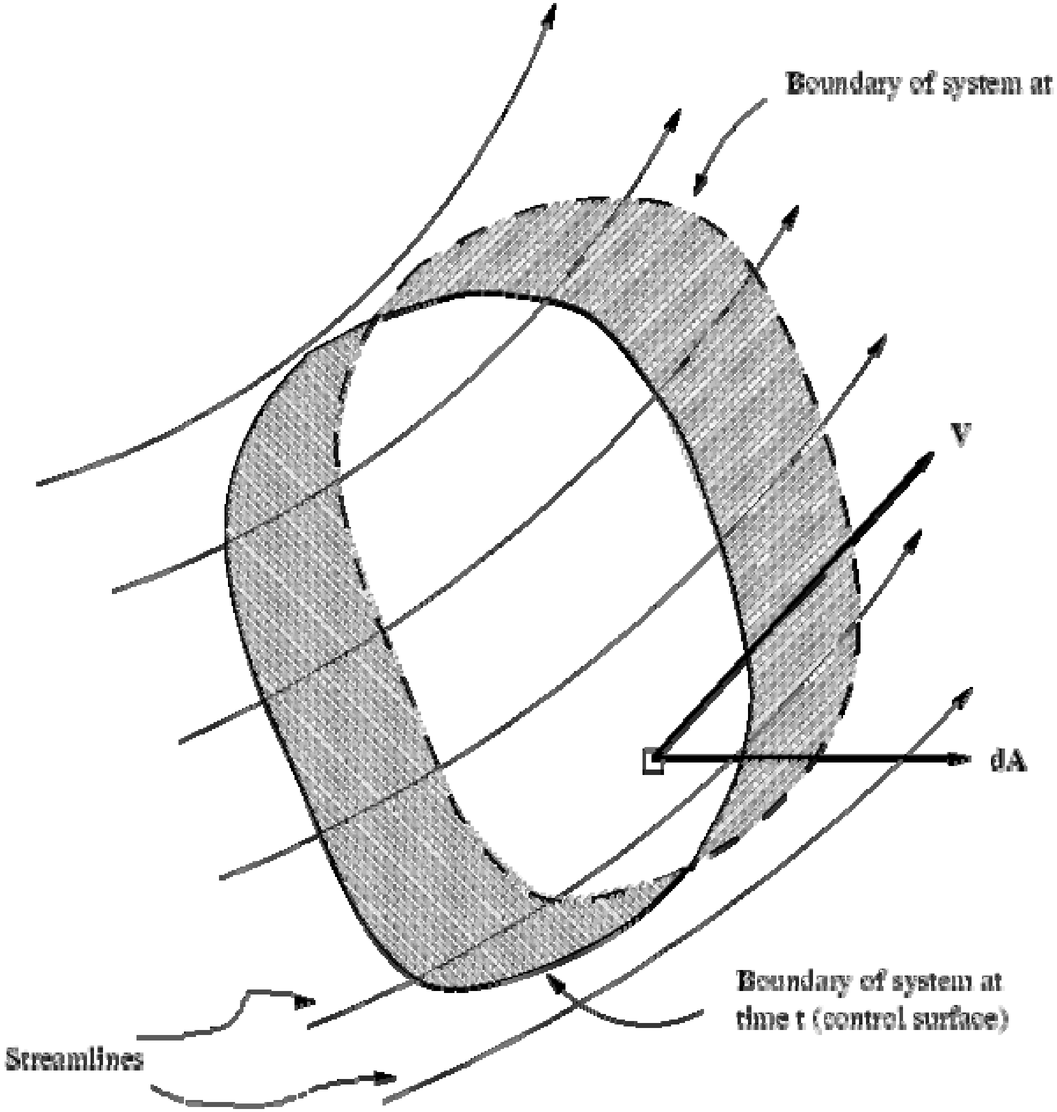
$$= 0.0540\ m$$

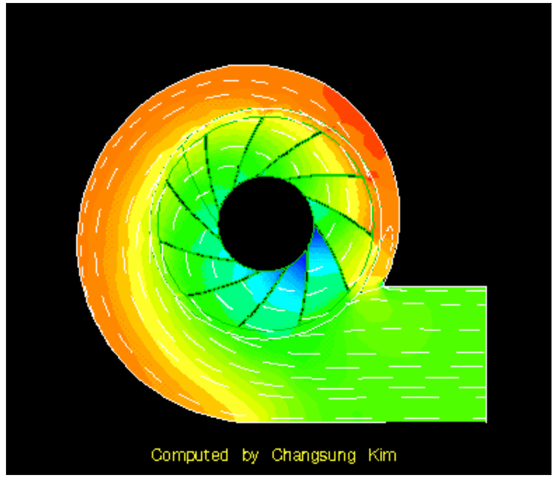
New differential reading, Δh, measured along inclined tube is equal to

$$\Delta h = \frac{a}{\sin 30°} + 0.05 + a$$

$$= \frac{0.0540\ m}{0.5} + 0.05\,m + 0.0540\ m = \underline{\underline{0.212\ m}}$$

Chapter 2 Basic Law of Fluid Flow





2-1 Steady and Unsteady Flow

2-2 Streamline 、 Pathline 、 Streamline tube and Streakline

2-3 Basic Laws for a Fluid Flow

2-4 System vs Control Volume

2-5 Mass Equation

2-6 Moment Equation

2-7 Energy Equation

2-8 Moment of Momentum

2-9 Vortex Motion

Velocity Field

- The velocity at a point C is defined as the instantaneous velocity of the fluid particle which is passing through point C.
  - the fluid particle is a small mass of fluid of fixed identity of volume.
- At a given instant the fluid velocity is a function of space and time.

$$\boxed{\vec{V} = \vec{V}(x, y, z, t)}$$

Representation of Velocity

- Velocity is a vector quantity, so the velocity field is a vector field.

$$\vec{V} = \vec{V}(x, y, z, t)$$

$$\boxed{\vec{V} = u(x, y, z, t)\,\hat{i} + \upsilon(x, y, z, t)\,\hat{j} + w(x, y, z, t)\,\hat{k}}$$

2-1 Steady and unsteady flow

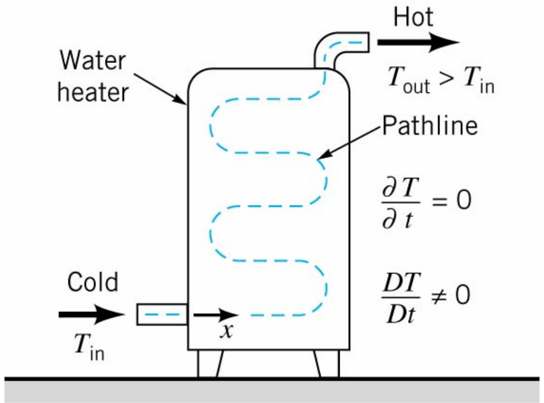
- **Steady flow** : Fluid properties at every point in a flow field do not change with time.

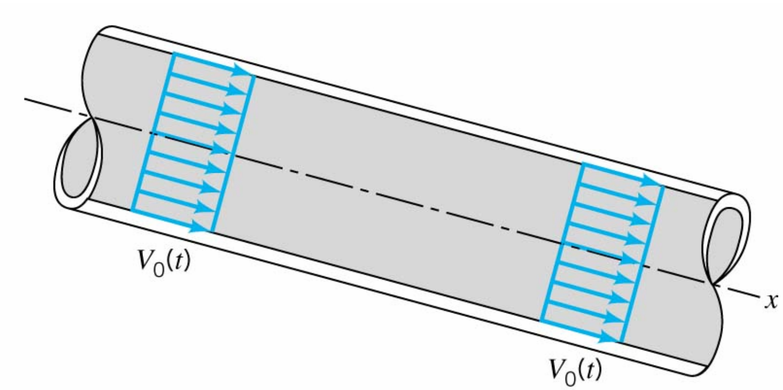
$$\boxed{\frac{\partial \eta}{\partial t} = 0}$$

<u>Mathematical definition of steady flow</u>

i.e $\quad \dfrac{\partial \rho}{\partial t} = 0 \qquad$ or $\qquad \rho = \rho(x,y,z)$

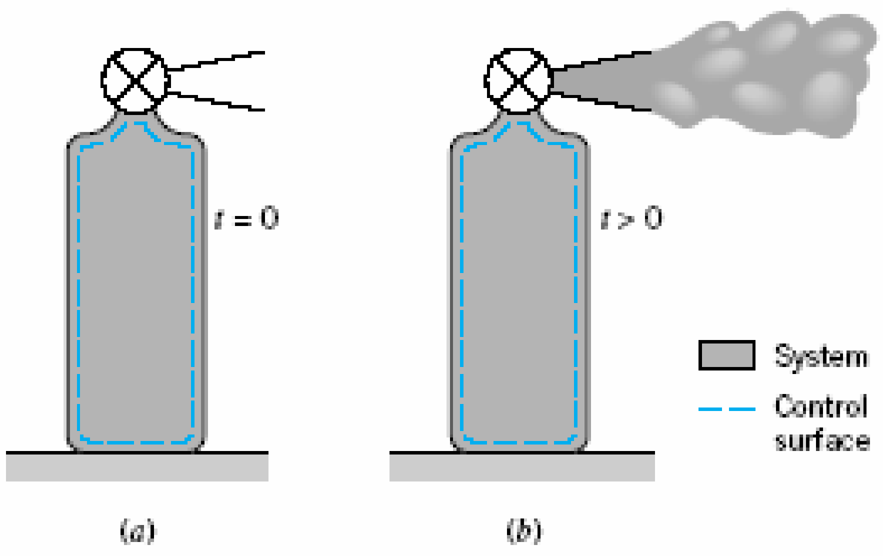
$$\frac{\partial \vec{V}}{\partial t} = 0 \qquad \text{or} \qquad \vec{V} = \vec{V}(x,y,z)$$





● **Unsteady flow** : Fluid properties at every point in a flow field change with time

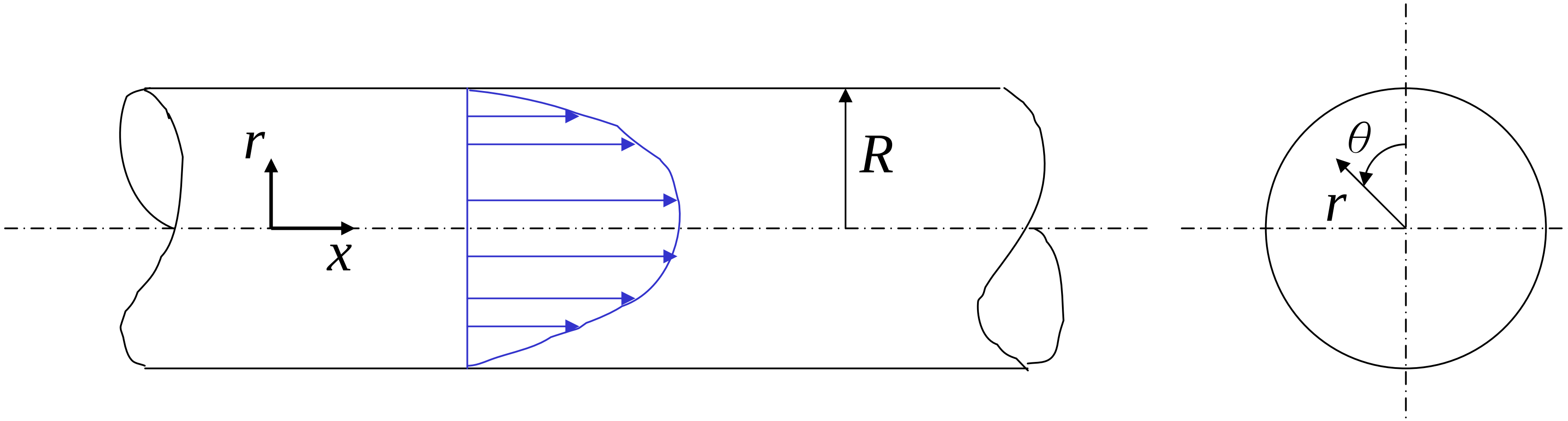
<u>Mathematical definition of unsteady flow:</u> $$\frac{\partial \eta}{\partial t} \neq 0$$



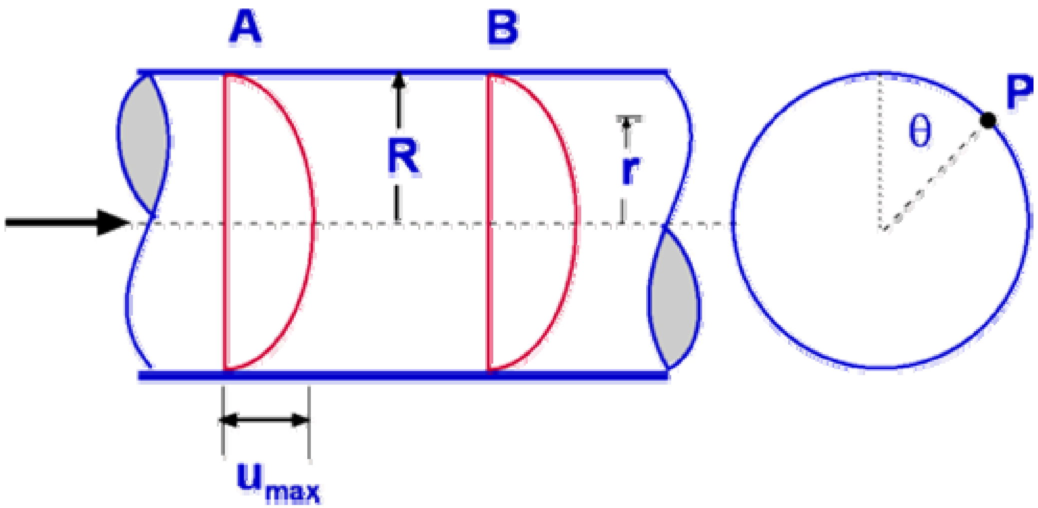
One-, Two-, and Three-Dimensional Flows

- Flow dimension is depending on the number of space coordinates required to specify the velocity field.
  - Most flow field are inherently three-dimensional, analysis based on a fewer dimensions is frequently meaningful.

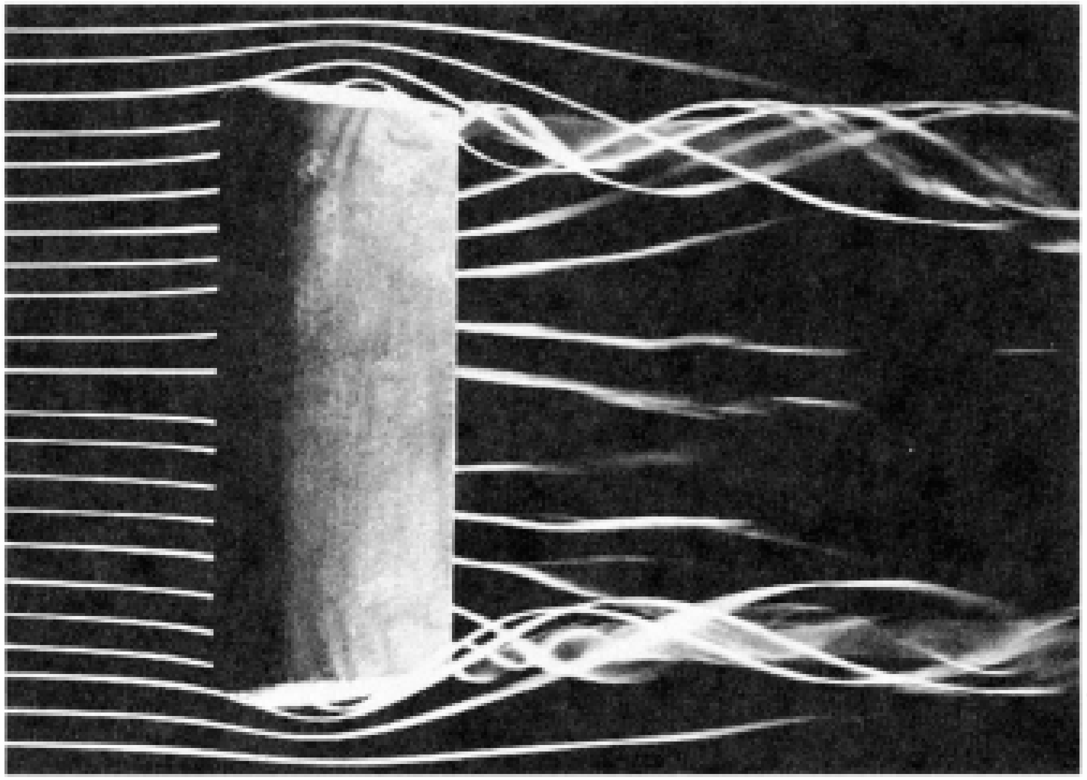
One-dimensional flow



$$u = u_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]$$

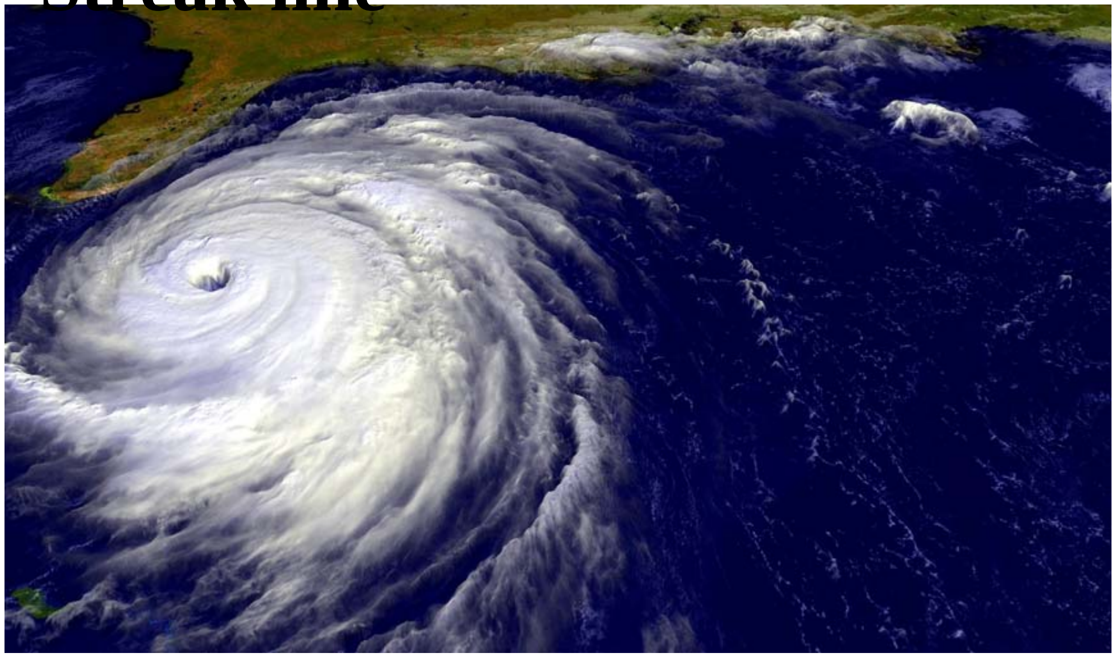


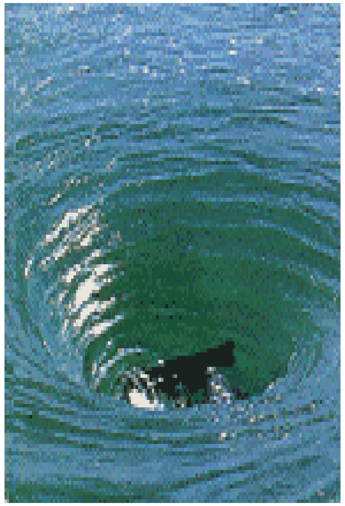
Three-dimensional flow

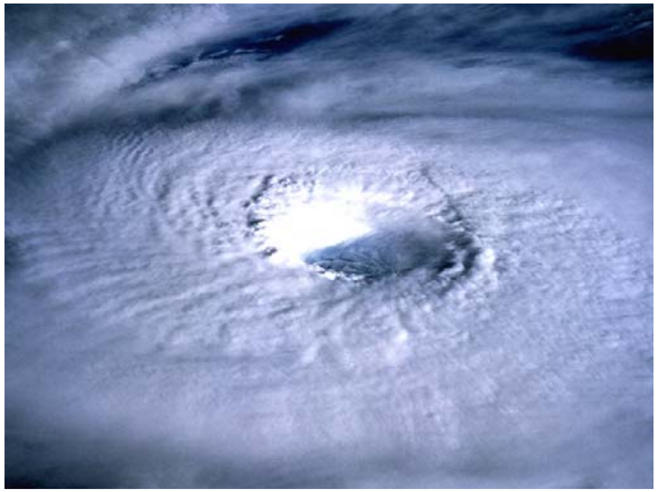


Flow visualization of the complex three-dimensional flow past a model airfoil. (Photograph by M. R. Head.)

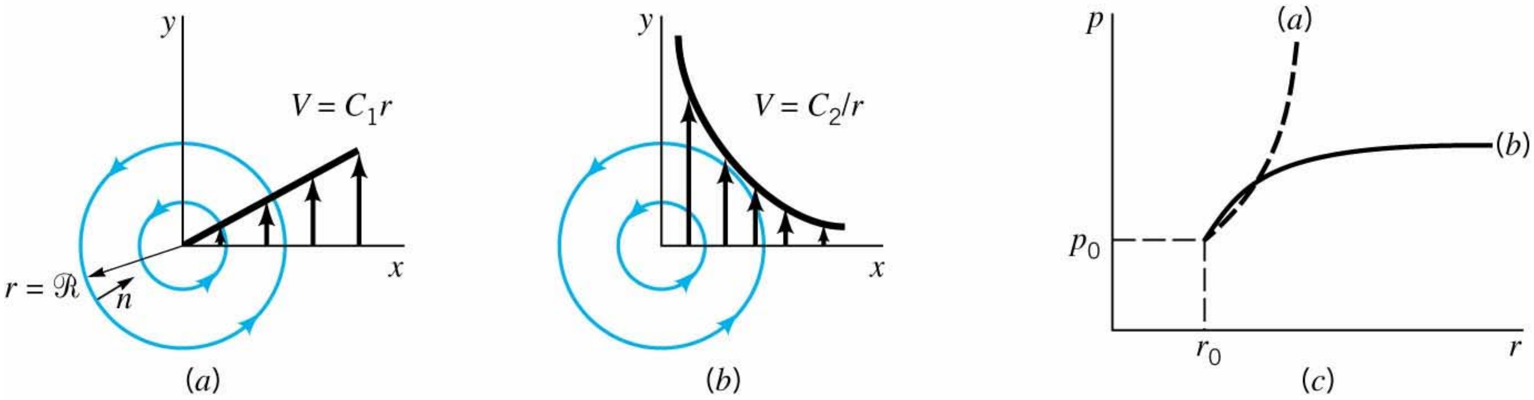
2-2 Streamline 、 Streamline tube 、 Pathlineand Streak line

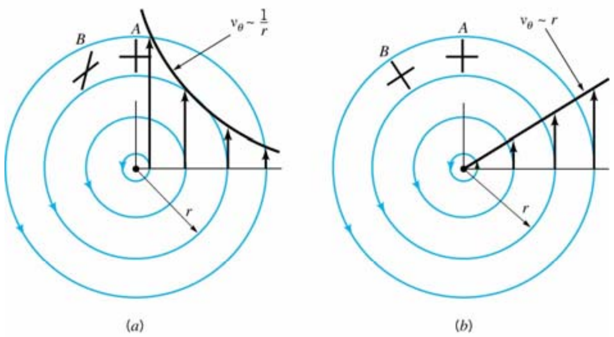


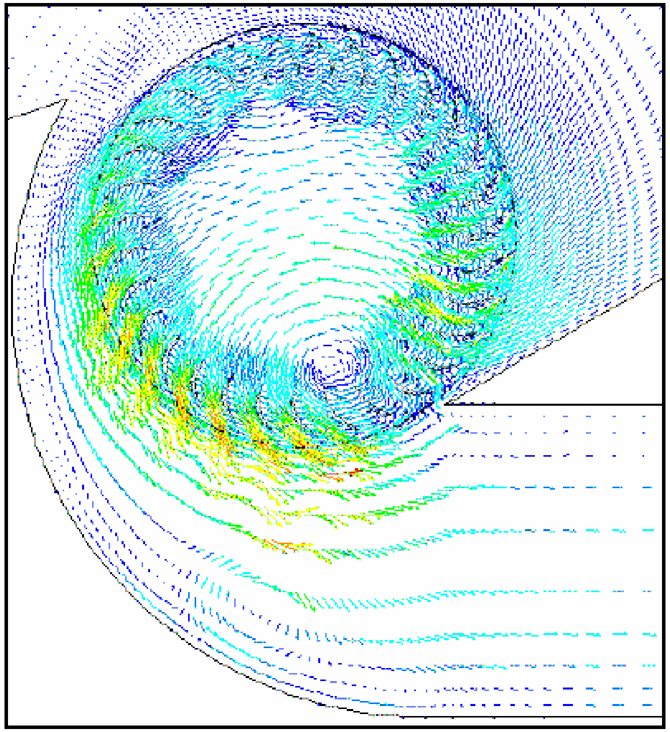


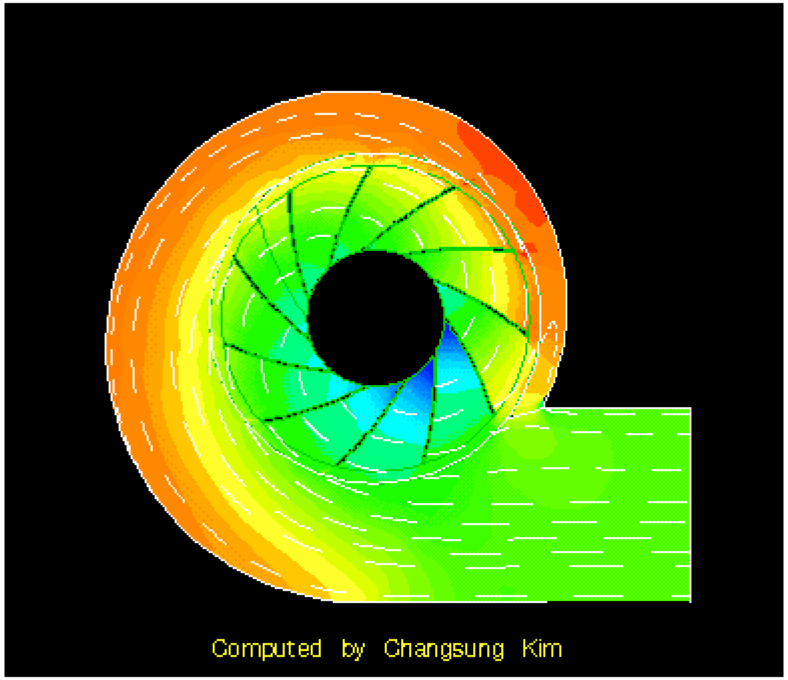


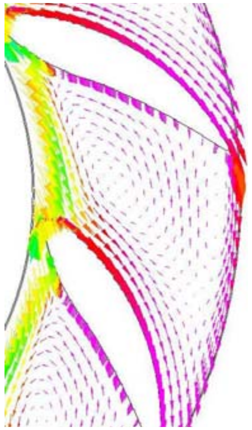
The streamline pattern for a vortex: (a) free vortex (b)rotational vortex





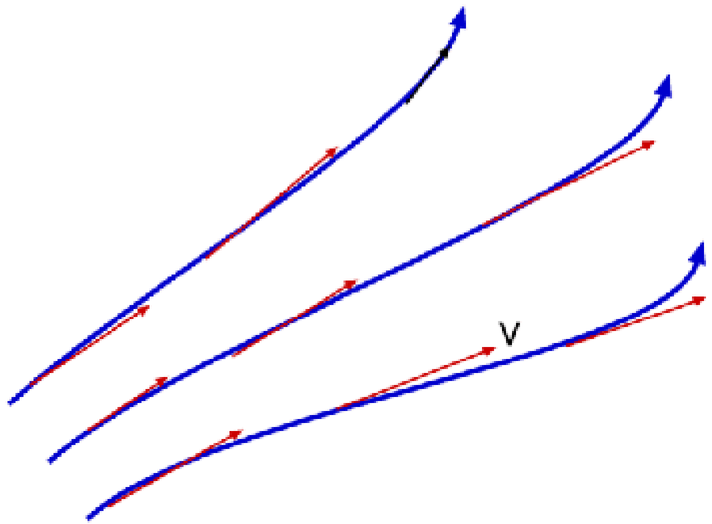


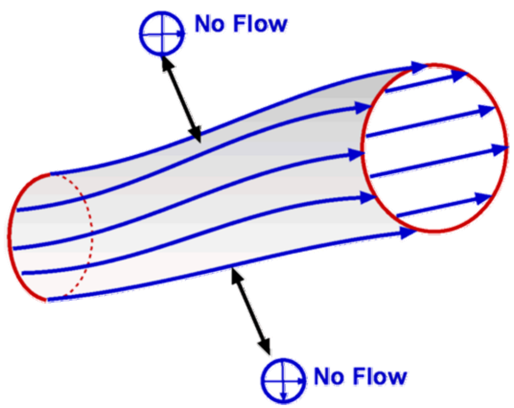

Computed by Changsung Kim



2-2 Streamline 、 Streamline tube 、 Pathline and Streak line

- Definition : **Streamlines**
  - Lines whose tangents are everywhere parallel to the velocity vector.





Streamline tube

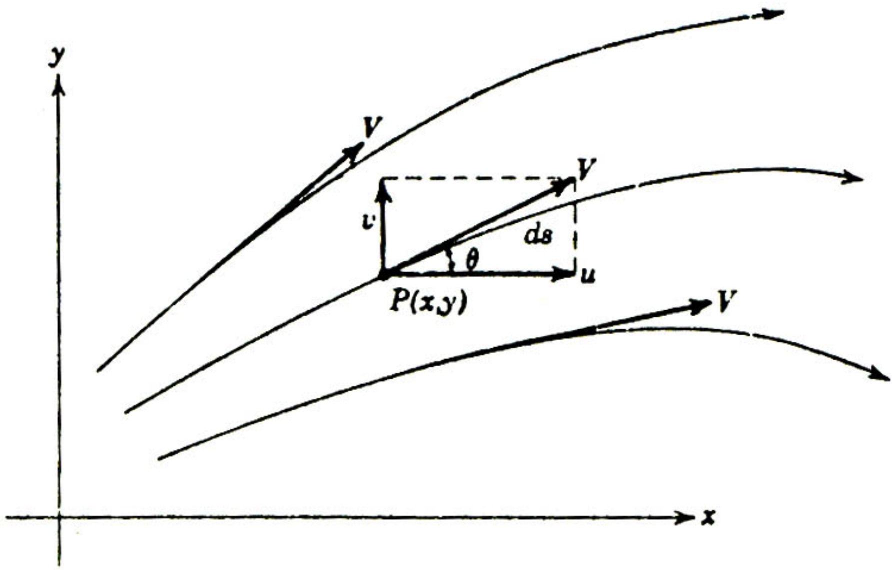
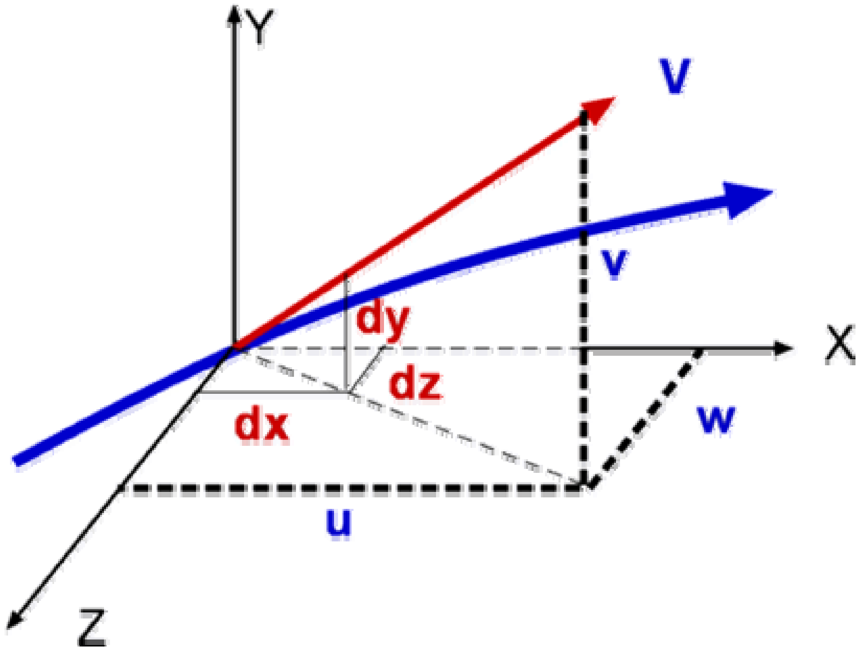


圖 2.1 流線與速度向量



$$\frac{dx}{dy}=\frac{v}{u} \qquad \frac{dz}{dx}=\frac{w}{u} \qquad \frac{dz}{dy}=\frac{w}{v}$$

$$\frac{dy}{v}=\frac{dx}{u} \qquad \frac{dz}{w}=\frac{dx}{u} \qquad \frac{dz}{w}=\frac{dy}{v}$$

$$\frac{dy}{v}=\frac{dx}{u}=\frac{dz}{w}=ds$$

Example1. Streamline for a Given Flow Field

Consider a two-dimensional flow field

$$u = u(x, y, z, t) = Ax$$
$$\upsilon = -Ay$$
$$w = 0$$

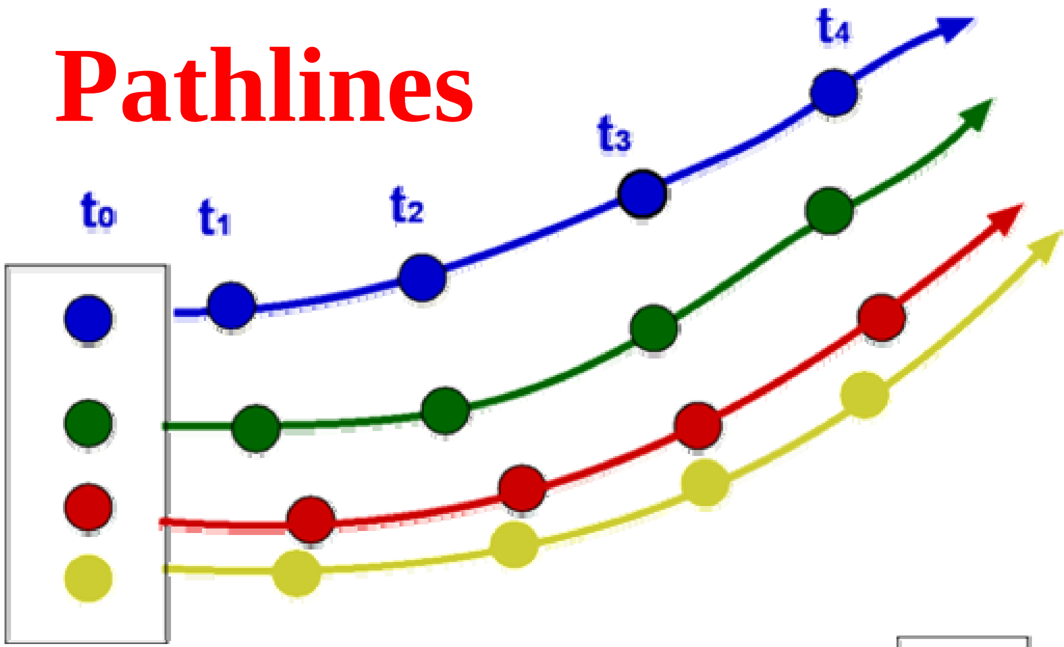
$$\left.\frac{dy}{dx}\right)_{\text{streamline}} = \frac{\upsilon}{u} = \frac{-y}{x}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x} \quad \Longrightarrow \quad \ln y = -\ln x + c_1 \quad \text{or} \quad xy = c$$

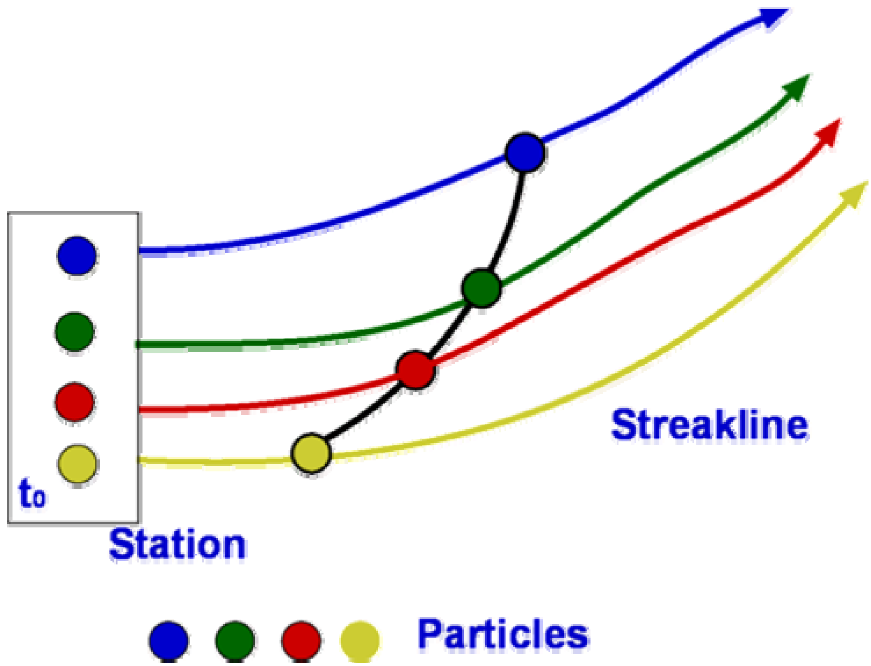
For the streamline passing through (x_0, y_0),

$$xy = x_0 y_0$$

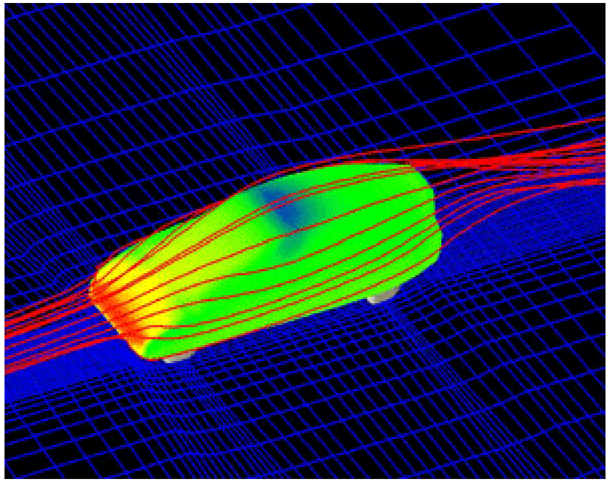
● Pathline and streakline (與實驗有關)



Streakline



Pathlines



- Definition
  - line traced out in time by a given fluid particle as it flows

$$\frac{dx_i}{dt} = u_i(x_i, t)$$

Under the initial conditions

$$x_i = x_i(x_0, y_0, z_0, t)$$

Pathline for a Given Flow Field (與實驗有關)

Example. Consider a two-dimensional flow field

$$u = u(x, y, z, t) = Ax$$
$$v = -Ay$$
$$w = 0$$

$$u = \frac{dx}{dt} = Ax$$

$$v = \frac{dy}{dt} = -Ay$$

$$x = c_1 e^{At}$$
$$y = c_2 e^{-At}$$

For the pathline passing through (x_0, y_0),

$$x = x_0 e^{At}; \qquad y = y_0 e^{-At}$$

For the streamline passing through (x_0,y_0),

$$xy = x_0 y_0$$

For the pathline passing through (x_0,y_0),

$$x = x_0 e^{At}; \qquad y = y_0 e^{-At}$$

$$\Downarrow$$

$$xy = x_0 e^{At} \times y_0 e^{-At} = x_0 y_0$$

Under steady flow condition, streamline and pathline are the same.

Streakline for a Given Flow Field

$$\frac{dx}{dt} = Ax(1+2t); \quad \frac{dy}{dt} = -Ay \qquad x = c_1 e^{A(1+t)t} \qquad y = c_2 e^{-At}$$

initial conditions: $x = x_0; \;\; y = y_0$ when $t = \tau$

Solution: $x = x_0 e^{A(1+t)t - A\tau(1+\tau)} \qquad y = y_0 e^{-A(t-\tau)}$

$$xy = x_0 y_0 e^{A\left(t^2 - \tau^2\right)}$$

2-3 Basic Laws for a Fluid Flow

- Conservation of Mass
- Newton’s Second law
- The Angular Momentum Principle
- The First Law of Thermodynamics
- The Second Law of Thermodynamic

※

- Conservation of Mass $\boxed{\left.\frac{dM}{dt}\right)_{system} = 0}$

$$M_{system} = \int_{system} dm = \int_{system} \rho d\forall$$

- Newton's Second law $\boxed{\vec{F} = \left.\frac{d\vec{P}}{dt}\right)_{system}}$

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V})$$

$$\vec{P}_{system} = \int_{system} \vec{V} dm = \int_{system} \vec{V} \rho d\forall$$

- The Angular Momentum Principle $\boxed{\vec{T} = \left.\frac{d\vec{H}}{dt}\right)_{system}}$

$$\vec{H}_{system} = \int_{system} \vec{r} \times \vec{V} dm = \int_{system} \vec{r} \times \vec{V} \rho d\forall$$

$$\vec{T} = \vec{r} \times \vec{F}_s + \int_{system} \vec{r} \times \vec{g} dm + \vec{T}_{shaft}$$

- The First Law of Thermodynamics $\boxed{\frac{\delta Q}{dt} - \frac{\delta W}{dt} = \dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right)_{system}}$

$$E_{system} = \int_{system} e dm = \int_{system} e \rho d\forall \quad e = u + \frac{V^2}{2} + gz$$

- The Second Law of Thermodynamic $\boxed{dS \geq \frac{\delta Q}{T}}$

$$S_{system} = \int_{system} s dm = \int_{system} s \rho d\forall$$

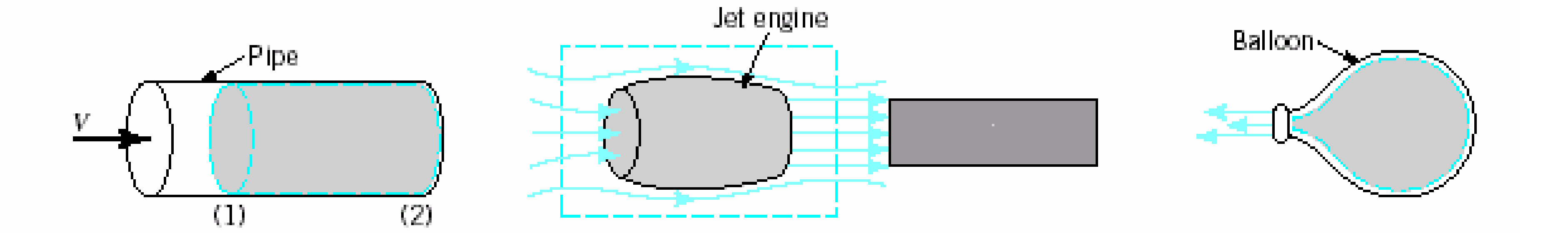
2-4 System vs Control Volume

Methods of Analysis

- System and Control Volume
- Differential versus Integral Approach
- Methods of description

System

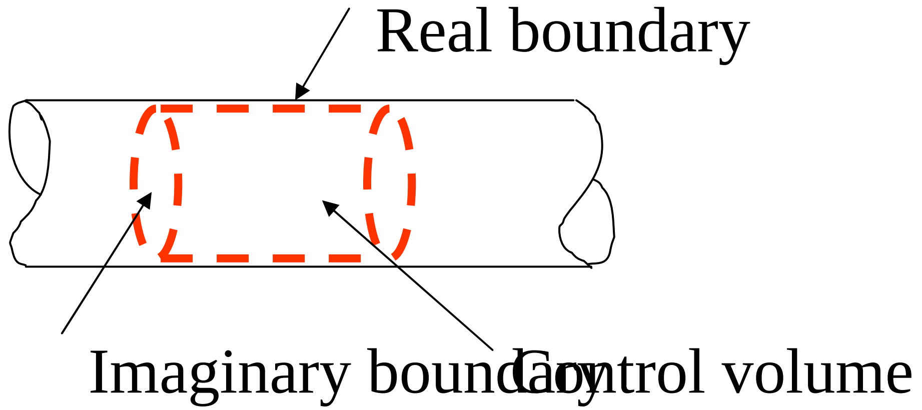
- A *system* is defined as <u>a fixed, identifiable quantity of mass</u>.
- The *system boundaries* separate the system from the *surroundings*.
- The *boundaries* of the system <u>may be fixed or movable</u>.



Typical control volumes: (*a*) fixed control volume, (*b*) fixed or moving control volume, (*c*) deforming control volume

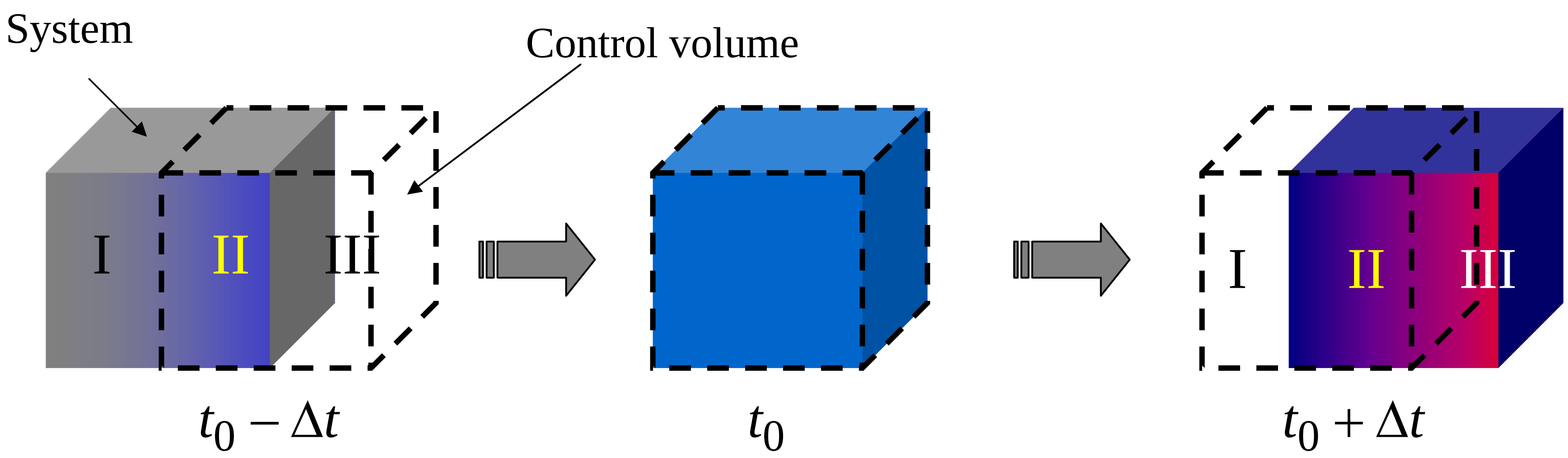
Control Volume

- An arbitrary volume in space through which fluid flows.
- The geometric *boundary* of the *control volume* is called the *control surface*.
- The *control surface* may be real or imaginary; it may be at rest or in motion.



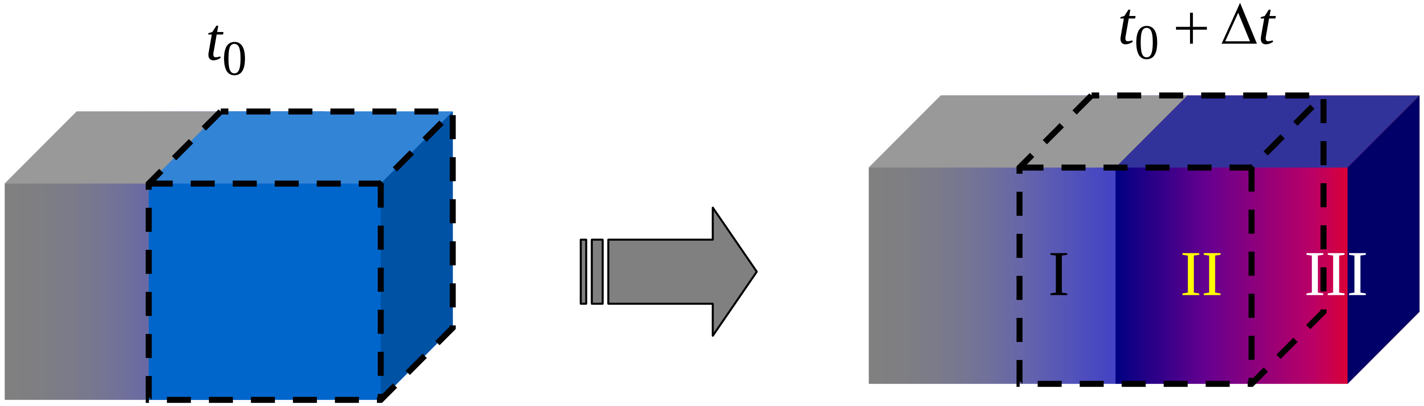
2-4 System vs Control Volume
Rate of Change of System Property

- Relationship between System and Control Volume Formulation



$$\left.\frac{dN}{dt}\right)_{\text{system}} \equiv \lim_{\Delta t \to 0} \frac{N_s)_{t_0} - N_s)_{t_0-\Delta t}}{\Delta t}$$

$$\left.\frac{dN}{dt}\right)_{\text{system}} \equiv \lim_{\Delta t \to 0} \frac{N_s)_{t_0+\Delta t} - N_s)_{t_0}}{\Delta t}$$

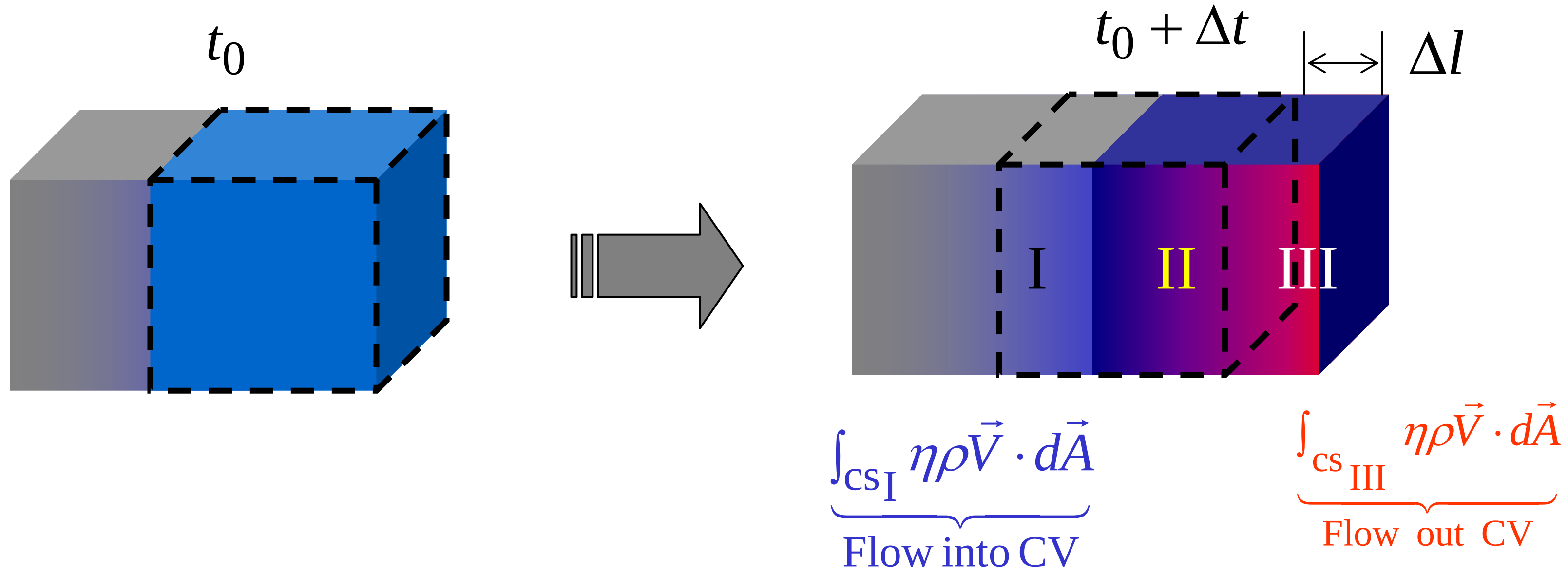


$$N_s\big)_{t_0} = \left(N_{CV}\right)_{t_0}$$

$$\begin{aligned} N_s\big)_{t_0+\Delta t} &= \left(N_{\mathrm{II}} + N_{\mathrm{III}}\right)_{t_0+\Delta t} \\ &= \left(N_{CV} - N_{\mathrm{I}} + N_{\mathrm{III}}\right)_{t_0+\Delta t} \end{aligned}$$

$$\left.\frac{dN}{dt}\right)_{\text{system}} \equiv \lim_{\Delta t \to 0} \frac{\left(N_{\mathrm{CV}} - N_{\mathrm{I}} + N_{\mathrm{III}}\right)_{t_0+\Delta t} - \left(N_{\mathrm{CV}}\right)_{t_0}}{\Delta t}$$

$$\left.\frac{dN}{dt}\right)_{\text{system}} \equiv \lim_{\Delta t \to 0} \frac{\left(N_{\mathrm{CV}}\right)_{t_0+\Delta t} - \left(N_{\mathrm{CV}}\right)_{t_0}}{\Delta t} + \lim_{\Delta t \to 0} \frac{\left(N_{\mathrm{III}}\right)_{t_0+\Delta t}}{\Delta t} - \lim_{\Delta t \to 0} \frac{\left(N_{\mathrm{I}}\right)_{t_0+\Delta t}}{\Delta t}$$



$$\lim_{\Delta t\to 0}\frac{(N_{\mathrm{CV}})_{t_0+\Delta t}-(N_{\mathrm{CV}})_{t_0}}{\Delta t}=\frac{\partial N_{\mathrm{CV}}}{\partial t}=\frac{\partial}{\partial t}\int_{\mathrm{CV}}\eta\rho\, d\mathcal{V}$$

$$\lim_{\Delta t\to 0}\frac{(N_{\mathrm{III}})_{t_0+\Delta t}}{\Delta t}=\lim_{\Delta t\to 0}\frac{\int_{\mathcal{V}_{\mathrm{III}}}\eta\rho\, d\mathcal{V}}{\Delta t}=\lim_{\Delta t\to 0}\frac{\int_{\mathcal{V}_{\mathrm{III}}}\eta\rho\,\Delta l\, dA}{\Delta t}$$

$$=\lim_{\Delta t\to 0}\int_{\mathrm{cs}_{\mathrm{III}}}\eta\rho\frac{\Delta l}{\Delta t}dA=\int_{\mathrm{cs}_{\mathrm{III}}}\eta\rho\vec{V}\cdot d\vec{A}$$

$$-\lim_{\Delta t\to 0}\frac{(N_{\mathrm{I}})_{t_0+\Delta t}}{\Delta t}=\int_{\mathrm{cs}_{\mathrm{I}}}\eta\rho\vec{V}\cdot d\vec{A}$$

2-4 System vs Control Volume

The Reynolds Transport Theorem

$N = m\eta \text{ , } \eta = N/m$

$$m = \int_{CV} \rho \, d\forall \qquad N = \eta \, m = \int_{CV} \eta\rho \, d\forall$$

$$\boxed{\left.\frac{dN}{dt}\right)_{system} = \frac{\partial N_{CV}}{\partial t} + \dot{N}_{out} - \dot{N}_{in}}$$

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \eta\rho \, d\forall + \int_{cs_{III}} \eta\rho V dA - \int_{cs_{I}} \eta\rho V dA$$

$$= \frac{\partial}{\partial t}\int_{CV} \eta\rho \, d\forall + \int_{CS} \eta\rho \vec{V} \cdot d\vec{A}$$

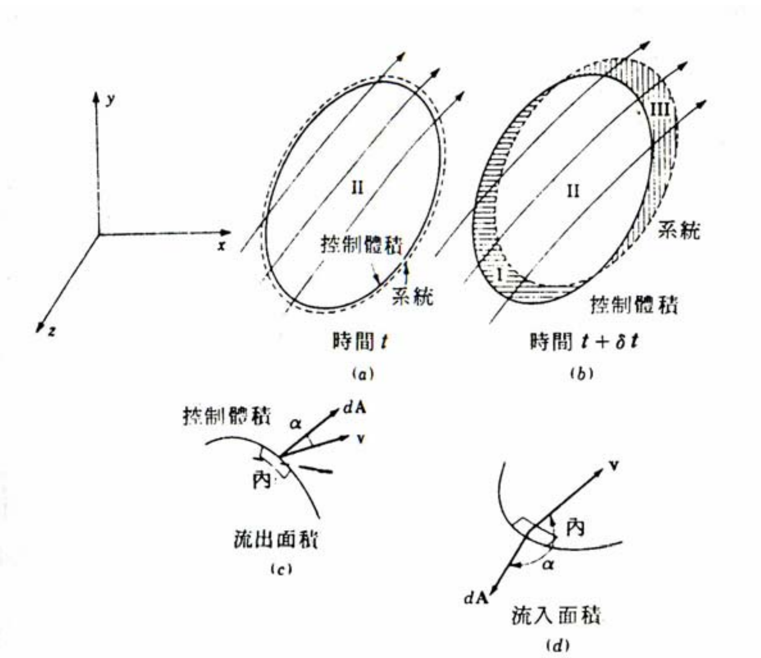


圖 2.3 系統與控制體積的關係

The General **Reynolds Transport Theorem** involves volume and surface integrals

N=extensive property , η=intensive property

N =mη , η = N /m

N=m(質量) η = m /m=1

$N=mV^2/2$(動能), $\eta = V^2/2$

N=mV (動量), $\eta = \vec{V}$

$N = m\vec{V} \times \vec{r}$ (動量矩), $\eta = \vec{V} \times \vec{r}$

$$N_{system} = \eta\ m = \int_{system} \eta\rho\, d\forall$$

❑ Special Case 1: Steady Flow

$$Steady\ flow: \qquad \frac{dN_{sys}}{dt} = \int_{CS} \rho\eta(\vec{V}\cdot\hat{n})dA$$

❑ Special Case 2: One-Dimensional Flow

$$One\text{-}dimensional\ flow:$$

$$\frac{dN_{sys.}}{dt} = \frac{d}{dt}\int_{CV}\rho\eta\, dV + \sum_{out}\underbrace{\rho_e\eta_e V_e A_e}_{\text{for each exit}} - \sum_{in}\underbrace{\rho_i\eta_i V_i A_i}_{\text{for each exit}}$$

$$\frac{dN_{sys.}}{dt} = \frac{d}{dt}\int_{CV}\rho\eta\, dV + \sum_{out}\dot{m}_e\eta_e - \sum_{in}m_i^{*}\eta_i$$

Physical Interpretation

$\left.\dfrac{dN}{dt}\right)_{\text{system}}$	Rate of change of any arbitrary extensive property, *N*, of the system
$\dfrac{\partial}{\partial t}\int_{\mathrm{CV}} \eta\rho\, d\forall$	• The time rate of change of arbitrary extensive property *N* within the control volume. • $\eta = N$ per unit mass.
$\rho\, d\forall$	An element of mass contained in the control volume
$\int_{\mathrm{CV}} \eta\rho\, d\forall$	The total amount of *N* in the control volume

Flux of System Property

$\int_{CS} \eta\rho\vec{V}\cdot d\vec{A}$	The net rate of flux of N out through the control surface.
$\rho\vec{V}\cdot d\vec{A}$	The rate of mass flux through an area element $d\vec{A}$
$\eta\rho\vec{V}\cdot d\vec{A}$	The rate of flux of N through area $d\vec{A}$

Conservation of Mass

$$\boxed{\left.\frac{dM}{dt}\right)_{system} = 0}$$

$$M_{system} = \int_{system} dm = \int_{system} \rho d\forall$$

$$\left.\frac{dN}{dt}\right)_{system} = \frac{d}{dt}\int_{system} \eta dm = \frac{d}{dt}\int_{system} \eta\rho d\forall$$

$$= \frac{\partial}{\partial t}\int_{cv} \eta\rho d\forall + \int_{CS} \eta\rho \vec{V}\cdot d\vec{A}$$

$$N = m\eta\ , \ \eta = N/m$$

$$\frac{dN_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho\eta\, dV + \int_{CS} \rho\, \eta(\vec{V}\cdot\hat{n})\, dA$$

$$N_{sys} = m_{sys} \qquad \eta = 1 \qquad \eta = 1$$

$$\overset{0}{\cancel{\frac{dm_{sys}}{dt}}} = \frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho(\vec{V}\cdot\hat{n})\, dA$$

N=M and $\eta = 1$

$$\boxed{\left.\frac{dM}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{CS} \rho\vec{V}\cdot d\vec{A} = 0}$$

Conservation of Mass--Special cases

- Incompressible flow (ρ =const)

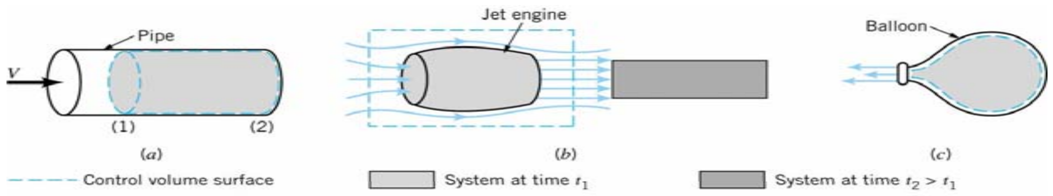
$$\boxed{\rho\left[\frac{\partial}{\partial t}\int_{CV} d\forall + \int_{CS}\vec{V}\cdot d\vec{A}\right]=0}$$

$$\boxed{\frac{\partial \forall}{\partial t} + \int_{CS}\vec{V}\cdot d\vec{A} = 0}$$

- And for nondeformable control volume (or steady state),

$$\forall = \text{constant} \qquad \frac{\partial \forall}{\partial t} = 0 \qquad \rho\int_{CS}\vec{V}\cdot d\vec{A} = 0$$

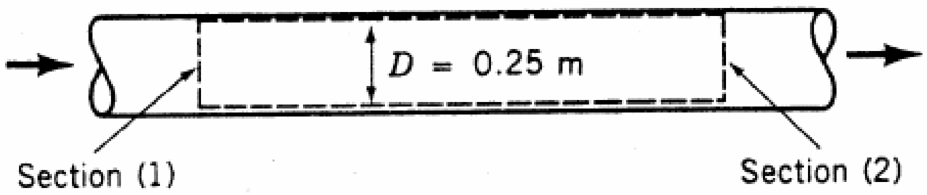
Mass flow rate in = Mass flow rate out $\boxed{\dot{m}_i = \dot{m}_o}$



(a) Fixed CV (b) Fixed or moving CV (c) Deforming CV

例題 一

Air flows steadily between two cross sections in a long, straight section of 0.25-m inside diameter pipe. The static temperature and pressure at each section are indicated in Fig. P5.3. If the average air velocity at section (2) is 320 m/s, determine the average air velocity at section (1).



$p_1 = 690$ kPa (abs)
$T_1 = 300$ K

$p_2 = 127$ kPa (abs)
$T_2 = 252$ K
$V_2 = 320$ m/s

For steady flow between sections (1) and (2)

$$\dot{m}_1 = \dot{m}_2$$

or

$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2$$

Thus

$$\bar{V}_1 = \frac{\rho_2}{\rho_1}\frac{A_2}{A_1}\bar{V}_2 \qquad (1)$$

Assuming that under the conditions of this problem, air behaves as an ideal gas we use the ideal gas equation of state (Eq. 1.7) to get

$$\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1}\frac{T_1}{T_2} \qquad (2)$$

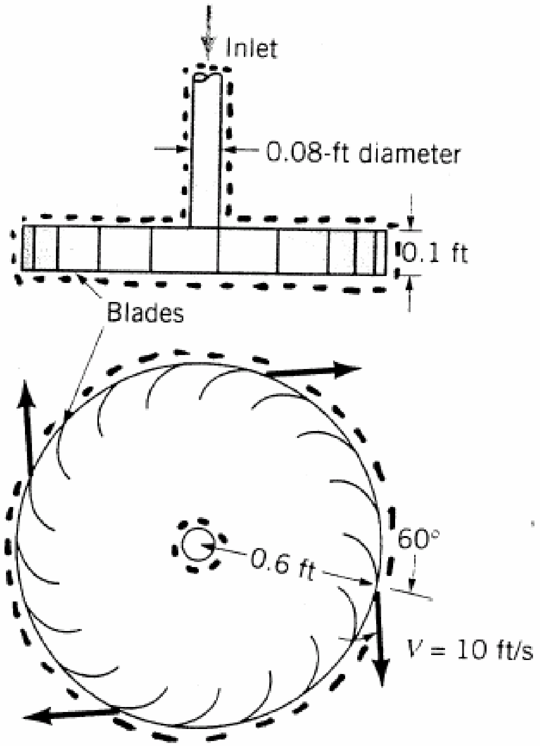
Combining Eqs. 1 and 2 and observing that $A_1 = A_2$ we get

$$\bar{V}_1 = \frac{P_2}{P_1}\frac{T_1}{T_2}\bar{V}_2 = \frac{[127\ kPa(abs)](300\,K)}{[690\ kPa(abs)](252\,K)}\left(320\ \frac{m}{s}\right)$$

$$\bar{V}_1 = \underline{\underline{70.1\ \frac{m}{s}}}$$

例題 二

Water flows out through a set of thin, closely spaced blades as shown in Fig. P5.5 with a speed of $V = 10$ ft/s around the entire circumference of the outlet. Determine the mass flowrate through the inlet pipe.



Use the control volume contained within the broken lines shown in the sketch above.

From the conservation of mass principle

$$\dot{m}_{inlet} = \dot{m}_{outlet}$$

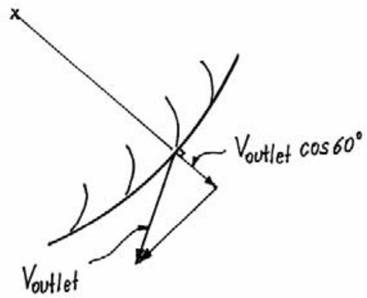
Also

$$\dot{m}_{outlet} = \rho A_{outlet} V_{outlet} \cos 60^\circ$$

$$= \rho \, 2\pi r_{outlet} h V_{outlet} \cos 60^\circ$$

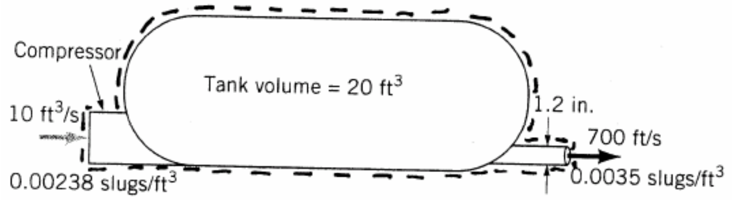
$$= \left(1.94 \frac{slugs}{ft^3}\right) 2\pi (0.6 ft)(0.1 ft)\left(10 \frac{ft}{s}\right) \cos 60^\circ$$

$$= \underline{\underline{3.66 \frac{slugs}{s}}}$$



例題 三

Air at standard conditions enters the compressor in Fig. P5.8 at a rate of 10 ft^3/s. It leaves the tank through a 1.2-in.-diameter pipe with a density of 0.0035 $slugs/ft^3$ and a uniform speed of 700 ft/s. **(a)** Determine the rate (slugs/s) at which the mass of air in the tank is increasing or decreasing. **(b)** Determine the average time rate of change of air density within the tank.



Use the control volume within the broken lines.

(a) From the conservation of mass principle we get

$$\frac{DM_{sys}}{Dt} = \dot{m}_{in} - \dot{m}_{out} = \rho_{in} Q_{in} - \rho_{out} A_{out} V_{out}$$

$$\frac{DM_{sys}}{Dt} = \left(0.00238 \frac{slug}{ft^3}\right)\left(10 \frac{ft^3}{s}\right) - \left(0.0035 \frac{slug}{ft^3}\right) \frac{\pi (1.2\ in.)^2}{\left(144 \frac{in^2}{ft^2}\right)} \left(700 \frac{ft}{s}\right)$$

$$\frac{DM_{sys}}{Dt} = \underline{\underline{0.00456 \frac{slug}{s}}} \quad \text{increasing}$$

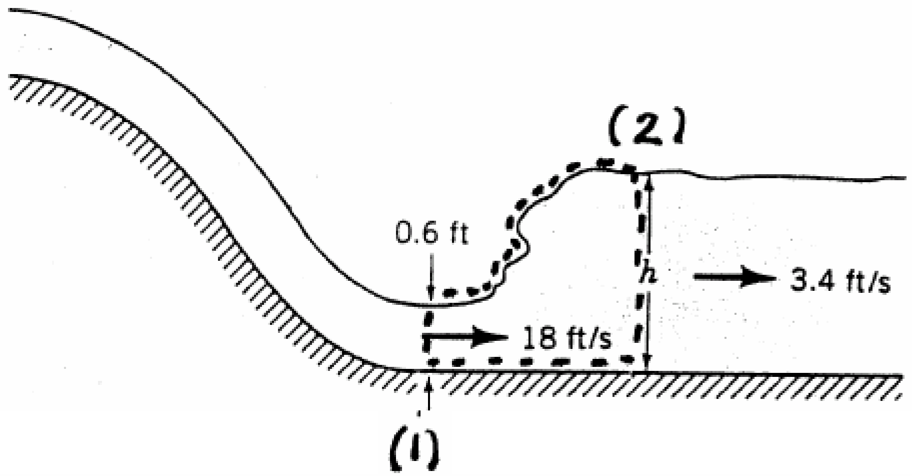
b) $$\frac{DM_{sys}}{Dt} = \frac{D(\rho V_{sys})}{Dt} = V_{sys} \frac{D\rho}{Dt} = 0.00456 \frac{slug}{s}$$

So

$$\frac{D\rho}{Dt} = \frac{0.00456 \frac{slug}{s}}{V_{sys}} = \frac{0.00456 \frac{slug}{s}}{20 ft^3} = \underline{\underline{2.28 \times 10^{-4} \frac{slug}{ft^3\, s}}}$$

例題 四

A hydraulic jump (see Video V10.5) is in place downstream from a spill-way as indicated in Fig. P5. Upstream of the jump, the depth of the stream is 0.6 ft and the average stream velocity is 18 ft/s. Just downstream of the jump, the average stream velocity is 3.4 ft/s. Calculate the depth of the stream, h, just downstream of the jump.



For steady incompressible flow between sections (1) and (2)

$Q_1 = Q_2$

or

$V_1 A_1 = V_2 A_2$

Thus

$V_1 h_1 = V_2 h_2$

and

$$h_2 = \frac{V_1 h_1}{V_2} = \frac{(18\ \frac{ft}{s})(0.6\ ft)}{(3.4\ \frac{ft}{s})} = \underline{\underline{3.18\ ft}}$$

Momentum Equation

$$\frac{dN_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho\eta \, dV + \int_{CS} \rho\eta(\vec{V}\cdot\hat{n})\, dA$$

$$N = m\vec{V} \qquad \eta = \vec{V} \qquad \eta = \vec{V}$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V} \, dV + \int_{CS} \rho\vec{V}(\vec{V}\cdot\hat{n})\, dA$$

- For Inertial control volume

$$\boxed{\sum \vec{F} = \vec{F}_S + \vec{F}_B = \left.\frac{d\vec{P}}{dt}\right)_{system}}$$

F_S:surface force

F_B:Body force

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\forall + \int_{CS} \eta\rho\vec{V}\cdot d\vec{A}$$

$$\vec{P}_{system} = \int_{system} \vec{V} dm = \int_{system} \vec{V}\rho d\forall$$

$$N = m\eta \ , \quad \eta = N/m$$

$$N = \vec{P} = m\vec{V} \quad \text{and} \quad \eta = \vec{V}$$

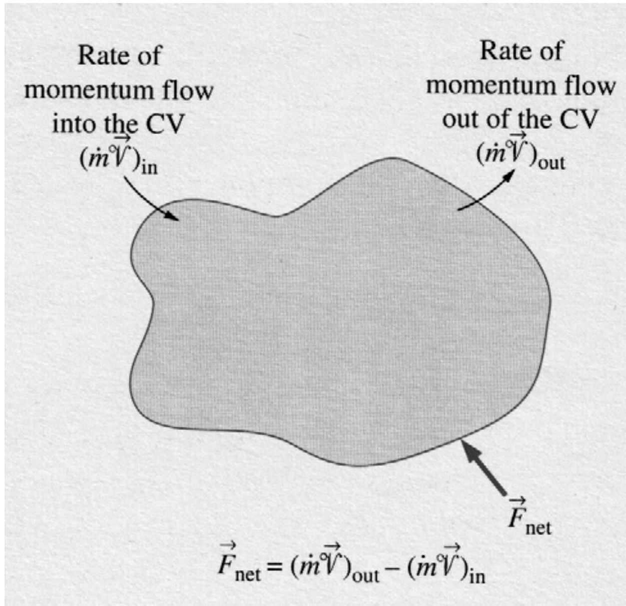
$$\boxed{\sum \vec{F} = \vec{F}_S + \vec{F}_B = \left.\frac{d\vec{P}}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}}$$

$$\vec{F} = \vec{F}_S + \vec{F}_B = \left.\frac{d\vec{P}}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}$$

$$\vec{F}_x = \vec{F}_{S_x} + \vec{F}_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\rho d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

$$\vec{F}_y = \vec{F}_{S_y} + \vec{F}_{B_y} = \frac{\partial}{\partial t}\int_{CV} \upsilon\rho d\forall + \int_{CS} \upsilon\rho\vec{V}\cdot d\vec{A}$$

$$\vec{F}_z = \vec{F}_{S_z} + \vec{F}_{B_z} = \frac{\partial}{\partial t}\int_{CV} w\rho d\forall + \int_{CS} w\rho\vec{V}\cdot d\vec{A}$$

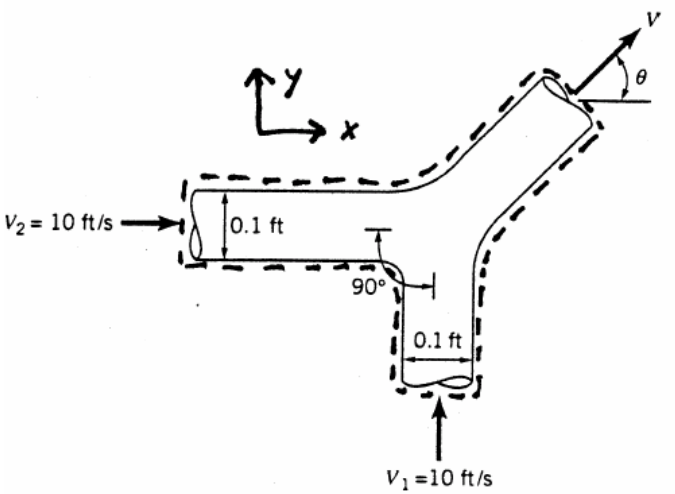


Steady, one - dimensional flow

(one - inlet, one - exit) : $\sum \vec{F} = \dot{m}(\vec{V}_2 - \vec{V}_1)$

Along x coordinate : $\sum \vec{F}_x = \dot{m}(\vec{V}_{2,x} - \vec{V}_{1,x})$

例題 五 Two water jets of equal size and speed strike each other as shown in Fig. P5.32. Determine the speed, V, and direction, θ, of the resulting combined jet. Gravity is negligible.



For the control volume shown in the sketch above the linear momentum equation for the x and y directions are, for the x direction

$$-V_2 \rho V_2 A_2 + (V\cos\theta)\rho V A = 0 \qquad (1)$$

and for the y direction

$$-V_1 \rho V_1 A_1 + (V\sin\theta)\rho V A = 0 \qquad (2)$$

Also for conservation of mass we have

$$\rho_1 V_1 A_1 + \rho V_2 A_2 - \rho V A = 0 \qquad (3)$$

From Eqs. 1 and 2 we get

$$\frac{V_2^2 A_2}{V_1^2 A_1} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

so

$$\theta = \cot^{-1}\frac{V_2^2 A_2}{V_1^2 A_1} = \cot^{-1}\left[\frac{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}\right] = 45^\circ$$

Now, combining Eqs. 2 and 3 we get

$$-V_1^2 A_1 + V\sin\theta\,(V_1 A_1 + V_2 A_2) = 0$$

or

$$V = \frac{V_1^2 A_1}{\sin\theta\,(V_1 A_1 + V_2 A_2)}$$

$$V = \frac{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1\,ft)^2}{4}}{(\sin 45^\circ)\left[\left(10\,\frac{ft}{s}\right)\frac{\pi (0.1\,ft)^2}{4} + \left(10\,\frac{ft}{s}\right)\frac{\pi (0.1\,ft)^2}{4}\right]}$$

and

$$V = 7.07\,\frac{ft}{s}$$

例題 六 A circular plate having a diameter of 300 mm is held perpendicular to an axisymmetric horizontal jet of air having a velocity of 40 m/s and a diameter of 80 mm as shown in Fig. P5.21. A hole at the center of the plate results in a discharge jet of air having a velocity of 40 m/s and a diameter of 20 mm. Determine the horizontal component of force required to hold the plate stationary.

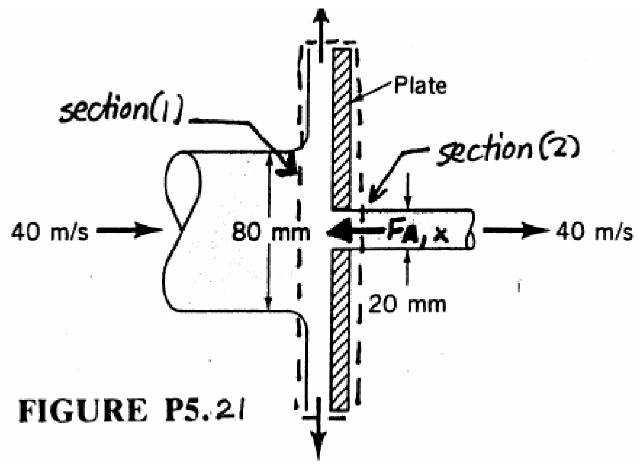


FIGURE P5.21

The control volume contains the plate and flowing air as indicated in the sketch above. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

or

$$F_{A,x} = u_1^2 \rho \frac{\pi D_1^2}{4} - u_2^2 \rho \frac{\pi D_2^2}{4} = u_1^2 \rho \frac{\pi}{4}\left(D_1^2 - D_2^2\right)$$

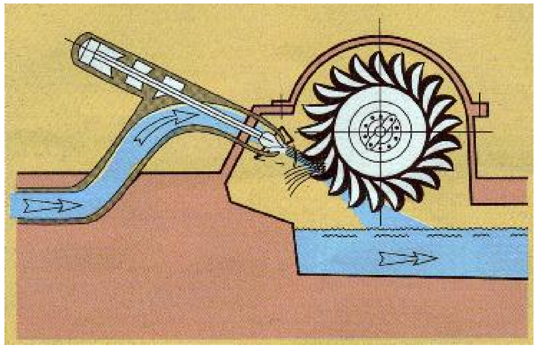
Thus

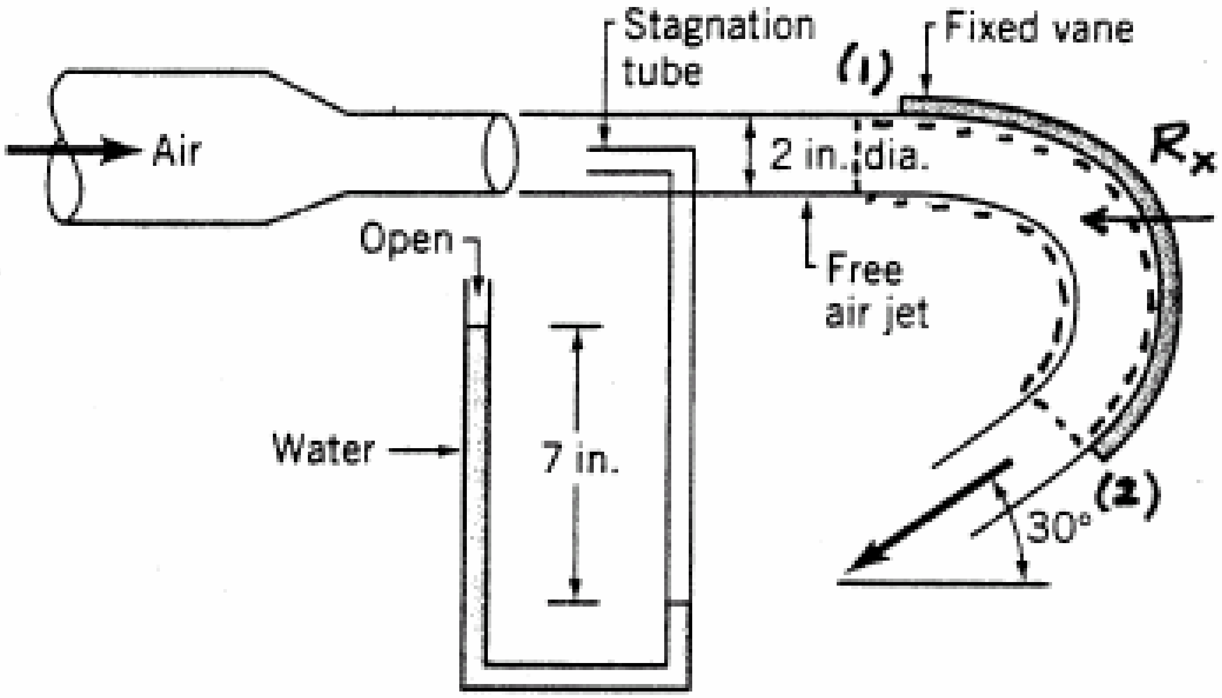
$$F_{A,x} = \left(40\ \frac{m}{s}\right)^2 \left(1.23\ \frac{kg}{m^3}\right) \frac{\pi}{4}\left[\frac{(80\ mm)^2 - (20\ mm)^2}{\left(1000\ \frac{mm}{m}\right)^2}\right]\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

and

$$F_{A,x} = \underline{\underline{9.27\ N}}$$

5.73 Air discharges from a 2-in.-diameter nozzle and strikes a curved vane, which is in a vertical plane as shown in Fig. P5.73. A stagnation tube connected to a water U-tube manometer is located in the free air jet. Determine the horizontal component of the force that the air jet exerts on the vane. Neglect the weight of the air and all friction.





例題 2.10 如圖 2.12 所示，一噴射流以流速 46m/s，流量 $0.0566\,m^3/s$ 之水衝擊在一固定葉上，並經偏角 35° 後，試求作用在此固定葉上之力。

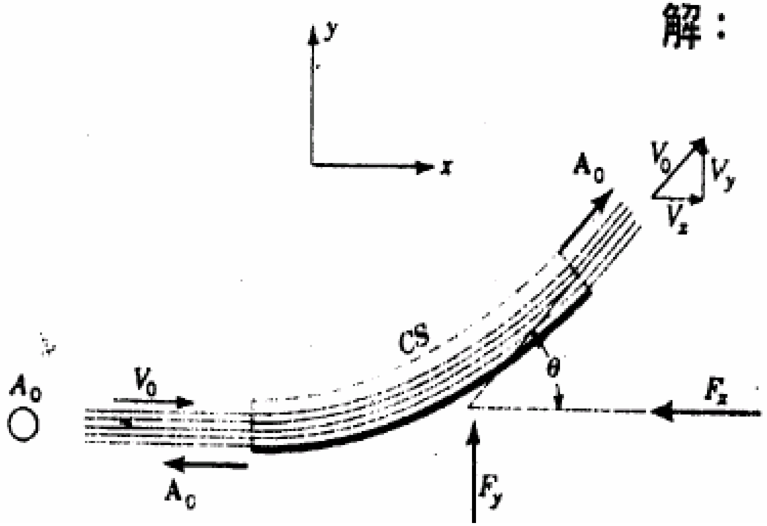


圖 2.12 噴射流衝擊平滑的固定葉片

解： 如圖 2.12 所示，在 x 及 y 方向上之作用力可用 (2.37) 式求得爲

$$-F_x=\rho V_0\cos\theta V_0A_0+\rho V_0(-V_0A_0)$$

$$F_y=\rho V_0\sin\theta V_0A_0$$

現已知 $V_0A_0=Q=0.0566\,m^3/s$，$V_0=46\,m/s$，$\theta=35°$

故 $F_x=\rho QV_0(\cos\theta-1)$

$$=-\frac{1000}{9.806}\times0.0566\times46\times(\cos35°-1)$$

$$=48\ \text{kgf}$$

$$=470.6\ \text{N}$$

$$F_y=\frac{1000}{9.806}\times0.0566\times46\times\sin35°$$

$$=152.3\ \text{kgf}$$

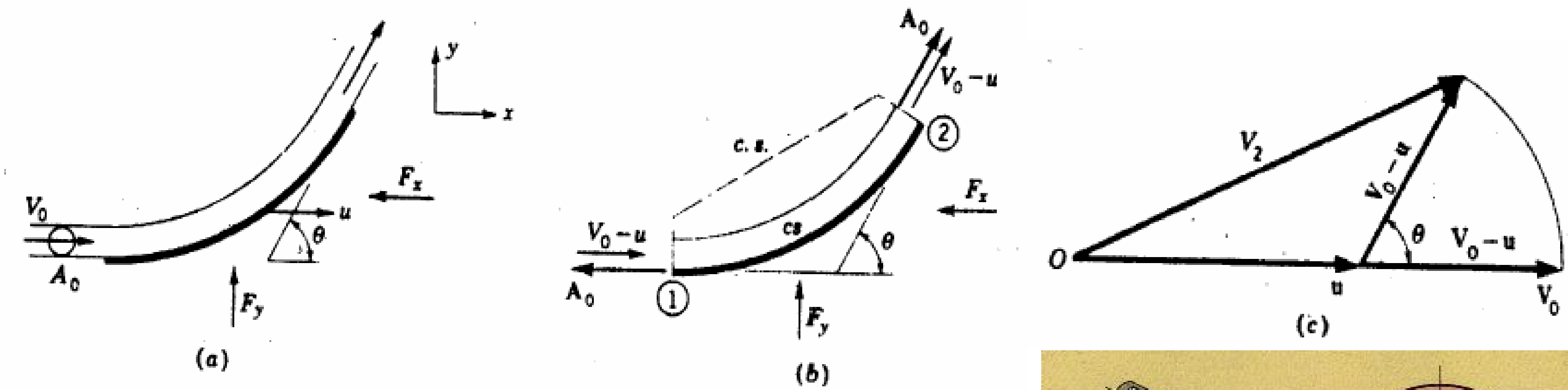
$$=1493.45\ \text{N}$$

$$\vec{F}_x=\vec{F}_{S_x}+\vec{F}_{B_x}=\frac{\partial}{\partial t}\int_{CV}u\rho d\forall+\int_{CS}u\rho\vec{V}\cdot d\vec{A}$$

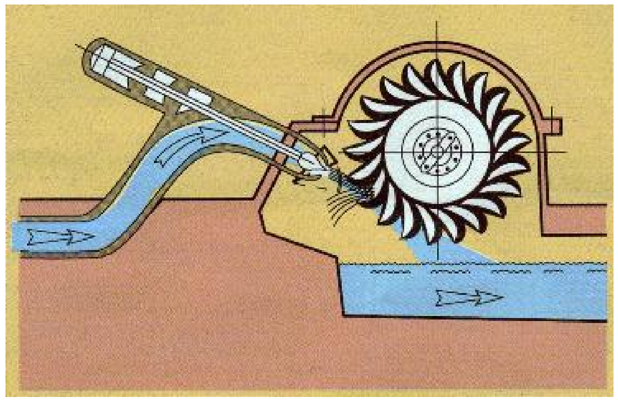
$$\vec{F}_y=\vec{F}_{S_y}+\vec{F}_{B_y}=\frac{\partial}{\partial t}\int_{CV}\upsilon\rho d\forall+\int_{CS}\upsilon\rho\vec{V}\cdot d\vec{A}$$

例題 2.12

如圖 2.14 所示，一噴射流打在一可動葉片上，試求葉片所受之力。



解：



設葉片對噴流無摩擦阻力，噴流以對於葉片的相對速度 V_0-u 流入，並以此速度流出。流出的絕對速度 V_2，爲此相對速度與葉片絕對速度 u 的向量和，參照圖(c)。因流動狀況爲穩定流態，應用 (2.37) 式於圖(b)控制體積，得

$$\sum F_x=\int_{cs}\rho v_x \mathbf{V}\cdot d\mathbf{A}=-F_x$$

$$=\rho(V_0-u)\cos\theta[(V_0-u)A_0]+\rho(V_0-u)[-(V_0-u)A_0]$$

或 $$F_x=\rho(V_0-u)^2A_0(1-\cos\theta)$$

$$\sum F_y=\int_{cs}\rho v_y \mathbf{V}\cdot d\mathbf{A}=F_y$$

$$=\rho(V_0-u)\sin\theta[(V_0-u)A_0]$$

或 $$F_y=\rho(V_0-u)^2A_0\sin\theta$$

$$\vec{F}_x=\vec{F}_{S_x}+\vec{F}_{B_x}=\frac{\partial}{\partial t}\int_{cv}u\rho d\forall+\int_{CS}u\rho\vec{V}\cdot d\vec{A}$$

$$\vec{F}_y=\vec{F}_{S_y}+\vec{F}_{B_y}=\frac{\partial}{\partial t}\int_{cv}\upsilon\rho d\forall+\int_{CS}\upsilon\rho\vec{V}\cdot d\vec{A}$$

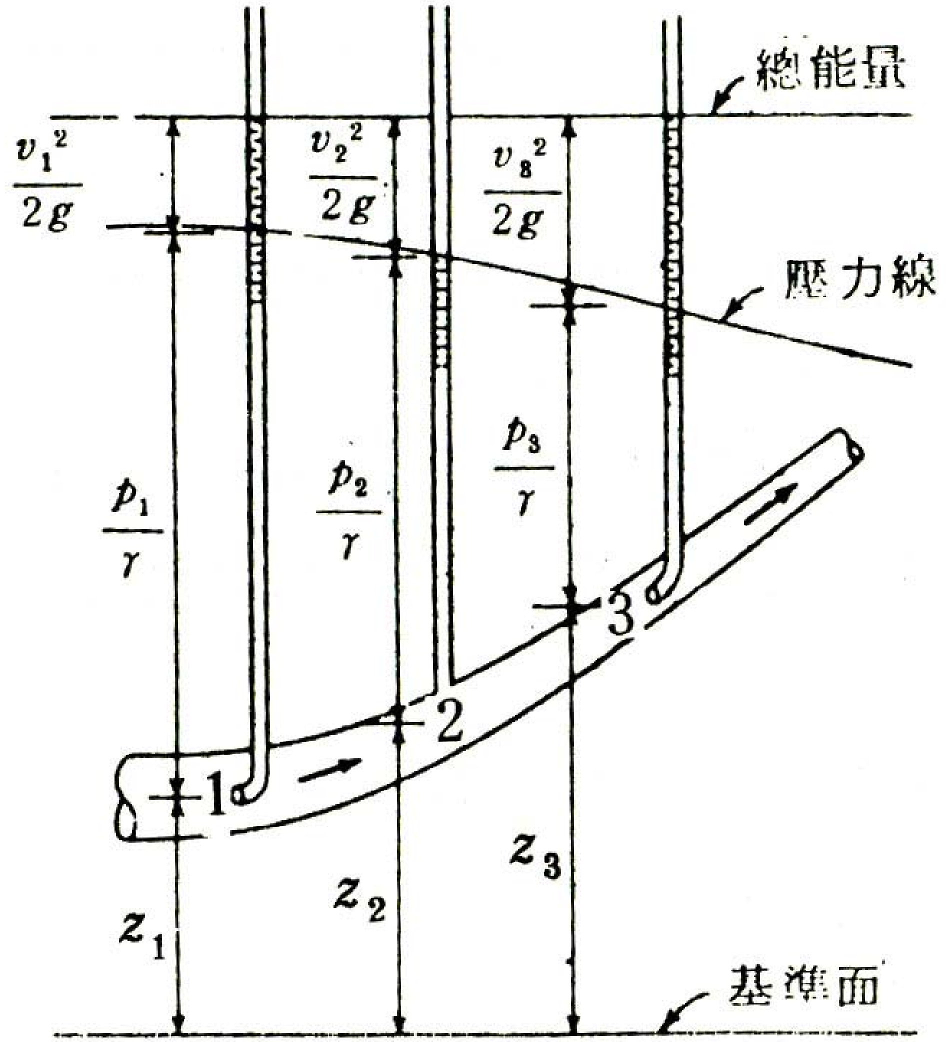


圖 2.10 柏努利方程式的圖解表示

The Angular Momentum Principle

2.8 動量矩方程式

$N = \vec{r} \times m\vec{V}$ (動量矩), $\eta = \vec{r} \times \vec{V}$

$$\boxed{\vec{T} = \left.\frac{d\vec{H}}{dt}\right)_{system}}$$

$$\vec{H}_{system} = \int_{M(system)} \vec{r} \times \vec{V} dm = \int_{\forall(system)} \vec{r} \times \vec{V} \rho d\forall$$

$$\vec{T} = \vec{r} \times \vec{F}_s + \int_{M(system)} \vec{r} \times \vec{g} dm + \vec{T}_{shaft} = \vec{r} \times \vec{F}_s + \int_{\forall(system)} \vec{r} \times \vec{g} \rho d\forall + \vec{T}_{shaft}$$

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t} \int_{CV} \eta \rho d\forall + \int_{CS} \eta \rho \vec{V} \cdot d\vec{A}$$

Set $\quad N = \vec{H} \quad$ and $\quad \eta = \vec{r} \times \vec{V}$

$$\boxed{\left.\frac{d\vec{H}}{dt}\right)_{system} = \frac{\partial}{\partial t} \int_{CV} \vec{r} \times \vec{V} \rho d\forall + \int_{CS} \vec{r} \times \vec{V} \rho \vec{V} \cdot d\vec{A}}$$

$$\frac{dN_{sys}}{dt} = \frac{d}{dt} \int_{CV} \rho \eta \, dV + \int_{CS} \rho \eta (\vec{V} \cdot \hat{n}) \, dA$$

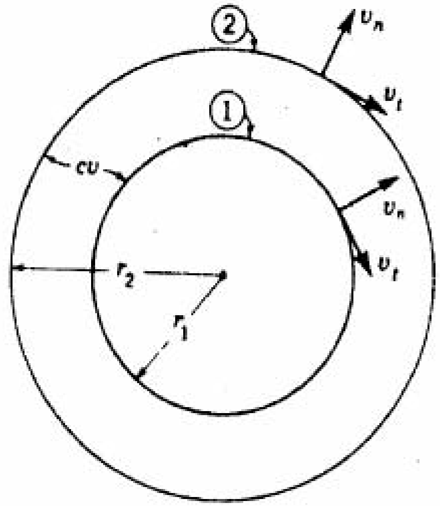
$N = \vec{r} \times m\vec{V} \qquad \eta = \vec{r} \times \vec{V} \qquad \eta = \vec{r} \times \vec{V}$

$$\sum \vec{T} = \vec{r} \times \vec{F}_s = \frac{\partial}{\partial t} \int_{CV} \rho \, \vec{r} \times \vec{V} \, dV + \int_{CS} \rho \, \vec{r} \times \vec{V} (\vec{V} \cdot \hat{n}) \, dA$$

$$\boxed{\vec{T} = \vec{r} \times \vec{F}_s + \int_{\forall(system)} \vec{r} \times \vec{g} \rho d\forall + \vec{T}_{shaft} = \frac{\partial}{\partial t} \int_{CV} \vec{r} \times \vec{V} \rho d\forall + \int_{CS} \vec{r} \times \vec{V} \rho \vec{V} \cdot d\vec{A}}$$

$$\sum \vec{T} = \vec{r} \times \vec{F}_s = \frac{\partial}{\partial t}\int_{CV} \rho\ \vec{r} \times \vec{V}\, dV + \int_{CS} \rho\ \vec{r} \times \vec{V} (\vec{V} \cdot \hat{n})\, dA$$

$$\boxed{Fr = T_z = \frac{\partial}{\partial t}\int_{cv} \rho r v_t\, d\forall + \int_{cs} \rho r v_t v_n\, dA}$$



式中T_z爲力矩。若流動爲穩定流則$\frac{\partial}{\partial t}\int_{cv} \rho r v_t\, d\forall = 0$

於是上式簡化爲

$$\boxed{T_z = \int_{A2} \rho_2 r_2 v_{t2} v_{n2}\, dA_2 - \int_{A1} \rho_1 r_1 v_{t1} v_{n1}\, dA_1} \qquad (2.42)$$

由連續方程式 $\int_{A1} \rho v_n\, dA = \int_{A2} \rho v_n\, dA = \rho Q$

故 $$\boxed{T_z = \rho Q[(r v_t)_2 - (r v_t)_1]} \qquad (2.43)$$

例題 2.13

圖 2.17 之灑水器每一噴嘴之流量為 283 cm^3/s，不計摩擦，試求其迴轉速。但每一噴嘴口之面積為 0.93 cm^2，$a=30.5$ cm，$b=20.3$cm。

解：

因流體進入灑水器無動量矩，又作用於系統上之外來扭矩為零，故流體離開時之動量矩亦應為零。

若令 ω 為旋轉速率，則水離開之動量矩為

$$\rho Q_1 r_1 v_{t1} + \rho Q_2 r_2 v_{t2}$$

其中 v_{t1}，v_{t2} 分別表示 1，2 處水的絕對速度，即

$$v_{t1} = v_{r1} - \omega r_1 = \frac{283}{0.93} - \omega \times 30.5$$

$$v_{t2} = v_{r2} - \omega r_2 = \frac{283}{0.93} - \omega \times 20.3$$

因動量矩為 0，故

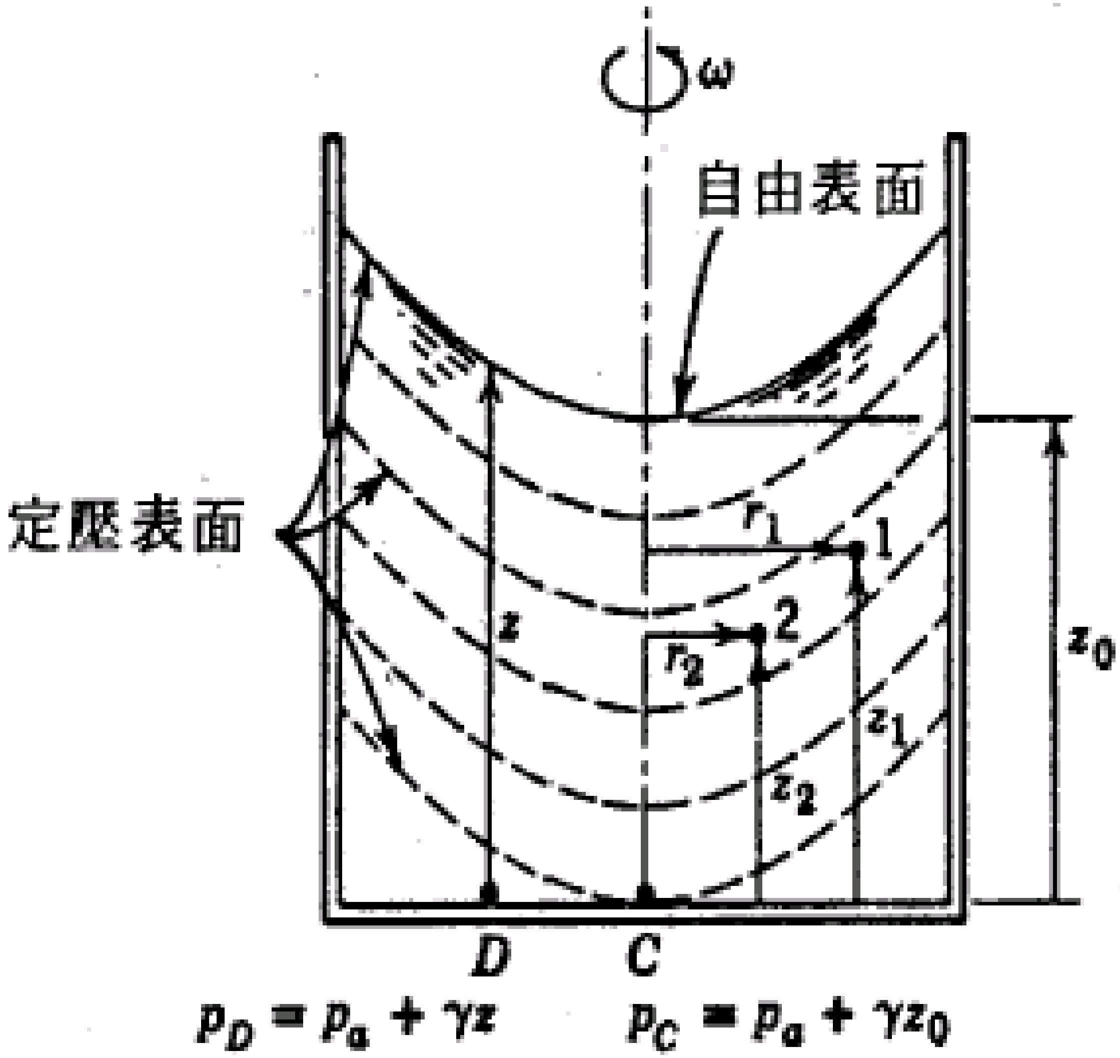
$$\rho Q (r_1 v_{t1} + r_2 v_{t2}) = 0$$

或

$$30.5 \left(\frac{283}{0.93} - 30.5\,\omega\right) + 20.3 \left(\frac{283}{0.93} - 20.3\,\omega\right) = 0$$

解之，得 $\omega \fallingdotseq 11.5$ rad/s 或 110 rpm

2.9 旋渦運動



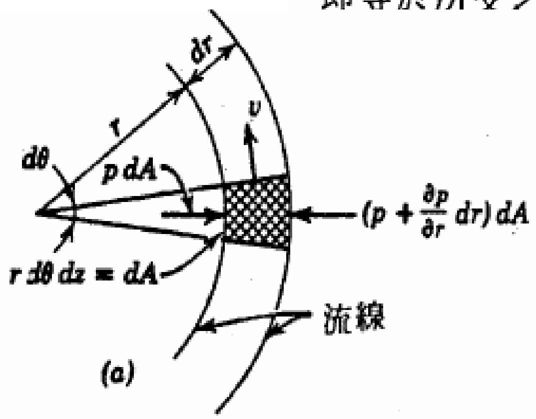
(b)

Washer, Pump, Water Turbine

(1) The pressure induced by centrifugal force (水平方向)

此流體元素在半徑方向兩個面上所受壓力 pdA 與 $\left[p+\left(\frac{\partial p}{\partial r}\right)dr\right]$ 之差

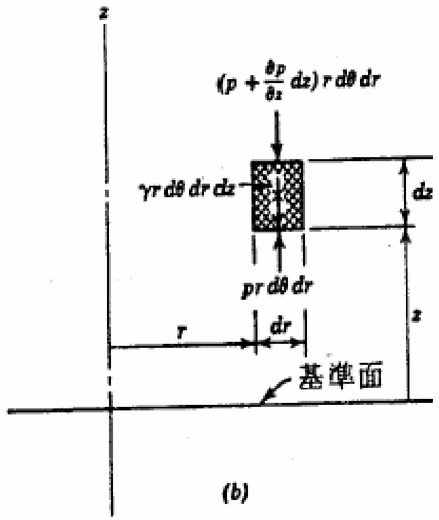
即等於所受之離心力，故



$$\sum F = ma$$

$$\left(p+\frac{\partial p}{\partial r}dr\right)dA - pdA = \rho dA dr \frac{v^2}{r}$$

$$\frac{\partial p}{\partial r} = \rho \frac{v^2}{r}$$



(2) The pressure induced by hydraulic static(垂直方向)

若在垂直方向無運動，則此方向上之壓力梯度與靜水壓之情況相同，

即

$$\frac{\partial p}{\partial z} = -\gamma \tag{2.45}$$

(a) 強制渦動

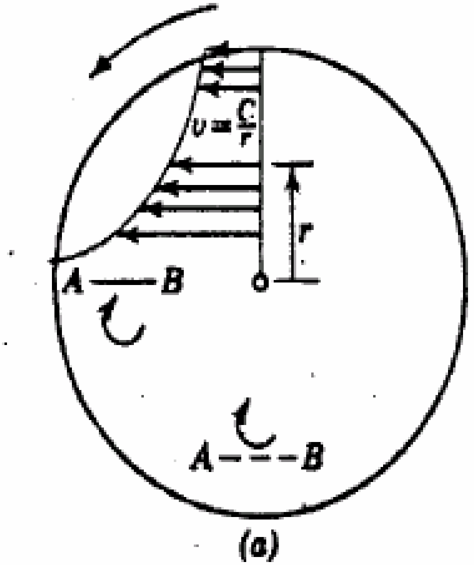
當流體受一定之扭力矩而旋轉時，即稱爲強制渦動，如圖 2.19，一液

$$\frac{\partial p}{\partial r}=\rho\frac{v^2}{r}=\rho r\omega^2 \tag{2.46}$$

$$\frac{P_2-P_1}{\rho}=\frac{1}{2}\omega^2\left(r_2^2-r_1^2\right)+g\left(z_1-z_2\right) \tag{2.47a}$$

$$\frac{P_2-P_1}{\gamma}=\frac{1}{2g}\omega^2\left(r_2^2-r_1^2\right)+\left(z_1-z_2\right) \tag{2.47b}$$

Free Vortex



V=C/r

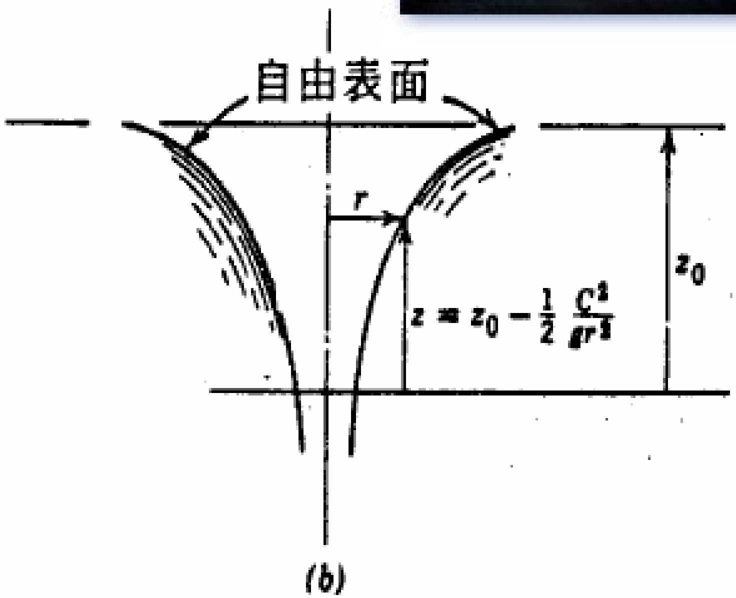


圖 2.20 自由渦旋運動

(b) 自由渦動

流體作曲線運動時，自然 發生自由渦動，此種運動不因外加之扭力而起，因此自由渦動之角動量變化率爲零。

$$r \times F = r \times \frac{\partial(mv)}{\partial t} = \frac{\partial(mvr)}{\partial t}$$

$\rightarrow$ $rv=$常數$=c$ $\qquad r_1 v_1 = r_2 v_2 = c$

因 $\dfrac{\partial p}{\partial r} = \rho \dfrac{v^2}{r} \qquad \dfrac{\partial p}{\partial r} = \dfrac{\rho c^2}{r^3}$ (2.49)

積分 $\dfrac{p_1 - p_1}{\rho} = \dfrac{1}{2} c^2 \left(\dfrac{1}{r_1^2} - \dfrac{1}{r_2^2} \right) + g(z_1 - z_2)$ (2.50)

$$r_1 v_1 = r_2 v_2 = c \qquad \frac{p}{\rho} + g z_1 + \frac{1}{2} v_1^2 = \frac{p_2}{\rho} + g z_2 + \frac{1}{2} v_2^2$$

$$\boxed{\frac{p_1 - p_1}{\rho} = \frac{1}{2} c^2 \left(\frac{1}{r_1^2} - \frac{1}{r_2^2} \right) + g(z_1 - z_2) \qquad (2.50)}$$

$$r_1 v_1 = r_2 v_2 = c$$

$$\boxed{\frac{p}{\rho} + g z_1 + \frac{1}{2} v_1^2 = \frac{p_2}{\rho} + g z_2 + \frac{1}{2} v_2^2}$$

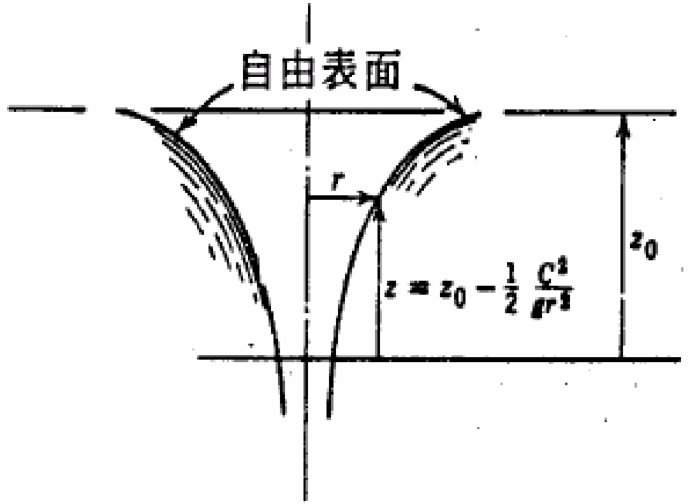
此指出無黏性理想流體之柏努利方程式，亦可適用於自由渦動。

自由渦動的自由表面形式可在 (2.50) 式設 $p_1 = p_2 = p_{atm} = 0$ 獲得，即

$$z_1 = z_2 = \frac{1}{2}\frac{C^2}{g}\left(\frac{1}{r_1^2} - \frac{2}{r_2^2}\right)$$

或

$$z = z_0 - \frac{1}{2}\frac{C^2}{gr^2}$$



(b)

如圖 2.20 (b)所示。水由容器底孔流出之現象即爲最常見之自由渦動例子。由 (2.48) 式在自由渦動中心點之流速應爲無限大，事實上這種現象並不存在。

例題 2.14

如圖 2.21 所示為一圓形彎曲管水道之橫斷面。假定流動為無摩擦，則經此彎道的流動屬於自由渦動形式，即流速為 $v=C/r$ ，C為常數，試以R，b，V及 ρ 表示彎道上內外緣之壓力差P_0-P_i ，V為水道中的平均速度。

$$\frac{p_1-p_1}{\rho}=\frac{1}{2}c^2\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)+g(z_1-z_2) \tag{2.50}$$

解：

就無摩擦的流動，壓力差P_0-P_i 可由 (2.50) ，以 $z_0=z_i$ 而得

$$P_0-P_i=\frac{1}{2}\rho C^2\left[\frac{1}{(R-b)^2}-\frac{1}{(R+b)^2}\right]$$

由連續方程式可決定常數C，即

$$Q=VA=\int_A v\,dA=C\int_A\frac{dA}{r}$$

因此 $$C=\frac{VA}{\int_A\frac{dA}{r}}=\frac{VA}{H\int_{R-b}^{R+b}\frac{dr}{r}}=\frac{VA}{H[\ln r]_{R-b}^{R+b}}=\frac{VA}{H\ln[(R+b)/(R-b)]}$$

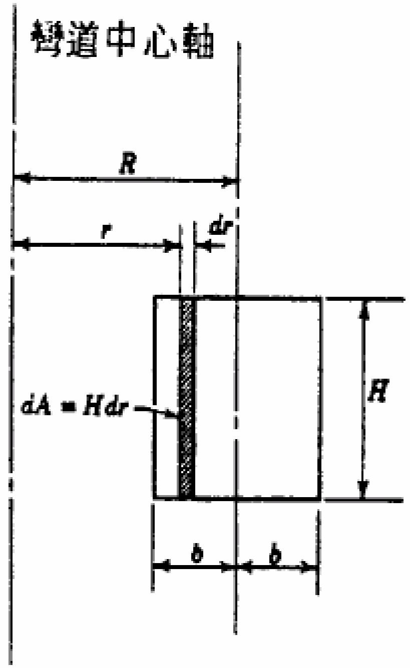


圖 2.21

但

$$H=\frac{A}{2b}$$

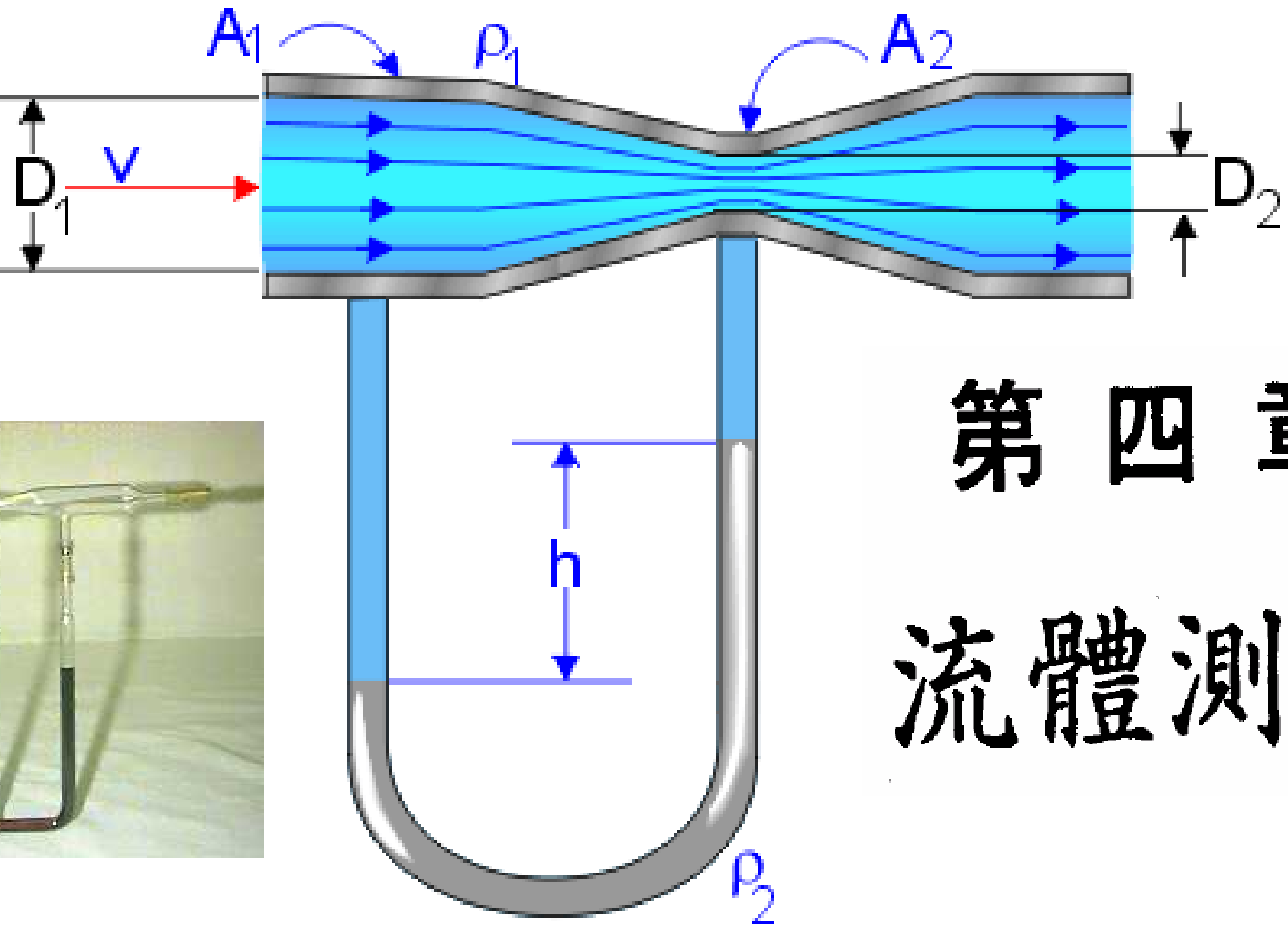
故

$$C=\frac{2bV}{\ln[(R+b)/(R-b)]}$$

於是

$$P_0-P_i=\frac{1}{2}\rho\left\{\frac{2bV}{\ln[(R+b)/(R-b)]}\right\}^2\cdot\frac{\text{[illegible]}}{(R^2-b^2)^2}$$

Chapter 4
Flow Measure



第 四 章

流體測量


3 8 2004


30 6 2004

4-1 Introduction

4-2 Measurement of Pressure

4-3 Measurement of Flow Speed

4-3 Measurement of Flowrate

4-5 Measurement of Viscosity

4-6 Measurement of Density

4-1 Introduction

客戶們所提供給我們的各種性能需求，都可利用電腦選機軟體程式爲我們精確地找到適合且效率高的機種．並可列出以下風機性能曲線：

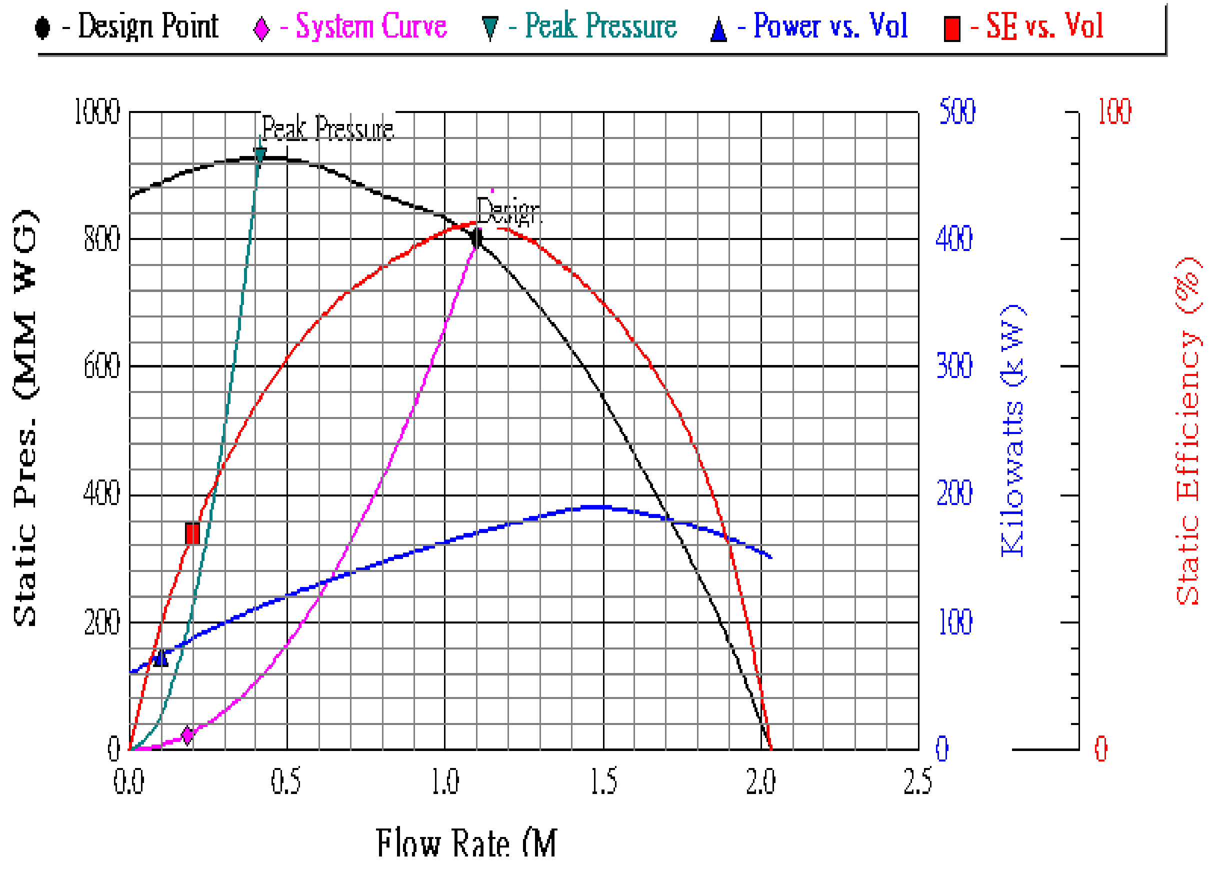
風量靜壓曲線

功率曲線

效率曲線

系統曲線性能曲線

八音頻譜



4-1 Introduction

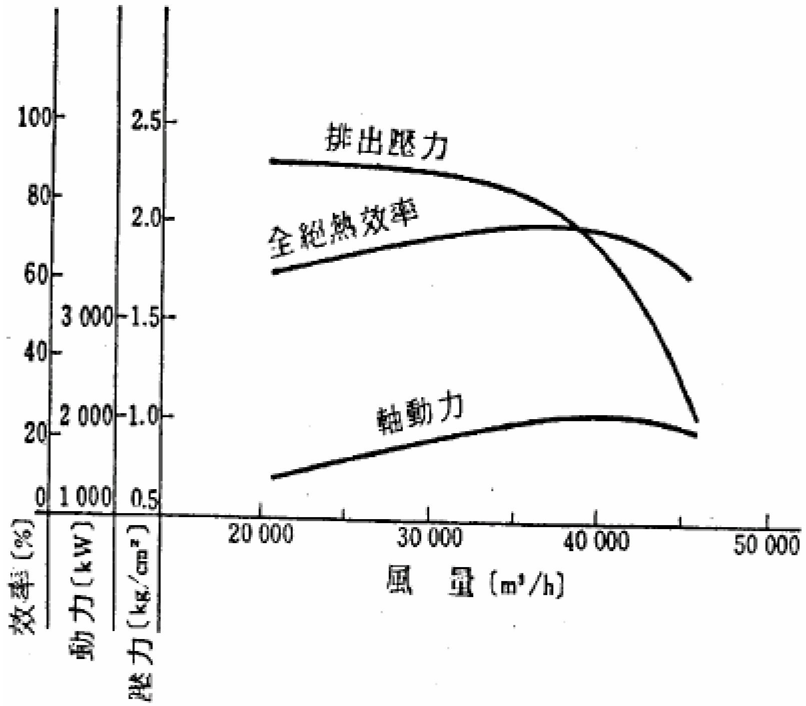


圖 10.27 離心壓縮機特性曲線一例

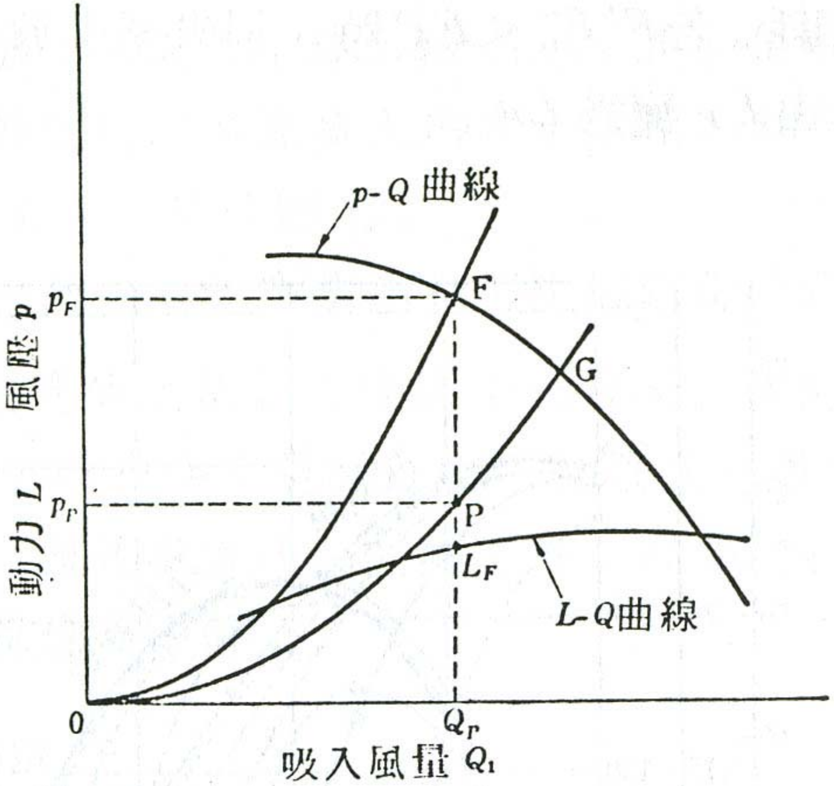


圖 10.28 排氣閥的風量調節

4-2 Measurement of Pressure 壓力量測

Bernoulli equation

$$\frac{P}{\gamma} + z + \frac{v^2}{2g} = \text{常數}$$

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$\frac{P}{\gamma}$ 稱爲壓力水頭 (pressure head)

z 爲位置水頭 (elevation head)

$\frac{v^2}{2g}$ 爲速度水頭 (velocity head)

三項能量之總和 H 稱爲全水頭 (total head)

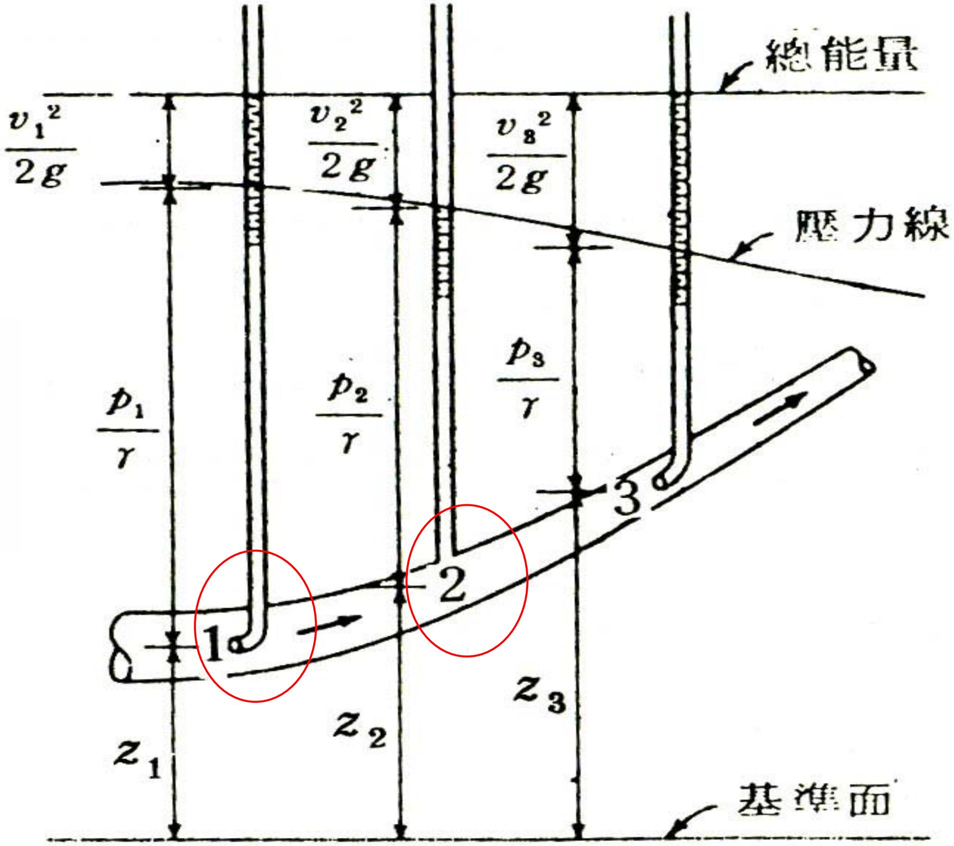


圖 2.10 柏努利方程式的圖解表示

4-2 Measurement of Pressure 壓力量測

4.2.1 Static pressure measurement(靜壓量測)– Piezometer tube

(1)靜止流體內壓力之量測

一般**靜止流體**壓力可以利用裝有水之彎管中顯示其高度，此高度為管內壓力與大氣壓間之差(第一章所提**U形管壓力計, 與<u>重力方向有關</u>**)。

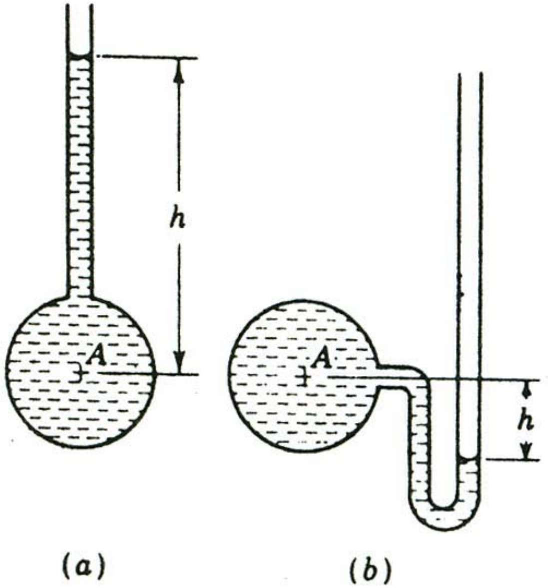
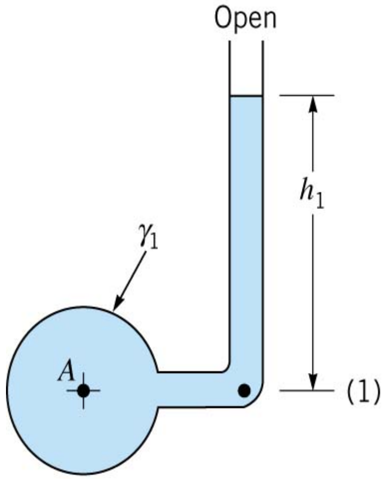


圖 1.5 靜壓管計



$$p = \gamma h + p_0$$

$$p_A = \gamma_1 h_1$$

4.2.1 Static pressure measurement – Piezometer tube

(2) 流動中流體內部的靜壓量測

一般來說，靜壓之測量，可分爲(1)流場中之靜壓測量；(2)固體表面處之靜壓測量。圖 4.1 所示爲測量靜壓的一種方法，當流體平行移動時，壓力之變化即爲垂直於流線方向的水壓變化，因此測量管壁之壓力，即可決定此斷面上任一點之壓力，但測量孔口必須很小，方可避免誤差。

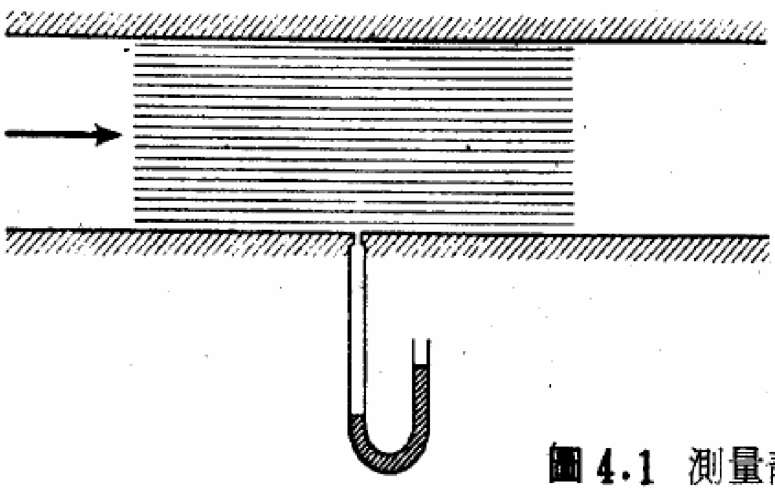


圖 4.1 測量靜壓之壓力管計

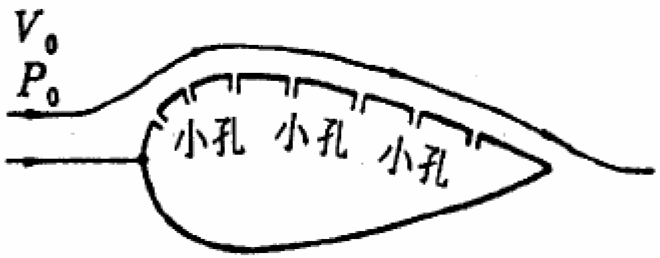


圖 4.3 固體表面之測壓口

4.2.2 The Measurement of total pressure

全壓力(Pt)的量測

當流體在管中流動時，會在管路中產生不同之壓力。一般壓力可分為三種，即靜壓(Ps)、動壓(P_v)與全壓(P_t)。

從垂直於管壁的孔中測出的壓力，若將之導到一個U型管裝置，如圖1，則可測出該管路之靜壓。而利用皮氏管面對風向所測得之壓力，導入U型管，即可獲得全壓。動壓則由全壓與靜壓之差求得。圖2為利用皮氏管將三種管壓力均以U型管水柱高表示。

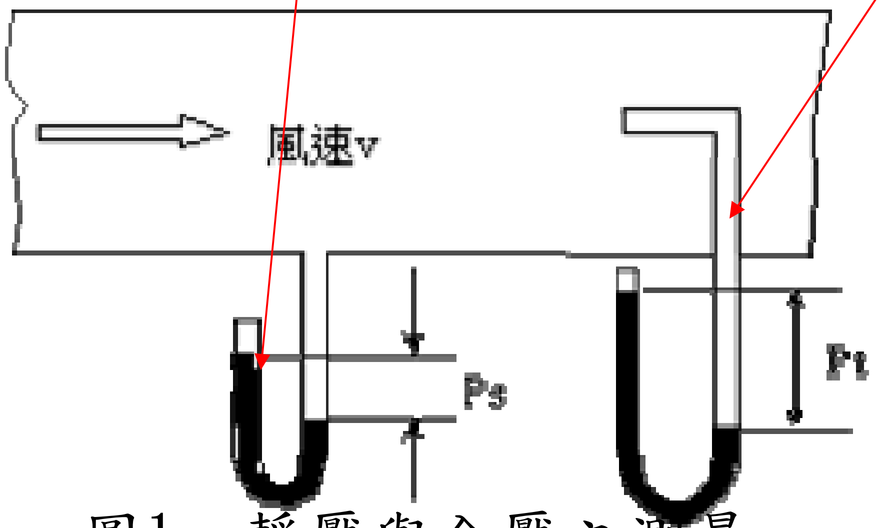


圖1. 靜壓與全壓之測量

A Piezometer and a Pitot tube

4.2.2 The Measurement of total pressure

全壓力(Pt)的量測

流動中流體的全壓力 p_t 又稱爲停滯壓力 p_{st} (stagnation pressure)，其測量可使用一有測壓小孔的測壓裝置，將其放入流體中，使小孔正對流向，此時小孔的位置即爲停滯點(stagnation poing)，所量得之壓力，即爲停滯壓力。圖 4.3 之皮托管(pitot tube)即利用此原理來測量停滯壓力，此法爲法國人皮托(Henri pitot)

(1932 年)所提出，係將一彎曲之玻璃管置於流速爲 V_0 的流場中，使其前端開口對正流向，則管中直立部份內的液面會上升 $V_0^2/2g$ 之高度，由柏努利方程式可得到下式關係

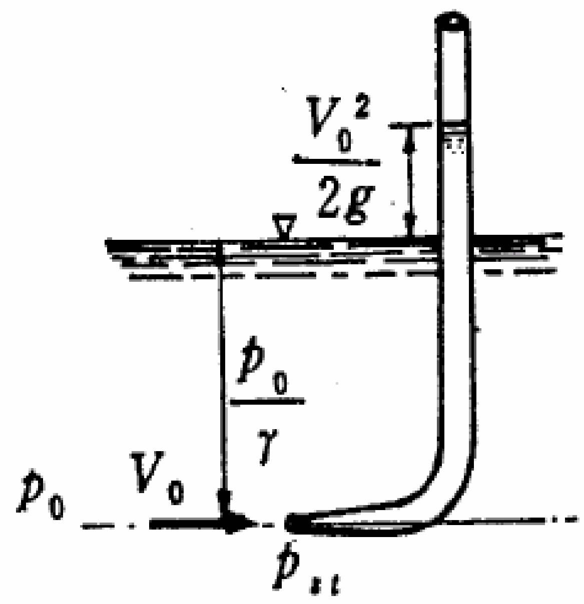


圖 4.3 皮托管

4.2.2 The Measurement total pressure

全壓力(Pt)的量測

移動中之流體內部壓力

Bernoulli's equation

$$p + \frac{1}{2}\rho V^2 + \gamma z = \text{constant along streamline}$$

$$\frac{p_{st}}{\gamma} = \frac{p_0}{\gamma} + \frac{V_0^2}{2g} \qquad (4.1)$$

$$p_{st} = p_0 + \frac{1}{2}\rho V_0^2 = p_s + p_d$$

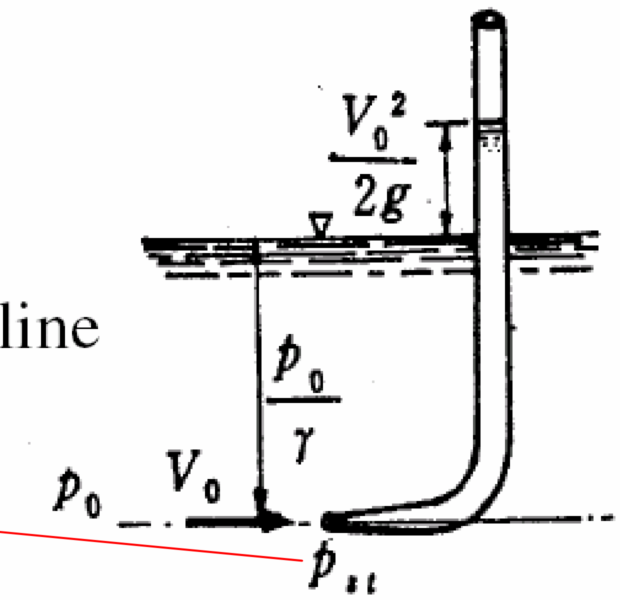


圖 4.3 皮托管

Total pressure=Stagnation pressure=Static pressure+Dynamic pressure

$$P_t = P_s + P_d = P_0 + \rho V_o^2/2$$

停滯壓力或全壓力＝靜壓＋動壓

Dynamic pressure 動壓(P_v)量測

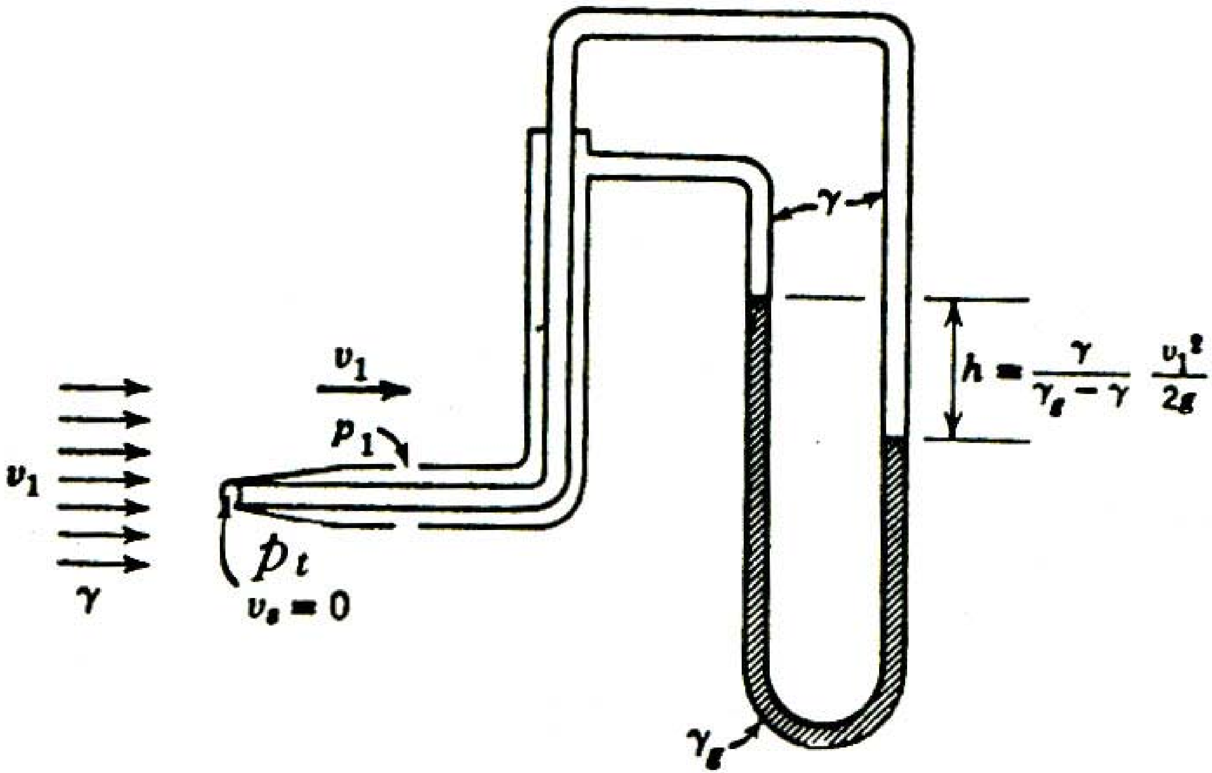
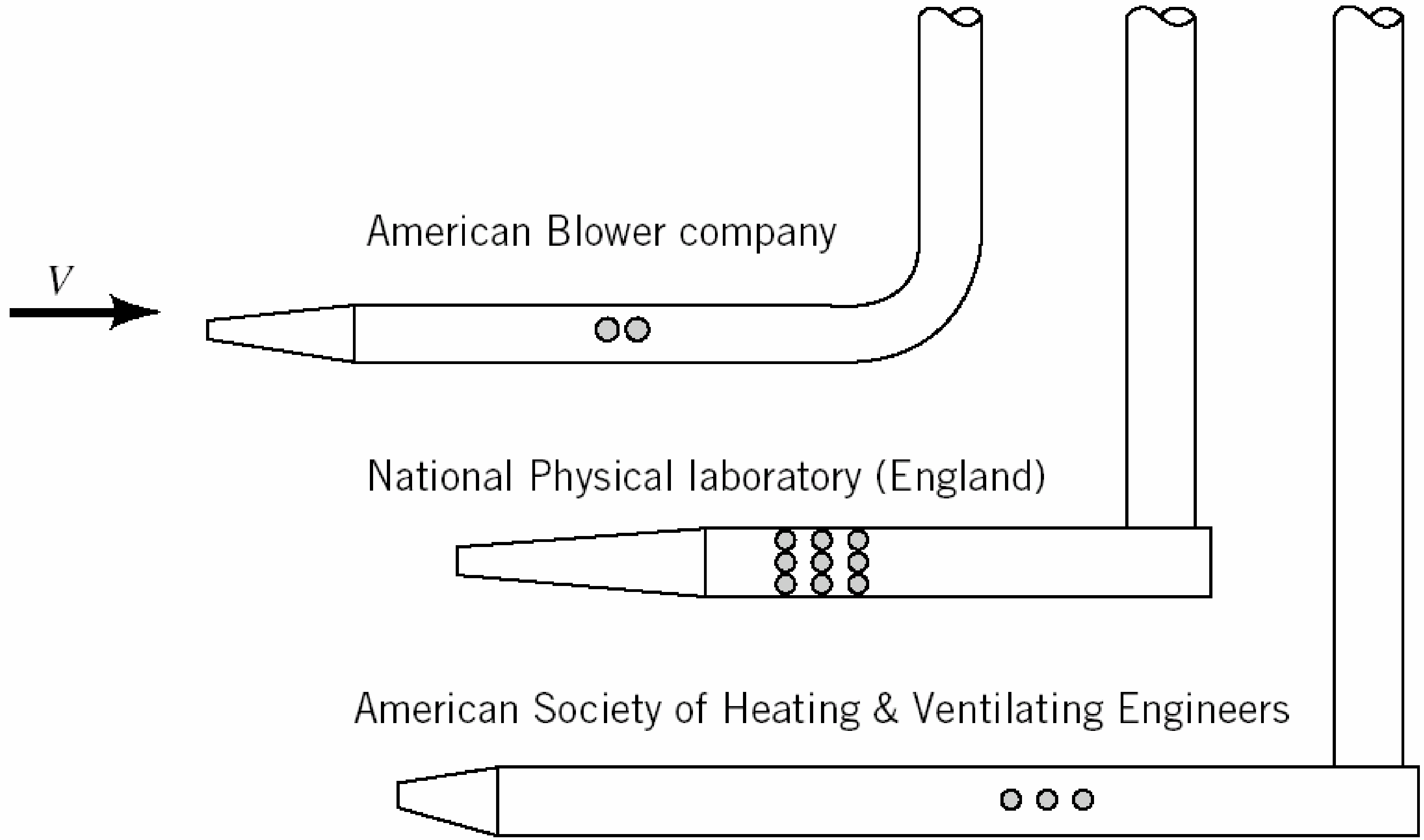
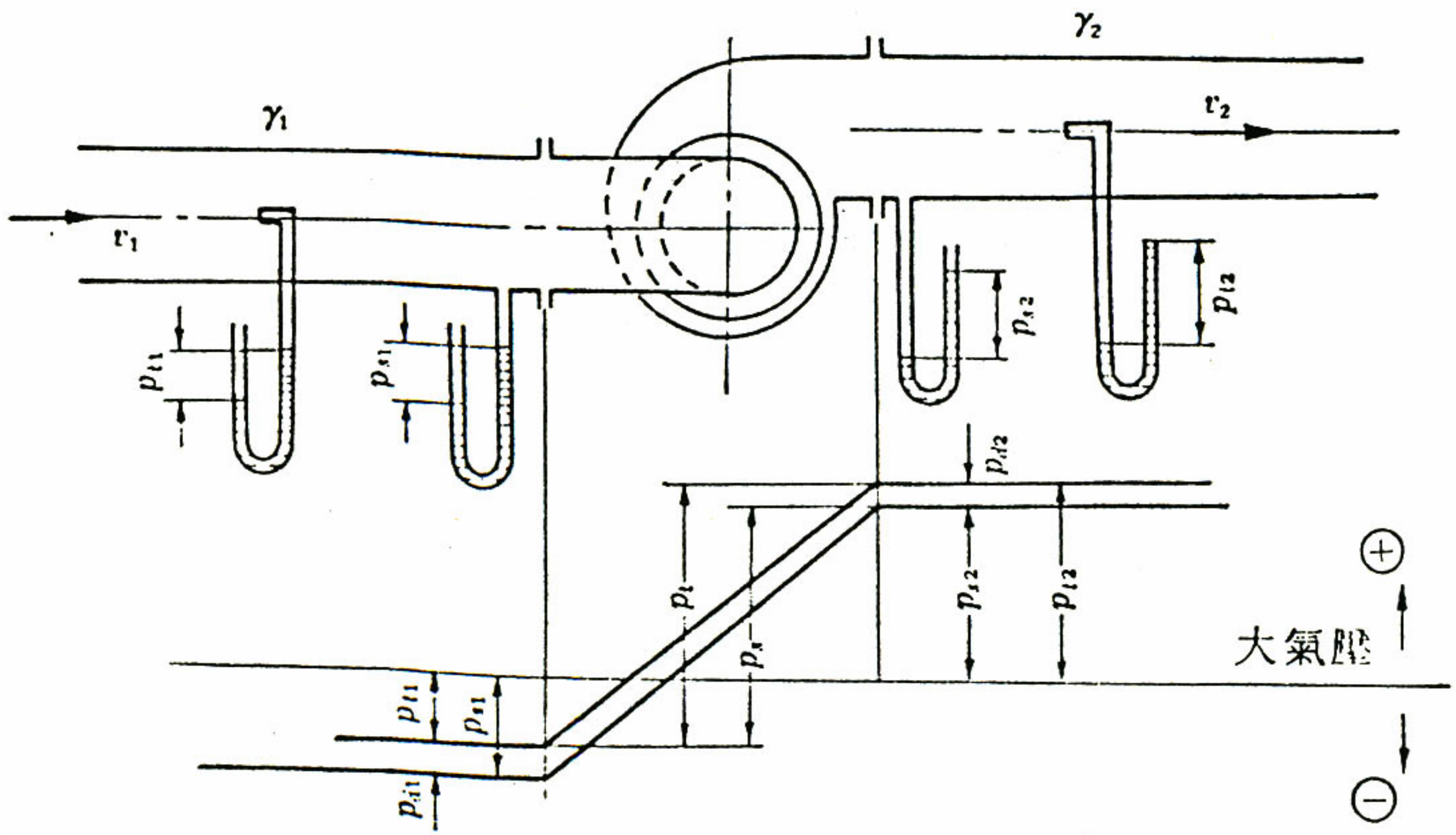


圖 4.5 皮托靜壓管

$$P_t - P_s = \rho V^2/2 = P_d$$



Pitot-static tube designs



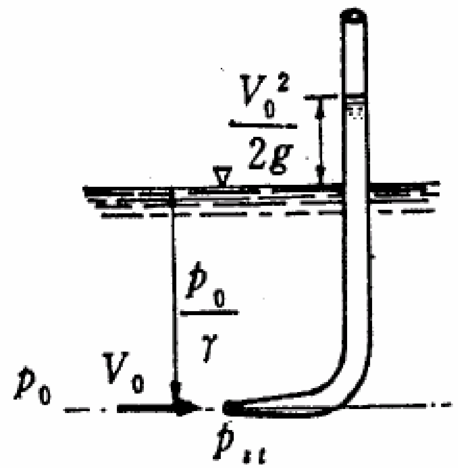
送風機壓力說明圖

4-3 Measurement of Flow Speed

Measurement of Airspeed

4.3.1 皮托管法(又稱滯流管法)

$$\frac{p_{st}}{\gamma}=\frac{p_0}{\gamma}+\frac{V_0^2}{2g} \qquad (4.1)$$



$$V_0=\sqrt{\frac{2g(p_t-p_1)}{\gamma}} \qquad (4.3)$$

$$V=\sqrt{\frac{2(p_0-p)}{\rho}}$$

不過由於流體流經靜壓管時，流速會增加，使靜壓管所量得之壓力會略小於實際壓力，因此 (4.3) 式必須乘一修正係數 C_p，即

$$V_1=C_p\sqrt{\frac{2g(p_t-p_1)}{\gamma}}$$

式中 C_p 值通常約為 0.97 至 1.0 之間。

例題 4.1

圖 4.4 所示之壓力計中液體爲水，用以測量管中空氣（p=16 lbf / in^2 ，t = 40°F）速度。設 h=1.2 in ，求空氣速度。

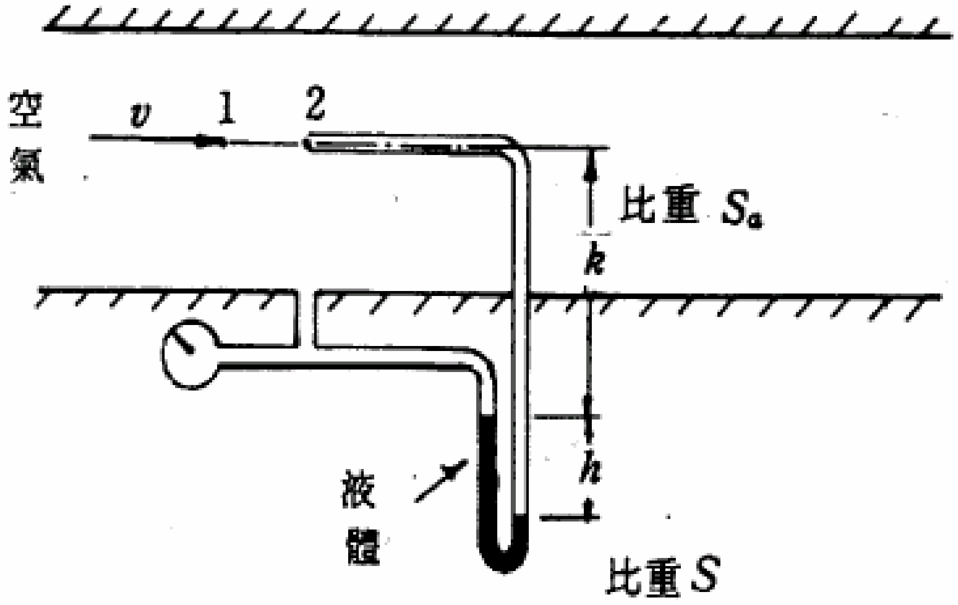


圖 4.4 皮托管與測壓計

解：

$$\text{空氣比重量 } \gamma_a = \frac{P}{RT} = \frac{16 \times 44}{53.34 \times (460 + 40)} = 0.0864 \text{ lbf / ft}^3$$

將柏努利方程式應用於 1 至 2，得

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma_a} = \frac{p_2}{\gamma_a}$$

又由圖中之壓力計知

$$p_2 + (k + h)\gamma_a - h\gamma_w - k\gamma_a = p_1$$

化簡得

$$\frac{p_2 - p_1}{\gamma_w} = h\left(\frac{\gamma_w}{\gamma_a} - 1\right) = h\left(\frac{s_w}{s_a} - 1\right) \qquad (2)$$

將(2)式代入(1)式，解之得 v 爲

$$v = \sqrt{2gh\left(\frac{s_w}{s_a} - 1\right)} = \sqrt{2 \times 32.174 \times \frac{1.2}{12} \times \left(\frac{62.4}{0.0864} - 1\right)}$$

$$= 68.124 \text{ ft / sec}$$

例題 4.1

圖 4.4 所示之壓力計中液體爲水，用以測量管中空氣（p=16 lbf / in²，t = 40°F）速度。設 h=1.2 in，求空氣速度。

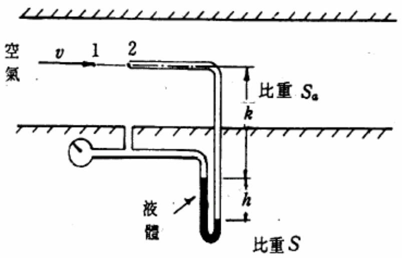
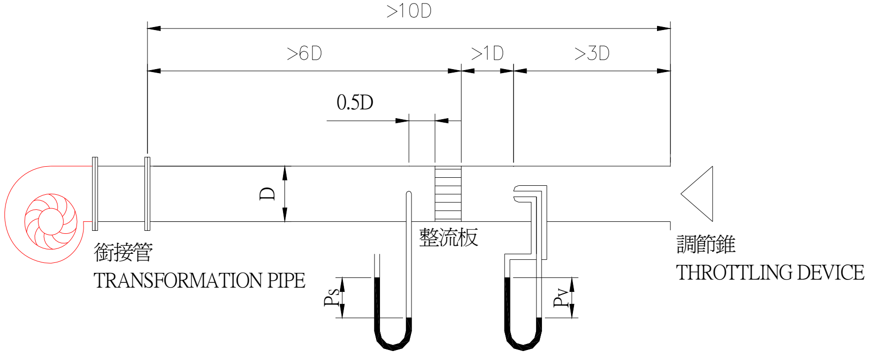


圖 4.4 皮托管與測壓計

4.3.2 皮托靜壓管



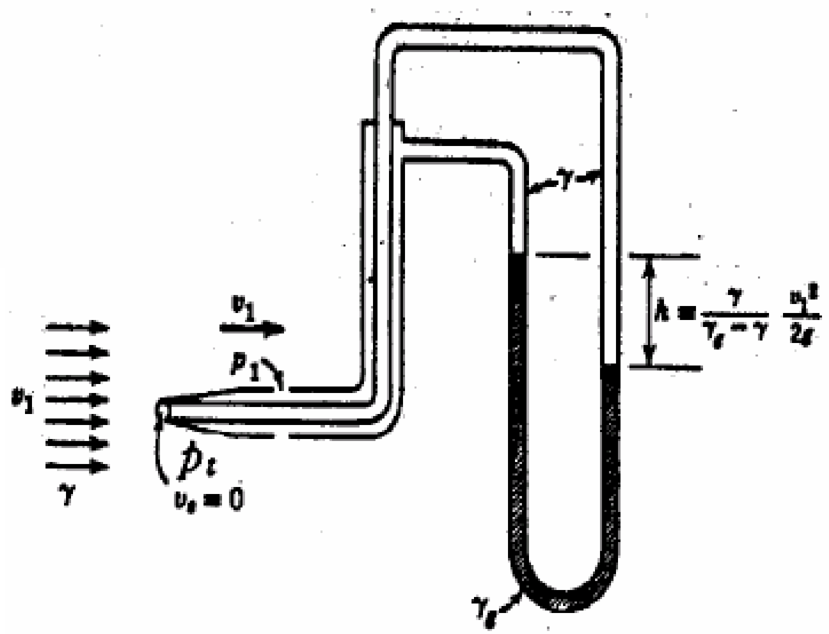


圖 4.5 皮托靜壓管

皮托靜壓管為將測量滯流點壓力之皮托管與測量靜壓之靜壓管組合而成的一種流速量測裝置，如圖 4.5 所示。由任一點至滯流點 T（同一高度）之柏努利方程式為

$$p_1 + \frac{1}{2}\rho V_1{}^2 + 0 = p_t + 0$$

$$p_1 + \frac{1}{2}\rho V_1^2 \quad 0 = p_t \quad 0$$

整理之，得

$$V_1 = \frac{\sqrt{2g(p_t - p_1)}}{\gamma}$$

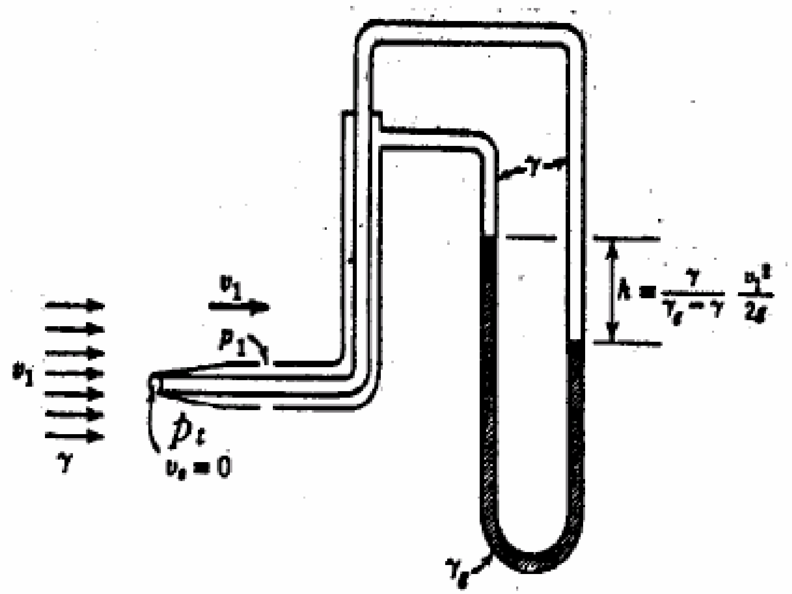


圖 4.5 皮托靜壓管

由U型壓力管計知 $\quad p_t + \gamma h - \gamma_g h = p_1$

$$p_t - p_1 = \gamma_g h - \gamma h$$

代入 (4.5) 式，並乘以一修正係數 c_p ，可得

$$V_1^2 = e_p \frac{\sqrt{2g(\gamma_g h - \gamma h)}}{\gamma} = c_p \frac{\sqrt{2g(\gamma_g - \gamma)h}}{\gamma} \tag{4.6}$$

從上式知，如測得壓力管計之讀數，h ，即可求得該處的流速。

例題 4.2

有一皮托管的修正係數爲 0.98 ，置於一管流中心，管中流體爲苯，與皮托靜壓管相連接之水銀差壓計讀數爲 75 mm，試求管中心處的苯流速。（水銀比重＝13.57 ，苯比重＝0.88 ）。

解：

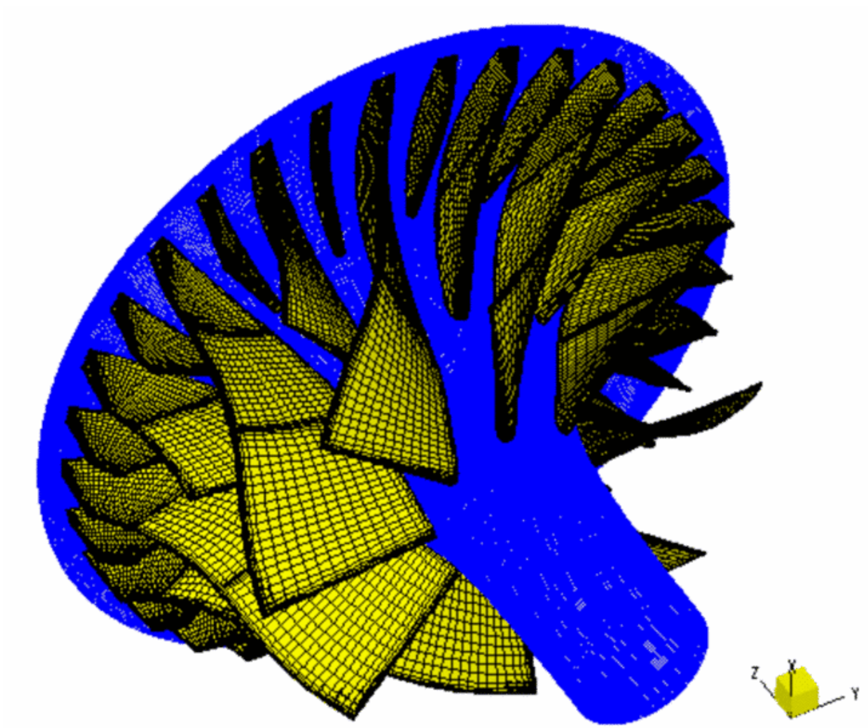
已知壓力計之讀數 $h=75\,\mathrm{mm}=0.075\,\mathrm{m}$

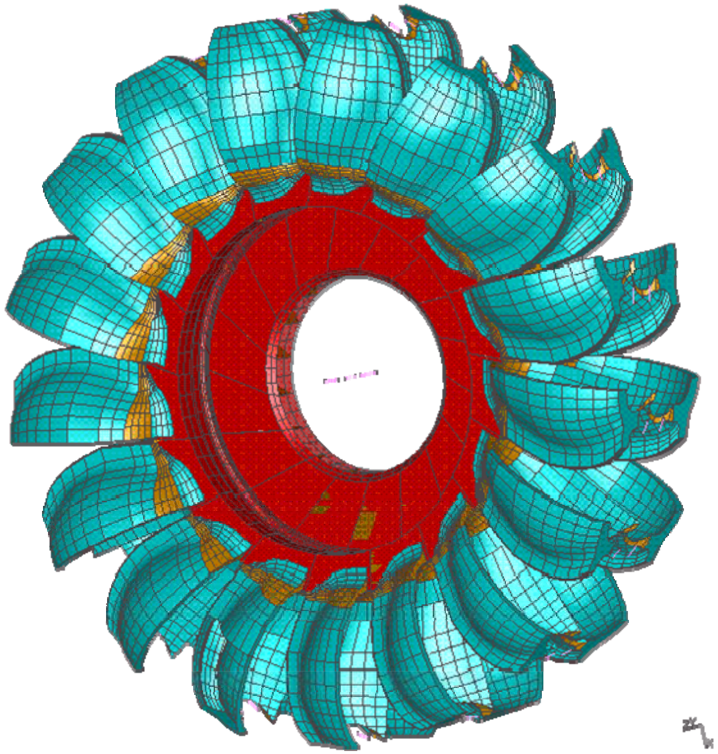
由 (4.6) 式知經管心處之流速

$$V=0.98\sqrt{\frac{2\times 9.8\times(13.57-0.88)\times 9806\times 0.075}{0.88\times 9806}}$$

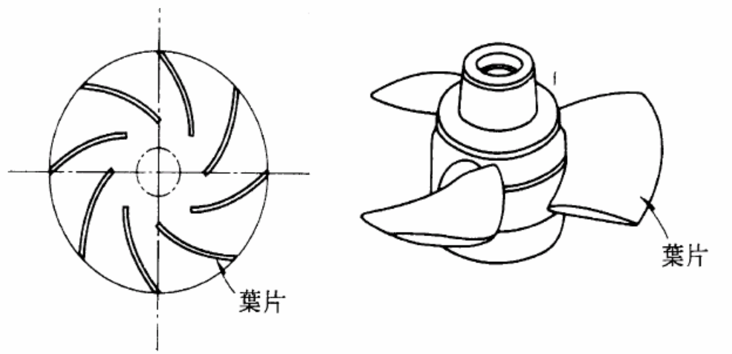
$$=4.51\,\mathrm{m/s}$$

Chap. 5 輪機機械的相似定理



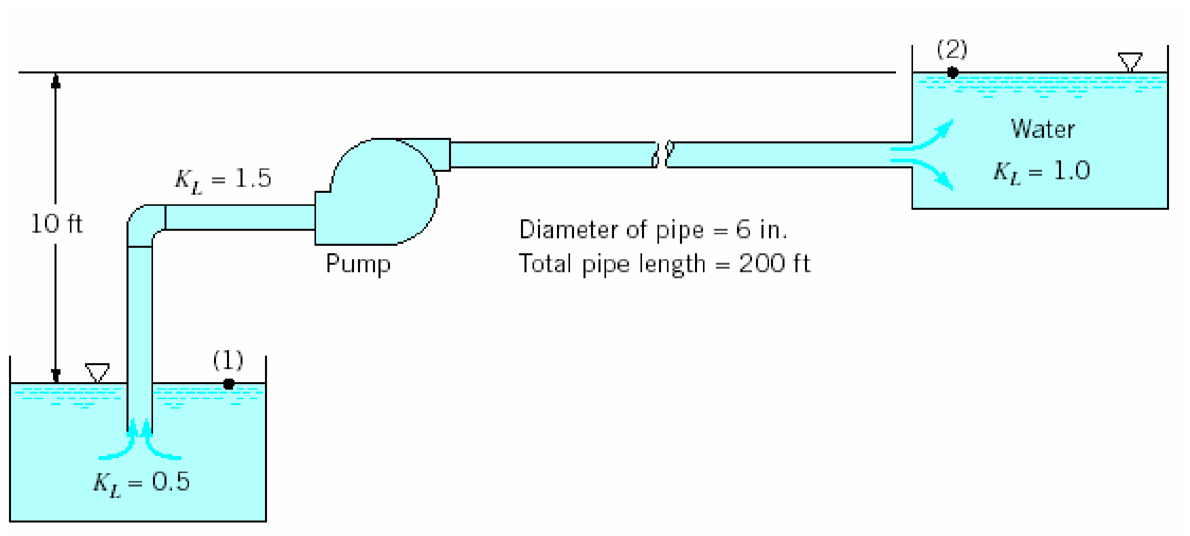


輪機機械（turbomachinery）係指利用於一平板圓周或一圓筒面上安裝呈輻射狀排列之葉片（vane）所形成之葉輪（如圖 2.1 所示），使葉輪旋轉且使流體流經其間而達成能量傳送之機械，如水輪機（water turbine）、泵（pump）、送風機（壓縮機）（blower, compressor）、蒸汽輪機（steam turbine）、與氣輪機（gas turbine）等。水輪機係將流體之位能轉換成機械功之機械；而泵與送風機（壓縮機）係將機械功轉換成流體壓力能之機械。蒸汽輪機與氣輪機則均係將流體所具有之能量(主要爲熱能)轉換成機械功,不屬於一般所謂的流體機械之範圍，另有專門書籍討論；本書係針對通稱之流體機械（包括水輪機、泵、送風機）作詳細說明。

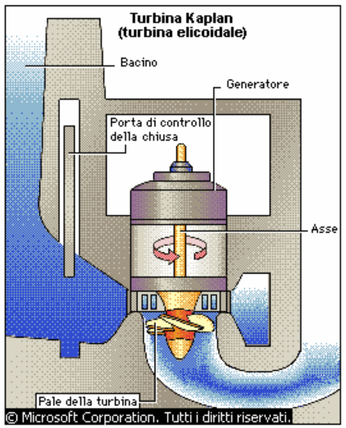


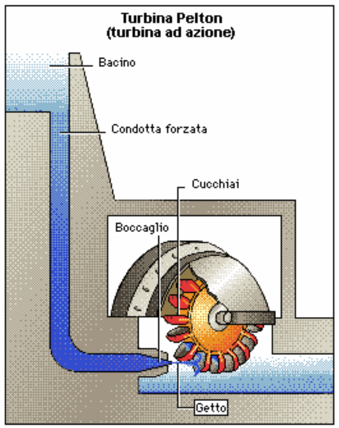
(a) 葉片排列圓周上之葉輪 (b) 葉片排列於圓筒面上之葉輪

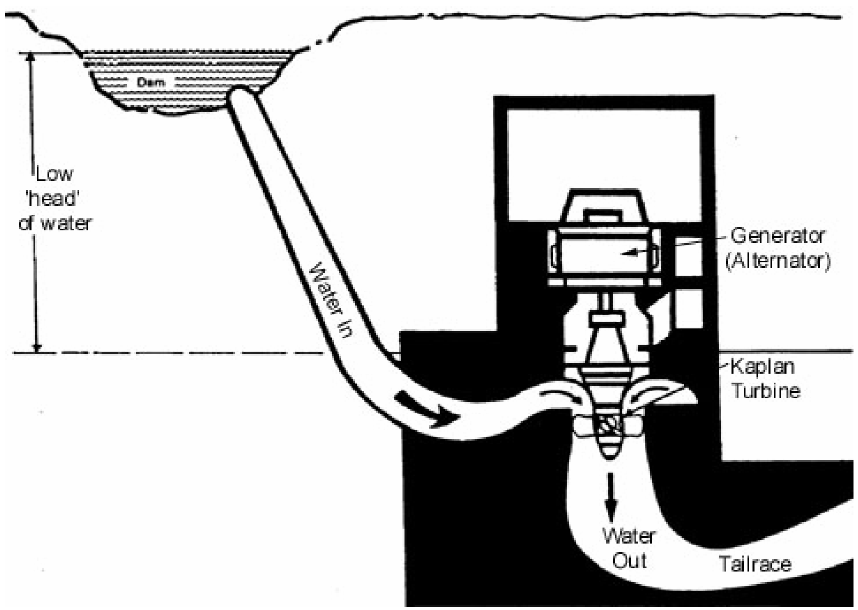
Pump



Water turbine







相似定理

目 的:

　　工程上常使用小尺寸的模型來協助瞭解複雜的問題，因尺寸較小的模型所需之經費較原型為小，故飛機、車輛、火箭、輪船、港口等設計、製造過程，常先以模型代替欲製造的原型加以實驗，性能達到預期目標之後才正式生產。

Similitude: Similarity of behavior of different systems.

A380 AIRBUS
380


Image © 2006 DigitalGlobe


Wind 1.5

GHAZI BAROTHA HYDRO-PROJECT
WATER AND POWER DEVELOPMENT
CONTRACT ME - 01
FRANCIS RUNNER 295 MW - 100 rpm
More. Powerful. Solutions.

Similarity Law 相似定理

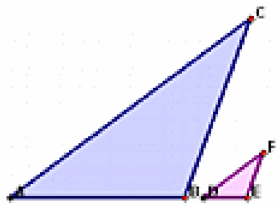
從模型(model)實驗所得到的數據，欲正確地應用到原型(Prototype)上，兩者間需符合三個基本條件：(1)幾何相似；(2)運動相似；(3)動力相似。

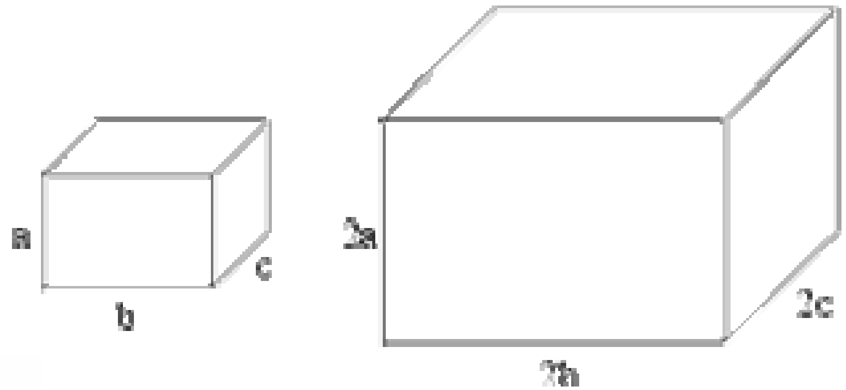
幾何相似：模型與原型之所有各對應部份之幾何形狀必須相同，即**模型與原型間對應部分的線性因次之比必須為常數**，包括粗糙度在內。

運動相似：模型與原型間運動情形應相似，**即兩者間流體流動的型式，如流線、速度、加速度之比均為一常數**。

動力相似：為力的相似，**系作用於模型與原型對應位置處之流體質點或對應邊界表面所有力之比均為常數**。動力相似包括下列各項：

(1) **幾何相似：**爲模型與原型之所有各對應部份之幾何形狀必須確實相同，故模型與原型間對應部份的線性因次(linear dimension)之比必須爲常數，即





$$L_P = \lambda_g L_m$$

P: prototype

M: model

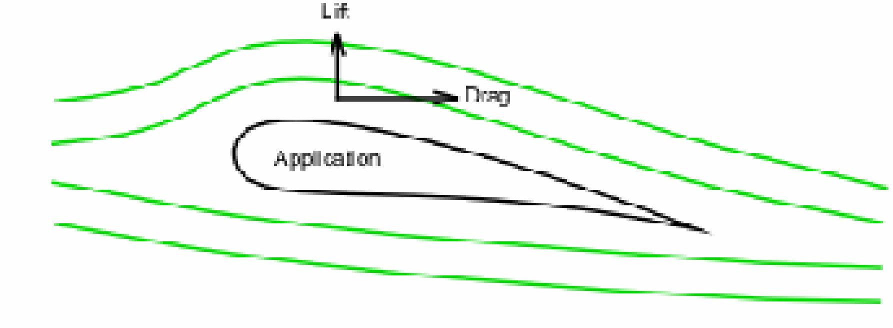
$$\frac{L_p}{L_m} = \lambda_l$$ 尺度比(scale ratio)

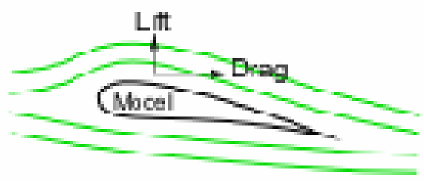
因尺度的縮小等因素，小尺寸之模型表面常較其原體爲光滑，須用人工糙率，如以水泥漿塗成粗糙面，或用砂石粒代表模型之糙體等以校正，故幾何相似性應包括粗糙度在內。

Kinematic similarity(運動相似)

爲其運動情形應相似。爲滿足模型及其原體之運動相似，流體流動之型式，如流線等亦須有幾何相似，故模型及原型二者間對應位置之流體速度、加速度之比亦均爲一常數，即

•**Kinematic similarity** - Fluid flow of both the model and real application must undergo similar time rates of change motions. (fluid streamlines are similar)



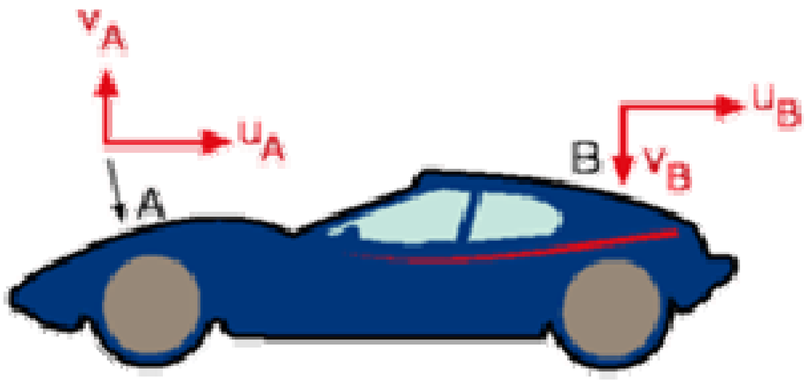


$$\frac{V_{1p}}{V_{1m}}=\frac{V_{2p}}{V_{2m}}=\lambda_v=\frac{L_p/T_p}{L_m/T_m}=\frac{L_p}{L_m}\times\frac{T_m}{T_p}=\lambda_g\frac{T_m}{T_p}=\text{常數} \qquad (5.2)$$

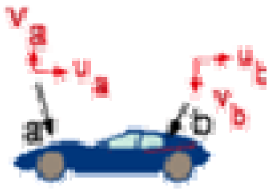
$$\frac{a_{1p}}{a_{1m}}=\frac{a_{2p}}{a_{2m}}=\lambda_a=\frac{L_p/T_p^2}{L_m/T_m^2}=\frac{L_p}{L_m}\times\frac{T_m^2}{T_p^2}=\lambda_g\frac{T_m^2}{T_p^2}=\text{常數} \qquad (5.3)$$

Dynamic similarity (動力相似)

動力相似：為力的相似，係指作用於模型與原型對應位置處之流體質點或對應邊界表面所有力之比均為定值。因作用於流體質點上之力可分為壓力F_P，重力F_G，黏滯力F_V，彈性力F_E及表面張力F_T等，此諸力的合力等於作用於流體質點之慣性力Ma，即



Prototype



Model

Dynamic similarity -
Ratios of all forces acting on corresponding fluid particles and boundary surfaces in the two systems are constant.

$$\sum F = Ma = [\rho][L^3]\left[\frac{V^2}{L}\right] = [\rho][V^2][L^2]$$

$$F_P = PA = [P][L^2]$$

$$F_G = Mg = [\rho][L^3][g]$$

$$F_V = \tau A = \mu \frac{du}{dy} A = [\mu]\frac{[V]}{[L]}[L^2] = [\mu][V][L]$$

$$F_E = E \cdot A = [E_V][L^2]$$

$$F_T = \sigma L = [\sigma][L]$$

E：容積彈性模數

σ：表面張力

L：長度

$$\Sigma F = F_p + F_G + F_V + F_E + F_T = Ma \qquad (5.4)$$

爲使模型和原型間能有完全的動力相似，不僅在此兩種流型內慣性力之比必須等於其合力之比，即

$$\frac{(\Sigma F)_p}{(\Sigma F)_m} = \frac{(F_P + F_G + F_V + F_E + F_T)_p}{(F_P + F_G + F_V + F_E + F_T)_m} = \frac{(M \cdot a)_p}{(M \cdot a)_m} \qquad (5.5)$$

且其慣性力之比必須與個別分力之比相等，即

$$\frac{(\Sigma F)_p}{(\Sigma F)_m}=\frac{(F_P)_p}{(F_P)_m}=\frac{(F_G)_p}{(F_G)_m}=\frac{(F_V)_p}{(F_V)_m}=\frac{(F_E)_p}{(F_E)_m}=\frac{(F_T)_p}{(F_T)_m} \tag{5.6}$$

$$=\frac{(M\cdot a)_p}{(M\cdot a)_m}$$

$$=\frac{\rho_p\cdot\dfrac{L_p}{T_p^2}}{\rho_m\dfrac{L_m}{T_m^2}}=\frac{L_p}{L_m}\cdot\frac{\rho_p}{\rho_m}\left(\frac{T_m}{T_p}\right)^2=\lambda_g\frac{\rho_p}{\rho_m}\left(\frac{T_m}{T_p}\right)^2 \tag{5.7}$$

（以單位體積力而言）

因作用於流體質點上之力可分

為壓力F_P ，重力F_G ，黏滯力F_V ，彈性力F_E及表面張力F_T等

的合力等於作用於流體質點之慣性力Ma

$$\Sigma F = F_p + F_G + F_V + F_E + F_T = Ma$$

$$\Sigma F = Ma = [\rho][L^3][\frac{V^2}{L}] = [\rho][V^2][L^2]$$

$$F_P = PA = [P][L^2]$$

$$F_G = Mg = [\rho][L^3][g]$$

$$F_V = \tau A = \mu \frac{du}{dy} A = [\mu]\frac{[V]}{[L]}[L^2] = [\mu][V][L]$$

$$F_E = E \cdot A = [E_V][L^2]$$

$$F_T = \sigma L = [\sigma][L]$$

E：容積彈性模數

σ ：表面張力

L：長度

$F_P = PA = [P][L^2]$ $F_G = Mg = [\rho][L^3][g]$ $F_E = E \cdot A = [E_V][L^2]$

$\Sigma F = Ma = [\rho][V^2][L^2]$ $F_V = \tau A = [\mu][V][L]$ $F_T = \sigma L = [\sigma][L]$

$$\frac{(Ma)_p}{(Ma)_m} = \frac{(F_P)_p}{(F_P)_m} \text{ 或 } \left(\frac{F_P}{Ma}\right)_p = \left(\frac{F_P}{Ma}\right)_m \text{ 即 } \left[\frac{P}{\rho V^2}\right]_p = \left[\frac{P}{\rho V^2}\right]_m$$

$$\frac{(Ma)_p}{(Ma)_m} = \frac{(F_G)_p}{(F_G)_m} \text{ 或 } \left(\frac{Ma}{F_G}\right)_p = \left(\frac{Ma}{F_G}\right)_m \text{ 即 } \left[\frac{gL}{V^2}\right]_p = \left[\frac{gL}{V^2}\right]_m$$

$$\frac{(Ma)_p}{(Ma)_m} = \frac{(F_V)_p}{(F_V)_m} \text{ 或 } \left(\frac{Ma}{F_V}\right)_p = \left(\frac{Ma}{F_V}\right)_m \text{ 即 } \left[\frac{\rho VL}{\mu}\right]_p = \left[\frac{\rho VL}{\mu}\right]_m$$

$$\frac{(Ma)_p}{(Ma)_m} = \frac{(F_E)_p}{(F_E)_m} \text{ 或 } \left(\frac{Ma}{F_E}\right)_p = \left(\frac{Ma}{F_E}\right)_m \text{ 即 } \left[\frac{\rho V^2}{E_V}\right]_p = \left[\frac{\rho V^2}{E_V}\right]_m$$

$$\frac{(Ma)_p}{(Ma)_m} = \frac{(F_T)_p}{(F_T)_m} \text{ 或 } \left(\frac{Ma}{F_T}\right)_p = \left(\frac{Ma}{F_T}\right)_m \text{ 即 } \left[\frac{\rho LV^2}{\sigma}\right]_p = \left[\frac{\rho LV^2}{\sigma}\right]_m$$

$$\left(\frac{F_P}{Ma}\right)_p=\left(\frac{F_P}{Ma}\right)_m \text{ 即 } \left[\frac{P}{\rho V^2}\right]_p=\left[\frac{P}{\rho V^2}\right]_m$$

$$\left[\frac{V}{\sqrt{P/\rho}}\right]_p=\left[\frac{V}{\sqrt{P/\rho}}\right]_m \text{，即 } [N_E]_p=[N_E]_m$$

$N_E=\dfrac{V}{\sqrt{P/\rho}}$ 稱爲歐拉數(Euler number)

是流體慣性力對壓力強度之比

值的平方根,於具有壓力梯度存在之流動此歐拉數十分重要

$$\Sigma F = Ma = [\rho][V^2][L^2] \qquad F_G = Mg = [\rho][L^3][g]$$

$$\left(\frac{Ma}{F_G}\right)_p = \left(\frac{Ma}{F_G}\right)_m \text{ 即 } \left[\frac{gL}{V^2}\right]_p = \left[\frac{gL}{V^2}\right]_m$$

$$\left[\frac{V}{\sqrt{gL}}\right]_p = \left[\frac{V}{\sqrt{gL}}\right]_m \text{ ，即 } [N_F]_p = [N_F]_m$$

$N_F = \dfrac{V}{\sqrt{gL}}$ 稱爲 **福祿數**（Froude number）

是流體的慣性力與重力比值的平方根，在藉重力流動之明渠，此福祿數甚爲重要

Froude number is a measure of the ratio of the inertia force on an element of fluid to the weight of the element. It will generally be important in problems involving flows with free surfaces since gravity principally affects this type of flow. Typical problems would include the study of the flow of water around ships (with the resulting wave action) or flow through rivers or open conduits.

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}} \qquad \frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} = \sqrt{\lambda_\ell}$$

$$\Sigma F=Ma=[\rho][V^2][L^2] \qquad F_V=\tau A\doteq[\mu][V][L]$$

$$\left(\frac{Ma}{F_V}\right)_p=\left(\frac{Ma}{F_V}\right)_m \text{ 即 } \left[\frac{\rho VL}{\mu}\right]_p=\left[\frac{\rho VL}{\mu}\right]_m$$

$$\left[\frac{\rho VL}{\mu}\right]_p=\left[\frac{\rho VL}{\mu}\right]_m \text{，即 } [N_R]_p\doteq[N_R]_m$$

$$\boxed{N_R=\frac{\rho VL}{\mu}\text{ 爲雷諾數}}$$

是流體慣性力與黏滯力之比，在任何眞實流體運動中雷諾數將影響流體的運動狀態

$$\frac{\rho_m V_m \ell_m}{\mu_m}=\frac{\rho V\ell}{\mu} \qquad \frac{V_m}{V}=\frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{\ell}{\ell_m}$$

$$\left(\frac{Ma}{F_E}\right)_p \doteq \left(\frac{Ma}{F_E}\right)_m \text{即} \left[\frac{\rho V^2}{E_V}\right]_p = \left[\frac{\rho V^2}{E_V}\right]_m$$

$$\left[\frac{V}{\sqrt{E_V/\rho}}\right]_p = \left[\frac{V}{\sqrt{E_V/\rho}}\right]_m \text{，即} \left[N_M\right]_p = \left[N_M\right]_m$$

$$\boxed{N_M = \frac{V}{\sqrt{E_V/\rho}} \text{稱爲}\textbf{馬赫數}\text{(Mach number)}}$$

是流體慣性力與彈性力比值的平方根，

亦等於流體運動速度與聲音在流體中傳播速度比，

$$\left(\frac{Ma}{F_T}\right)_p = \left(\frac{M_a}{F_T}\right)_m \text{ 即 } \left[\frac{\rho L V^2}{\sigma}\right]_p = \left[\frac{\rho L V^2}{\sigma}\right]_m$$

$$\left[\frac{V}{\sqrt{\sigma/\rho L}}\right]_p = \left[\frac{V}{\sqrt{\sigma/\rho L}}\right]_m \text{，即 } [N_W]_p = [N_W]_m$$

$$\boxed{N_W = \frac{V}{\sqrt{\sigma/\rho L}} \text{ 稱為韋伯數 (Weber number)}}$$

是慣性力對表面張力比值的平方根，在應考慮表面張力之任何處所，如在空氣與水之界面會產生甚小之表面波動，此時韋伯數具有重要影響，但通常對流體運動影響甚微。

上述換句話說，欲使模型與原型滿足相似定律，除了兩者幾何相似及運動相似外，尚須使兩者中之歐拉數、福祿數、雷諾數、馬赫數及韋伯數完全相同，以獲致動力相似。因不可能有兩種流體含有所有之特性以滿足運動相似的五個條件，故除模型與原型之尺度比相同之情形外（亦即除二者之形狀及大小完全相同之情形外），無法獲致完全的動力相似，幸在大部份流體力學或工程問題內，每種問題常有二、三種上述之力對問題較為重要，其餘較不重要之力則可略去不計。

例題 5.1

試計算模型與原型尺寸之比為 $\frac{1}{10}$，動力相似由福祿數控制時，其相對應之速度比，加速度比，角速度比，流量比和時間比。

解：

因 $\dfrac{V_m}{\sqrt{gL_m}}=\dfrac{V_p}{\sqrt{gL_p}}$，所以

速度比：$\dfrac{V_m}{V_p}=\sqrt{\dfrac{L_m}{L_p}}=\sqrt{\dfrac{1}{10}}=0.316$（因 $g_m=g_p$）

時間比：$\dfrac{T_m}{T_p}=\dfrac{L_m/V_m}{L_p/V_p}=\dfrac{L_m}{L_p}\cdot\dfrac{V_p}{V_m}=\dfrac{1}{10}\sqrt{10}=0.316$

加速度比：$\dfrac{a_m}{a_p}=\dfrac{V_m/T_m}{V_p/T_p}=\dfrac{V_m}{V_p}\cdot\dfrac{T_p}{T_m}=1$

角速度比：$\dfrac{\omega_m}{\omega_p}=\dfrac{V_m/D_m}{V_p/D_p}=\dfrac{V_m}{V_p}\cdot\dfrac{D_p}{D_m}=\sqrt{\dfrac{L_m}{L_p}}\cdot\dfrac{L_p}{L_m}=\sqrt{\dfrac{L_p}{L_m}}$

$=\sqrt{10}=3.16$

流量比：$\dfrac{Q_m}{Q_p}=\dfrac{L_m^2}{L_p^2}\cdot\dfrac{V_m}{V_p}=(\dfrac{1}{10})^2\sqrt{\dfrac{1}{10}}=0.00316$

例題 5.3

有一法蘭西水輪機原型直徑爲 20 呎，今以比例尺爲 $\frac{1}{30}$ 之模型在落差爲 5 呎，流量 8 呎^3 / 秒，速度 350 rpm 下運轉，可產生 2.5 HP 之動力，試求相對應原型之流量、落差、動力及轉速。（設摩擦效應不計，動力相似由福祿數與歐拉數控制）。

解：

已知 $\frac{D_m}{D_p}=\frac{1}{30}$ ，$D_p=20'$

(a) 因 $(N_F)_m=(N_F)_p$ 即 $\frac{V_m}{\sqrt{gD_m}}=\frac{V_p}{\sqrt{gD_p}}$

所以 $\frac{V_m}{V_p}=\sqrt{\frac{D_m}{D_p}}$ ，

流量比 $\frac{Q_m}{Q_p}=\frac{D_m^2V_m}{D_p^2V_p}=(\frac{1}{30})^2\times\sqrt{\frac{1}{30}}=\frac{\sqrt{30}}{27000}=2.03\times10^{-4}$

即 $Q_p=\frac{Q_m}{2.03\times10^{-4}}=\frac{8}{2.03\times10^{-4}}=3.941\times10^4\ \mathrm{ft^3/sec}$

(b) 因 $(N_E)_p=(N_E)_m$ ，即 $\dfrac{P_p}{\rho_p V_p^2}=\dfrac{P_m}{\rho_m V_m^2}$

$$\frac{P_m}{P_p}=\frac{\rho_m V_m^2}{\rho_p V_p^2}=\frac{D_m}{D_p}=\frac{1}{30}\quad(\text{因 }\rho_p=\rho_m)$$

而落差比：$\dfrac{H_m}{H_p}=\dfrac{P_m/\gamma_m}{P_p/\gamma_p}=\dfrac{P_m}{P_p}=\dfrac{1}{30}\quad(\gamma_m=\gamma_p)$

所以 $H_p=30\cdot H_m=150\text{ft}$

(c) 動力比：$\dfrac{L_m}{L_p}=\dfrac{P_m D_m^2 V_m}{P_p D_p^2 V_p}=\dfrac{1}{30}\times\left(\dfrac{1}{30}\right)^2\times\sqrt{\dfrac{1}{30}}=6.77\times10^{-6}$

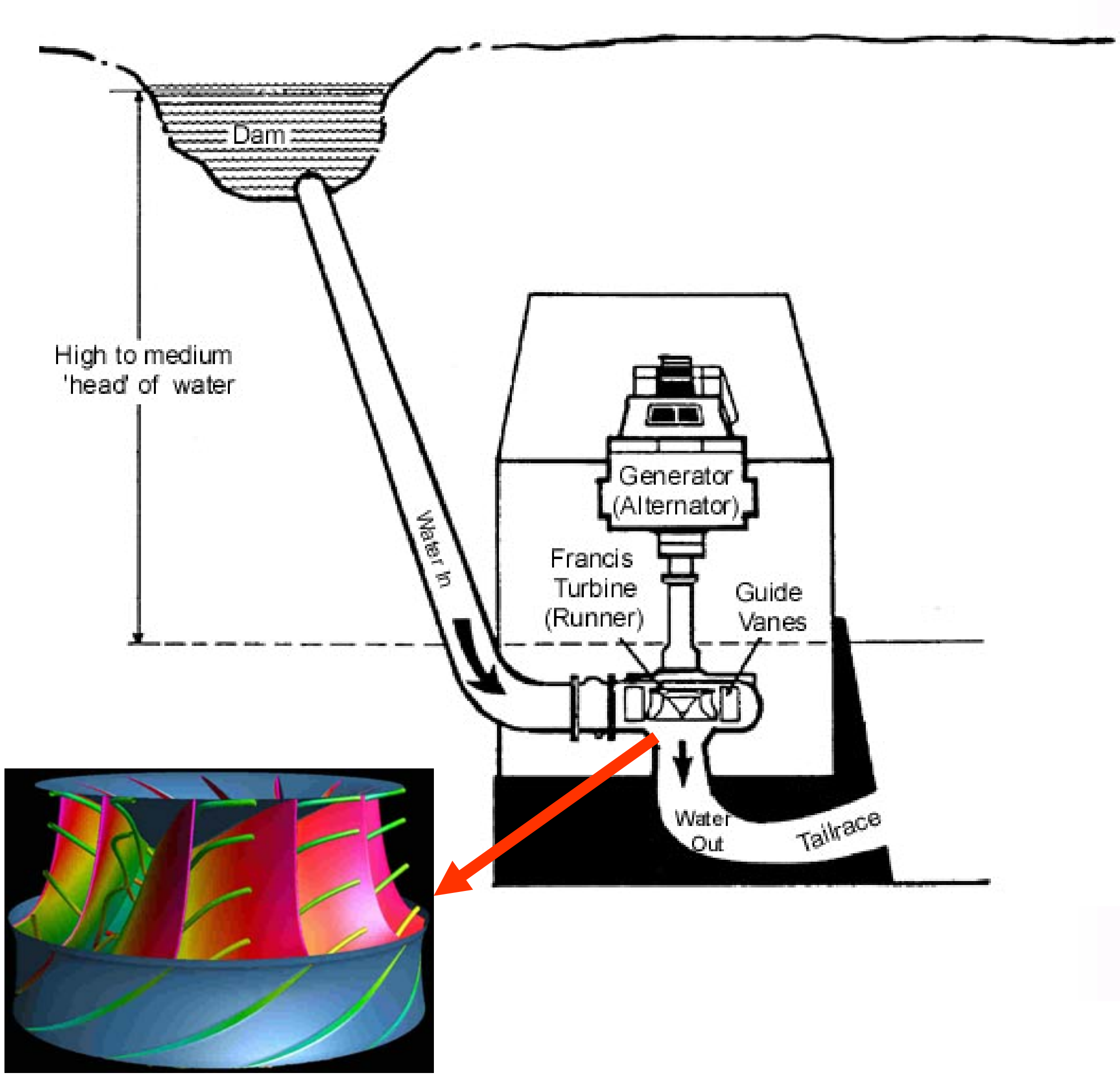
因此 $L_p=\dfrac{L_m}{6.77\times10^{-6}}=\dfrac{2.5}{6.77\times10^{-6}}=369276.22\text{HP}$

(d) 轉速比：$\dfrac{\omega_m}{\omega_p}=\dfrac{V_m/D_m}{V_p/D_p}=\dfrac{V_m}{V_p}\cdot\dfrac{D_p}{D_m}=\sqrt{\dfrac{D_m}{D_p}}\cdot\dfrac{D_p}{D_m}=\sqrt{\dfrac{D_p}{D_m}}=\sqrt{30}$

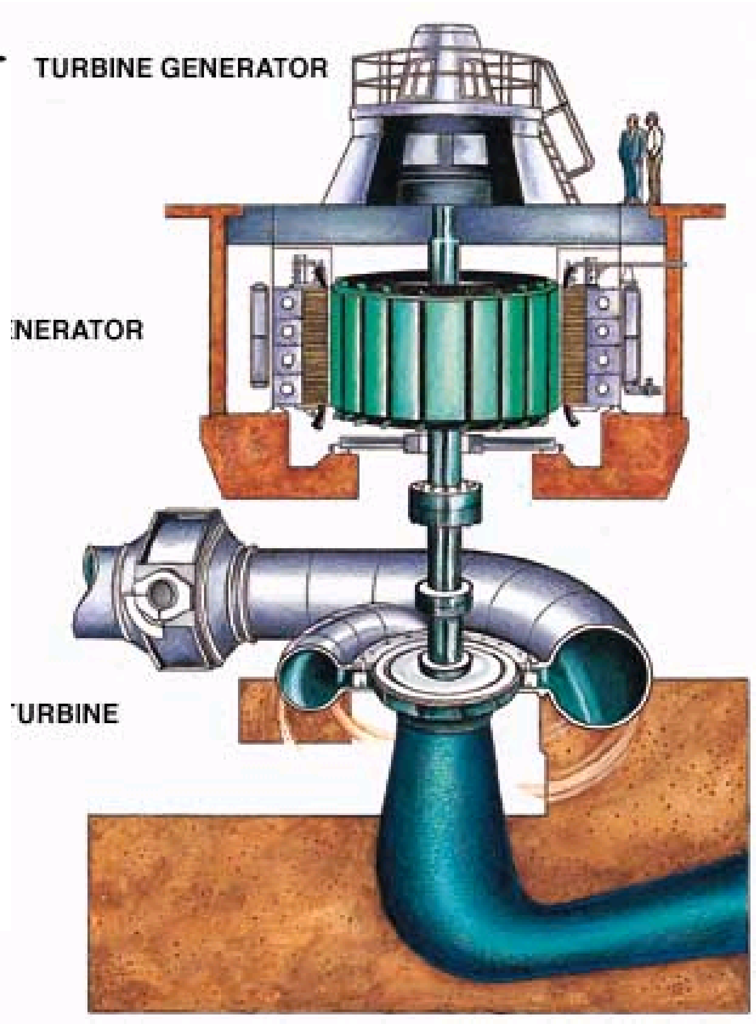
$=5.477$

因此 $\omega_p=\dfrac{350}{5.477}=63.9$ 轉/分

5.2 水輪機之相似定理



Dam
High to medium
'head' of water
Water In
Generator
(Alternator)
Francis
Turbine
(Runner)
Guide
Vanes
Water
Out
Tailrace



TURBINE GENERATOR

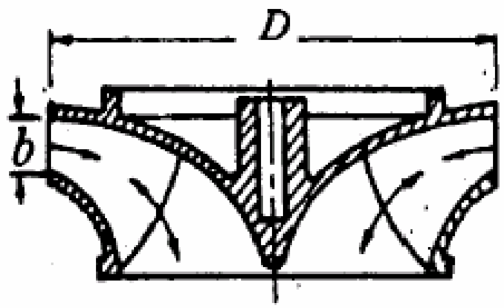


圖 5.1 水輪機轉輪或泵之葉輪

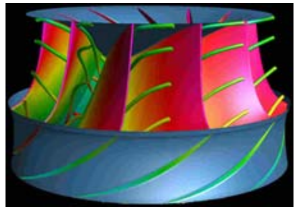
b 為轉輪入口處之寬度　D 為轉輪外徑

$$m=\frac{b}{D}$$ 寬徑比

k 為面積之修正因素（即自由流通所佔之部份約 0.95）

實際入口水流圓周面積 A_C 應為

$$\boxed{A_C = k\pi Db = k\pi mD^2}$$



H = 有效落差

C_r = 流量比（Flow ratio）

流入水流的徑向速度

$$\boxed{V_r = C_r\sqrt{2gH}}$$

水輪機之流量為

$$\boxed{\begin{aligned} Q &= V_r \cdot A_C = C_r\sqrt{2gH}\cdot k\pi mD^2 = (\sqrt{2g}\,k\pi mC_r)D^2\sqrt{H} \\ &= k_C D^2\sqrt{H} \qquad k_C = \sqrt{2g}\,k\pi\, mC_r \text{ 為一常數} \end{aligned}}$$

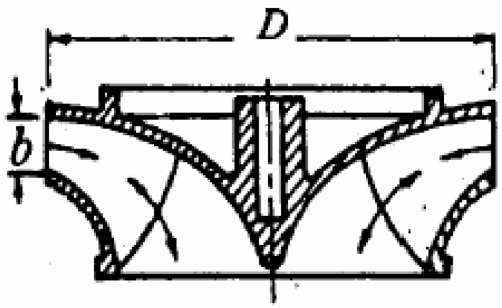


圖 5.1　水輪機轉輪或泵之葉輪

設轉輪外周的圓周速度

$$\boxed{U=\phi\sqrt{2gH}}$$

ϕ 為速度比（ speed ratie ）

轉輪每分鐘之轉速為

$$\boxed{N=\frac{60U}{\pi D}=\frac{60\phi\sqrt{2g}}{\pi}\left(\frac{\sqrt{H}}{D}\right)=K_V\frac{\sqrt{H}}{D}}$$

$$\sqrt{H}=\frac{ND}{K_V}$$

$K_V=\dfrac{60\phi\sqrt{2g}}{\pi}$ 為一常數

水輪機之流量為

$$Q\doteq k_C D^2\sqrt{H}$$

$$\boxed{Q=\frac{K_C}{K_V}(ND^3)=K_Q ND^3}$$

$K_Q=\dfrac{K_C}{K_V}$ 為一常數

因轉輪所受之轉矩(torque)T與D, U, Q三者乘積成比例

$$T \propto (Q.D.U) \qquad \boxed{T = K_T D^3 H}$$

.設轉輪的輸出動力(在泵的場合稱爲軸動力)爲L

$$L \propto N \cdot T \qquad \boxed{L = K_B D^2 H^{\frac{3}{2}}}$$

N, Q, T 及 L 均爲 D 和 H 的函數 且 常數 K_C , K_V , K_T 及 K_B 均爲定值

相似水輪機之關係式

$$\frac{Q}{D^2H^{1/2}}=\frac{Q'}{D'^2H'^{1/2}}=\cdots\cdots=K_C \qquad (5.23)$$

$$\frac{ND}{H^{1/2}}=\frac{N'D'}{H'^{1/2}}=\cdots\cdots=K_V \qquad (5.24)$$

$$\frac{T}{D^3H}=\frac{T'}{D'^3H'}=\cdots\cdots=K_T \qquad (5.25)$$

$$\frac{L}{D^2H^{3/2}}=\frac{L'}{D'^2H'^{3/2}}=\cdots\cdots=K_B \qquad (5.26)$$

(1) 兩部相似的水輪機在同一落差（$H=H'$）下運轉

$$\frac{Q}{Q'}=\left(\frac{D}{D'}\right)^2 \quad (5.27)$$

$$\frac{N}{N'}=\frac{D'}{D} \quad (5.28)$$

$$\frac{T}{T'}=\left(\frac{D}{D'}\right)^3 \quad (5.29)$$

$$\frac{L}{L'}=\left(\frac{D}{D'}\right)^2 \quad (5.30)$$

(2) 若兩部相似的水輪機其直徑相同（$D=D'$），但在不同之落差下運轉 則

$$\frac{Q}{Q'}=\left(\frac{H}{H'}\right)^{\frac{1}{2}} \tag{5.31}$$

$$\frac{N}{N'}=\left(\frac{H}{H'}\right)^{\frac{1}{2}} \tag{5.32}$$

$$\frac{T}{T'}=\frac{H}{H'} \tag{5.33}$$

$$\frac{L}{L'}=\left(\frac{H}{H'}\right)^{\frac{3}{2}} \tag{5.34}$$

例題 5.5

有效落差爲 5 m，流量爲 $100 m^3$ / sec，回轉速 80 rpm 的水輪機，效率爲 88%，今使效率維持一定，有效落差爲 6.5 m，試求此時水輪機的回轉速，流量與輸出動力（水比重量爲 1000 kgf / m^3）。

解：

因係同一水輪機，即 $D=D'$

由方程式 (5.32)，知運轉情況改變後之轉速爲

$$N=N'\left(\frac{H}{H'}\right)^{\frac{1}{2}}=80\left(\frac{6.5}{5}\right)^{\frac{1}{2}}=91.2\ \text{rpm}$$

由方程式 (5.31) 知流量變爲

$$Q=Q'\sqrt{\frac{H}{H'}}=100\cdot\sqrt{\frac{6.5}{5}}=114\,\text{m}^3/\text{sec}$$

又此設計點的輸出動力爲

$$L' = \frac{\eta Q' H'}{102}$$

$$= \frac{0.88 \times 1000 \times 100 \times 5}{102} = 4313.7 \text{ kw}$$

由方程式 (5.34) 得

$$L = L' \left(\frac{H}{H'}\right)^{\frac{3}{2}} \left(\frac{\eta}{\eta'}\right) = L' \left(\frac{H}{H'}\right)^{3/2} \quad (\eta = \eta')$$

$$= 4313.7 \times \left(\frac{6.5}{5}\right)^{\frac{3}{2}}$$

$$= 6393.9 \text{ kw}$$

5.3 泵之相似定理

● 水輪機的性能多由其落差大小來決定，其水流方向是由外向內，而泵的性能則通常由其轉速大小來決定，其水流方向與水輪機的水流方向剛好相反，由內向外，● 故水輪機以H表示Q,N,T及L，而泵則以N來表示Q,N,T及L等。 與水輪機相同，泵亦可建立相似定律 由(5.17)

設轉輪外周的圓周速度 $\boxed{U=\phi\sqrt{2gH}}$ ϕ 為速度比 (speed ratie)

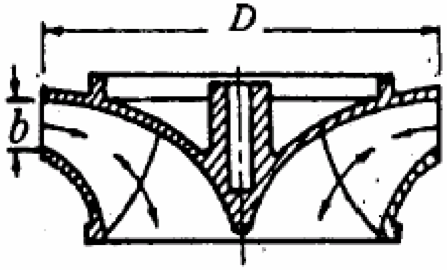
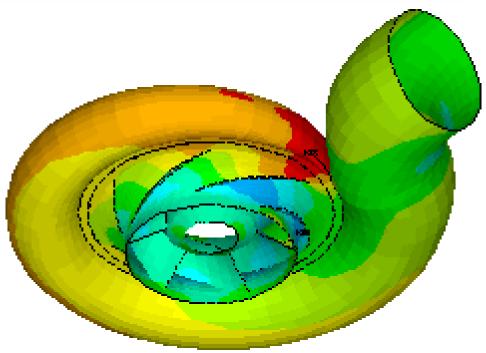


圖 5.1



$$\boxed{N=\frac{60U}{\pi D}=\frac{60\phi\sqrt{2g}}{\pi}\left(\frac{\sqrt{H}}{D}\right)=K_V\frac{\sqrt{H}}{D}}$$

$$\sqrt{H}=\frac{ND}{K_V}$$

$$\boxed{H=N^2D^2\left(\frac{1}{K_V}\right)^2 \text{ 即 } H=KN^2D^2}$$

Water turbine

$N, Q, T, L \propto D$和$\underline{H}$

$$\boxed{N} \boxed{= K_V \frac{\sqrt{H}}{D}}$$

$$\boxed{Q = \frac{K_Q}{K_V}(ND^3) = K_Q N D^3}$$

$$\boxed{T = K_T D^3 H}$$

$$\boxed{L = K_B D^2 H^{\frac{3}{2}}}$$

Pump

$H, Q, T, L \propto D$及$\underline{N}$

$$\boxed{H = K N^2 D^2}$$

$$\boxed{Q = \frac{K_Q}{K_V}(ND^3) = K_Q N D^3}$$

$$\boxed{T = K_T N^2 D^5}$$

$$\boxed{L = K_B N^3 D^5}$$

例題 5.6

設一離心泵的轉速爲 1250 rpm，其揚程爲 12 m，每分鐘排水量爲 2.5 m^3，葉輪直徑爲 25 cm，試計算另一相似泵的揚程及每分鐘的排水量。設其轉速爲 1000 rpm，葉輪直徑爲 20 cm。

解：

已知　$N = 1250$ rpm　　$H = 12$ m

　　　$Q = 2.5\ m^3/min$　　$D = 0.25$ m

又知　$N' = 1000$ rpm　　$D' = 0.2$ m

由（5.39）式知

$$\therefore \quad H' = \left(\frac{N'^2 D'^2}{N^2 D^2}\right) H = \left(\frac{1000^2 \times 0.2^2}{1250^2 \times 0.25^2}\right) \times 12 = 4.915\ \text{m}$$

$$Q' = Q\left(\frac{N'}{N}\right)\left(\frac{D'}{D}\right)^3$$

$$= 2.5\left(\frac{1000}{1250}\right)\left(\frac{0.2}{0.25}\right)^3$$

$$= 2.5\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)^3$$

$$= 1.024\,m^3/min$$

相似泵關係式 (Geometrically similar pumps) **Similarity Laws**

$$\frac{H}{D^2N^2}=\frac{H'}{D'^2N'^2} \tag{5.39}$$

$$\frac{Q}{ND^3}=\frac{Q'}{N'D'^3}=\cdots\cdots=K_Q \tag{5.40}$$

$$\frac{T}{N^2D^5}=\frac{T'}{N'^2D'^5}=\cdots\cdots=K_T \tag{5.41}$$

$$\frac{L}{N^3D^5}=\frac{L'}{N'^3D^5}=\cdots\cdots=K_B \tag{5.42}$$

Special Pump Scaling Laws *(pump affinity laws)*

(1) 若有兩部形狀相似，直徑相同 ($D=D'$) 之泵以不同轉速運轉，則

$N \neq N'$

$$\boxed{\begin{aligned} \frac{H}{H'} &= \frac{N^2}{N'^2} \\ \frac{Q}{Q'} &= \frac{N}{N'} \\ \frac{T}{T'} &= \left(\frac{N}{N'}\right)^2 \\ \frac{L}{L'} &= \left(\frac{N}{N'}\right)^3 \end{aligned}}$$

(5.43)

(5.44)

(5.45)

(5.46)

$$\left(\frac{L}{L'}\right)^2 = \left(\frac{T}{T'}\right)^3 = \left(\frac{Q}{Q'}\right)^6 = \left(\frac{H}{H'}\right)^3 \qquad (5.47)$$

例題 5.7

一單吸式渦卷泵的轉速爲 1450 rpm 。其特性如下：全揚程爲 25m ，流量爲 1.5m^3/min ，軸動力爲 8.7 kw ，試求轉速 750 rpm和 2940 rpm時之性能 。

解：

(1) 因爲使用相同的泵，即 $D=D'$

由 (5.39) 式知

$$H=H'\left(\frac{N}{N'}\right)^2=25\left(\frac{750}{1450}\right)^2=6.69$$

同理由 (5.44) 式及 (5.46) 式知

$$\text{流　量：}Q=Q'\left(\frac{N}{N'}\right)=1.5\left(\frac{750}{1450}\right)=0.776\ m^3/min$$

$$\text{軸動力：}L=L'\left(\frac{N}{N'}\right)^3=8.7\left(\frac{750}{1450}\right)^3=1.2\ kw$$

(2) 同理轉速爲 2940 rpm時

$$H=25\left(\frac{2940}{1450}\right)^2=102.8m$$

$$Q=1.5\,\frac{2940}{1450}=3.04\ m^3/min$$

$$L=8.7\left(\frac{2940}{1460}\right)^3=71.4\ kw$$

(2) 有兩部形狀相似，尺寸不同（$D \neq D'$）之泵，以相同轉速運轉 $N = N'$

$$\frac{H}{H'} = \frac{D^2}{D'^2} \qquad (5.48)$$

$$\frac{Q}{Q'} = \left(\frac{D}{D'}\right)^3 \qquad (5.49)$$

$$\frac{T}{T'} = \left(\frac{D}{D'}\right)^5 \qquad (5.50)$$

$$\frac{L}{L'} = \left(\frac{D}{D'}\right)^5 \qquad (5.51)$$

$$\frac{L}{L'} = \frac{T}{T'} = \left(\frac{Q}{Q'}\right)^{\frac{5}{3}} = \left(\frac{H}{H'}\right)^{\frac{5}{2}} \qquad (5.52)$$

例題 5.8

有一實際操作之泵，其全揚程為 241.4 m，流量為 $18.6 m^3/s$，轉速為 300 rpm，軸動力為 49000 kw，若維持幾何形狀相似，而將此泵縮小十倍，轉速則增加為 3000 rpm，試求此小泵的揚程、流量與軸動力。

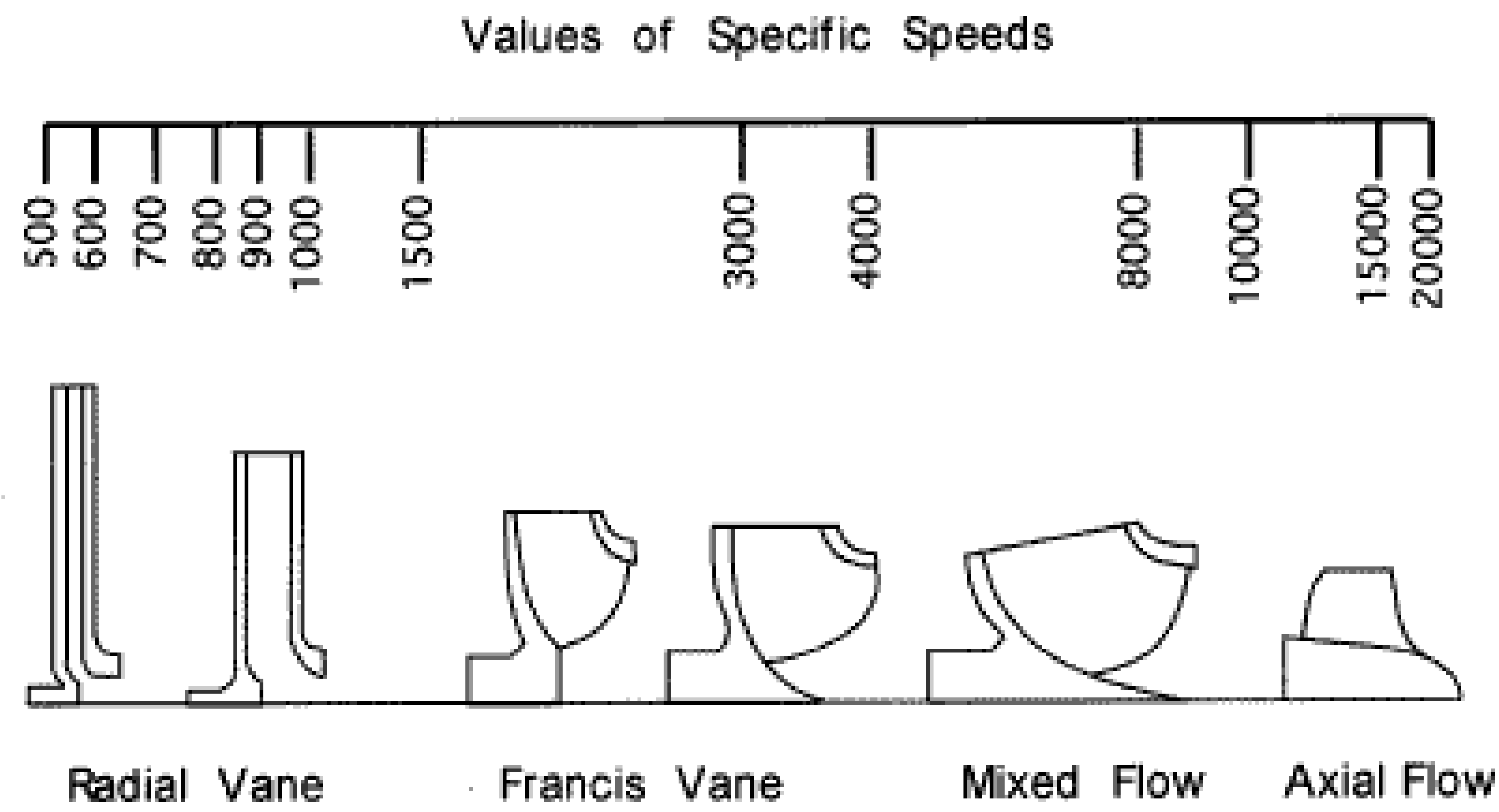
解： 由 (5.39) 式知小泵的揚程

$$H=H'\left(\frac{D}{D'}\right)^2\left(\frac{N}{N'}\right)^2=241.4\left(\frac{1}{10}\right)^2\left(\frac{3000}{300}\right)^2=241.4\,\text{m}$$

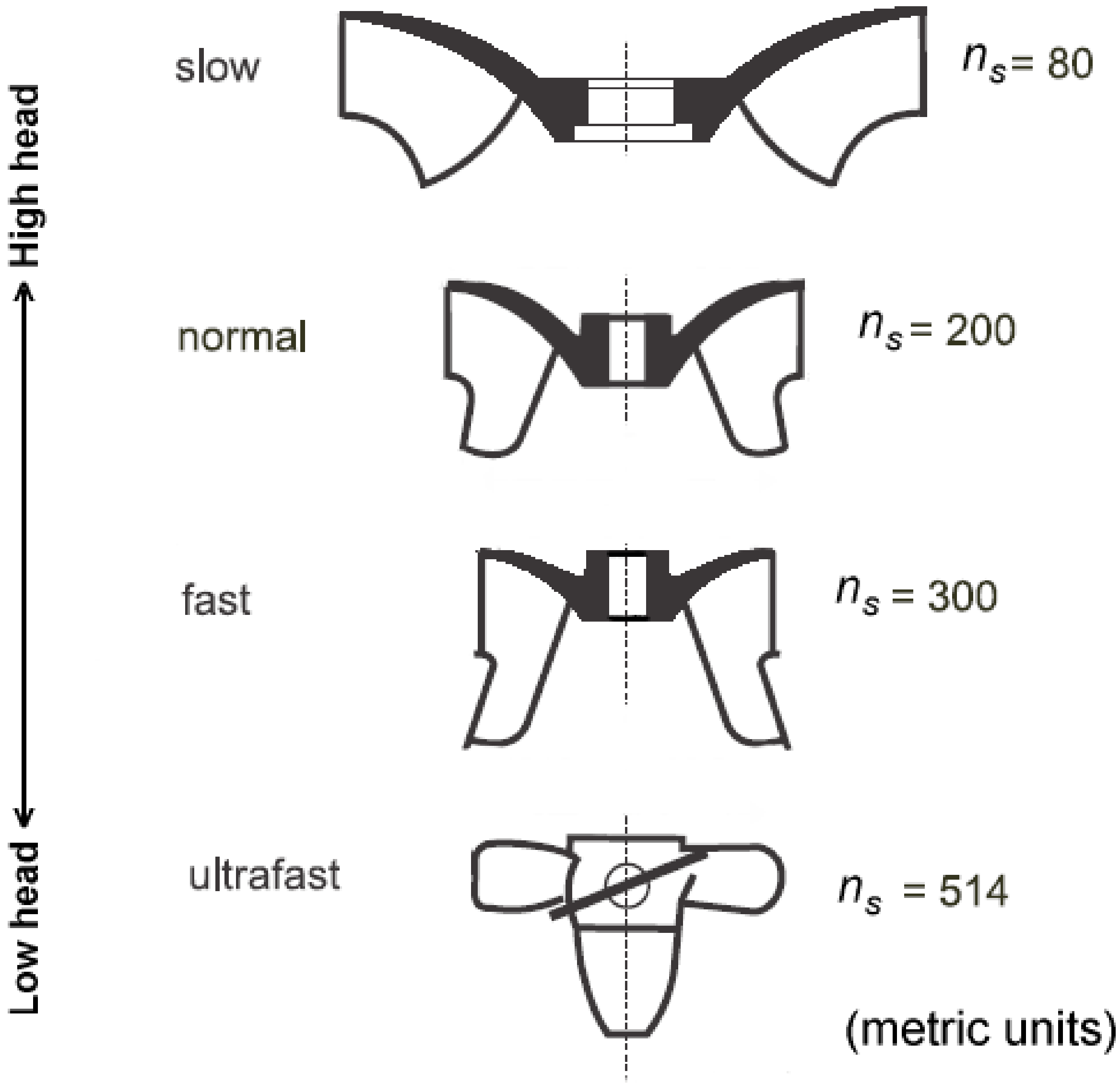
同理由 (5.40) 式及 (5.42) 式知

$$Q=Q'\left(\frac{N}{N'}\right)\left(\frac{D}{D'}\right)^3=18.6\times\left(\frac{3000}{300}\right)\left(\frac{1}{10}\right)^3\times 60=11.16\,\text{m}^3/\text{min}$$

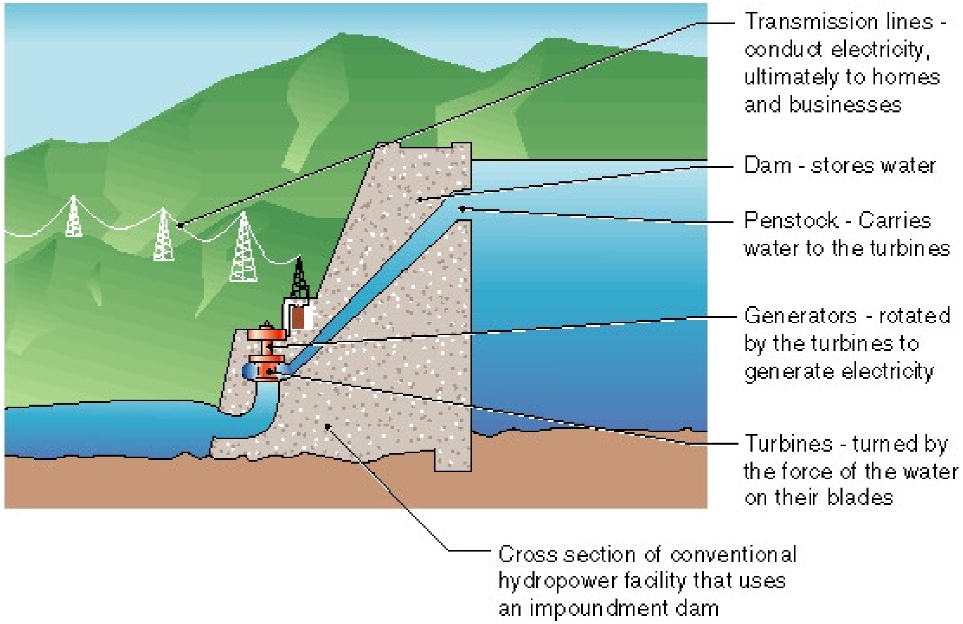
$$L=L'\left(\frac{N}{N'}\right)^3\left(\frac{D}{D'}\right)^5=49000\times\left(\frac{3000}{300}\right)^3\left(\frac{1}{10}\right)^5=490\,\text{kw}$$


Values of Specific Speeds
500
600
700
800
900
1000
1500
3000
4000
8000
10000
15000
20000
Radial Vane
Francis Vane
Mixed Flow
Axial Flow

Comparison of Turbine Shape vs. Specific Speed



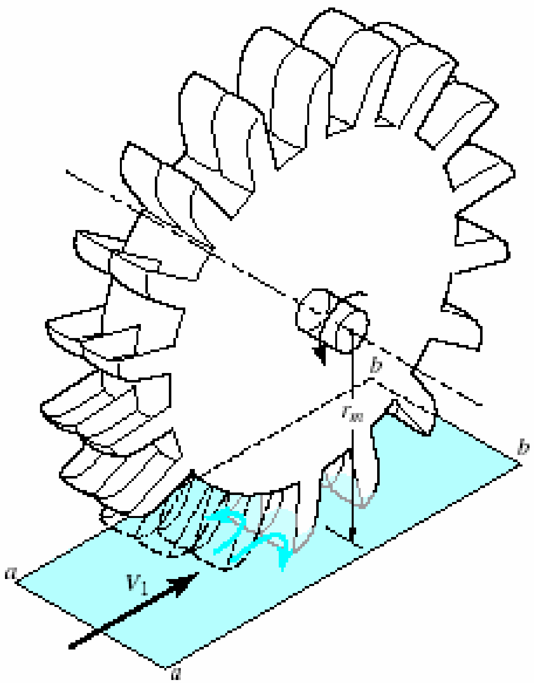
Chapter 6 Water Turbine

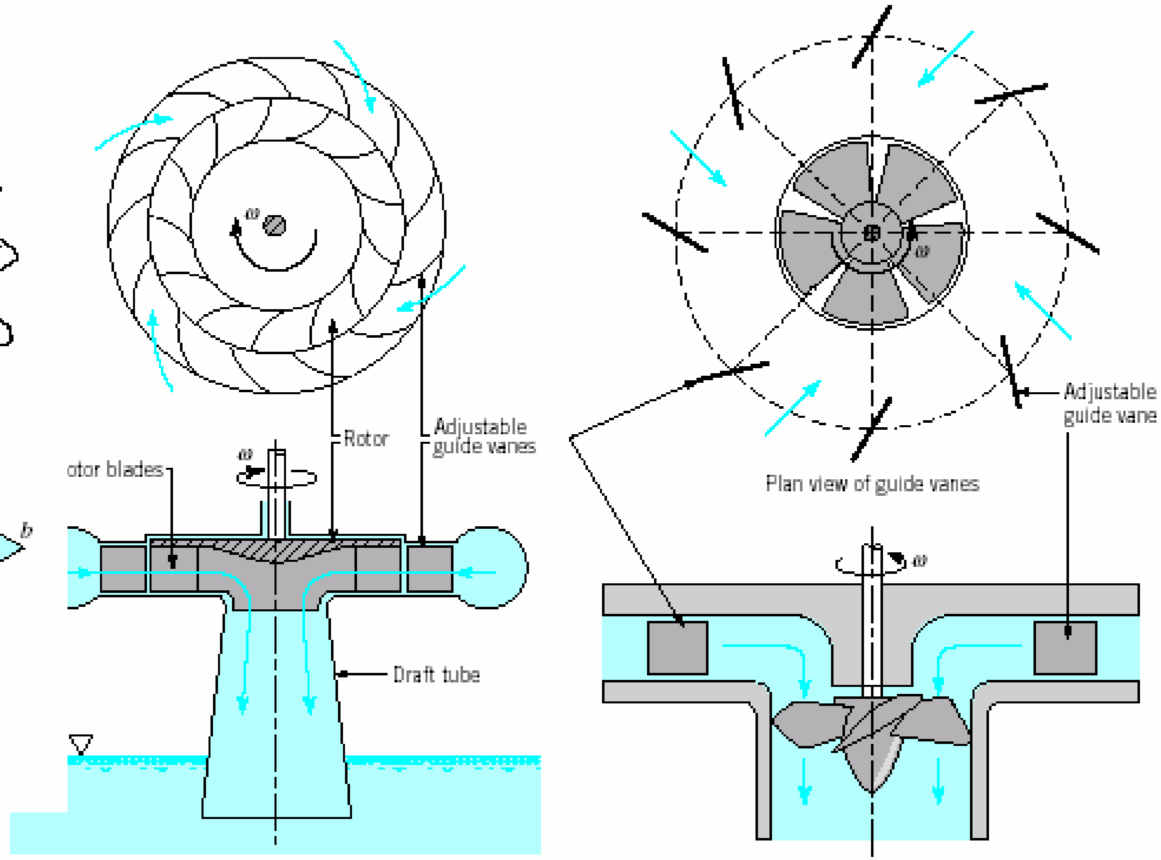


Classification of Water Turbine

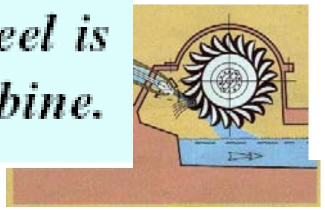
Impulse Turbines

Reaction Turbines





The Pelton wheel is an impulse turbine.



(a) Typical radial-flow Francis turbine, (b) typical axial-flow Kaplan tur

衝動式輪機（impulse turbine）
- 帕爾登水輪機（Pelton turbine）
- 橫流水輪機（cross-flow turbine）

適用於高落差的水源，其動作原理是以噴嘴將水之位能轉變爲動能而衝擊水輪機之輪葉，使之產生旋轉的機械能

反動式輪機（reaction turbine）
- 混流式（mixed-flow）
  - ——法式水輪機（Francis turbine）
- 斜流式（diagonal-flow）
  - ——達里斯水輪機（Deriaz turbine）
- 軸流式（axial-flow）
  - ——卡普蘭水輪機（Kaplan turbine）
  - 旋葉水輪機（propeller turbine）
- 泵水輪機（pump turbine）

適用於中、低落差的水源，其動作原理係將水壓入輪機內，再流過輪葉，將壓力能轉變爲動能而使轉輪旋轉

6.2 水輪機的要項

(a) 落差 (head)：

h_1 為上水路之損失落差

h_2 為水壓管（penstock）之損失落差

h_3 為放水路之損失落差及排水管之排棄落差

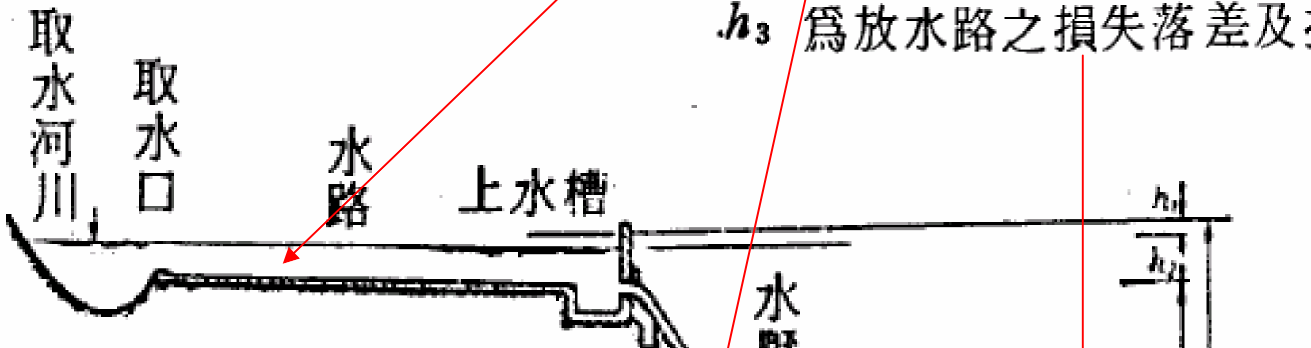
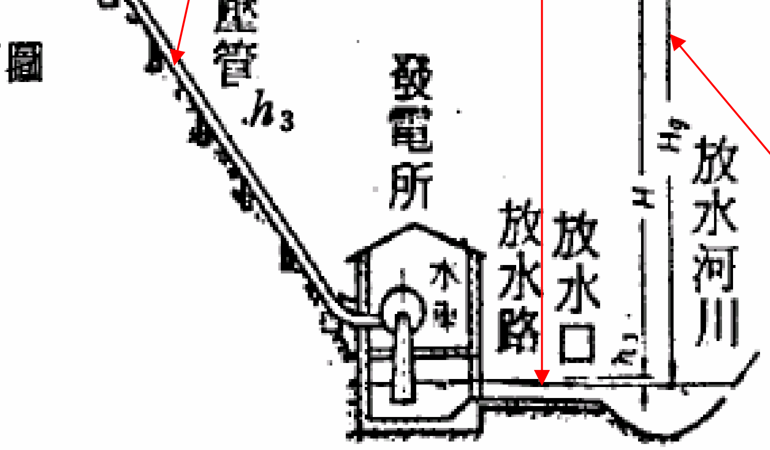
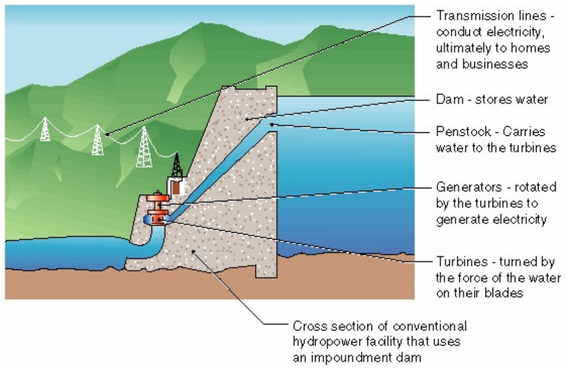



圖 6.1 水力發電廠之水路斷面圖



該取水河川之水面高度 H_g 即為自然落差（natural head）或總落差（gross head）

水輪機之有效落差 H（effective head） $H = H_g - (h_1 + h_2 + h_3)$

(b) 理論動力及效率：

水輪機所獲得之**理論動力**或**理論功率**(theoretical power)L_{th} 爲

$$L_{th}=\gamma QH\,[\,\mathrm{kgf\cdot m/sec}\,]=\frac{\gamma QH}{102}\,[\,\mathrm{kw}\,]$$

以公制馬力PS（1 PS=75 kgf・m/ sec ）表示爲

$$L_{th}=\frac{\gamma QH}{75}\,[\,\mathrm{PS}\,]$$

全效率 (total efficiency) η_t

$$\boxed{\eta_t = \frac{L}{L_{th}} = \eta_h \cdot \eta_v \cdot \eta_m}$$

η_h 水力效率 (hydraulic efficiency)

$$\eta_h = \frac{H - H_v}{H}$$

H: 實際有效落差

H_v 水力損失落差

如①水在水輪機外殼與導流葉片之水力損耗，

②水流入轉輪時的衝擊損耗及在轉輪內部的水力損失

③排水管內的水力損失及排棄損失

④洩漏損失

⑤機械摩擦損失

η_v 容積效率（volumetric efficiency）

$$\eta_v = \frac{Q-q}{Q}$$ q 爲洩漏（leakage）流量

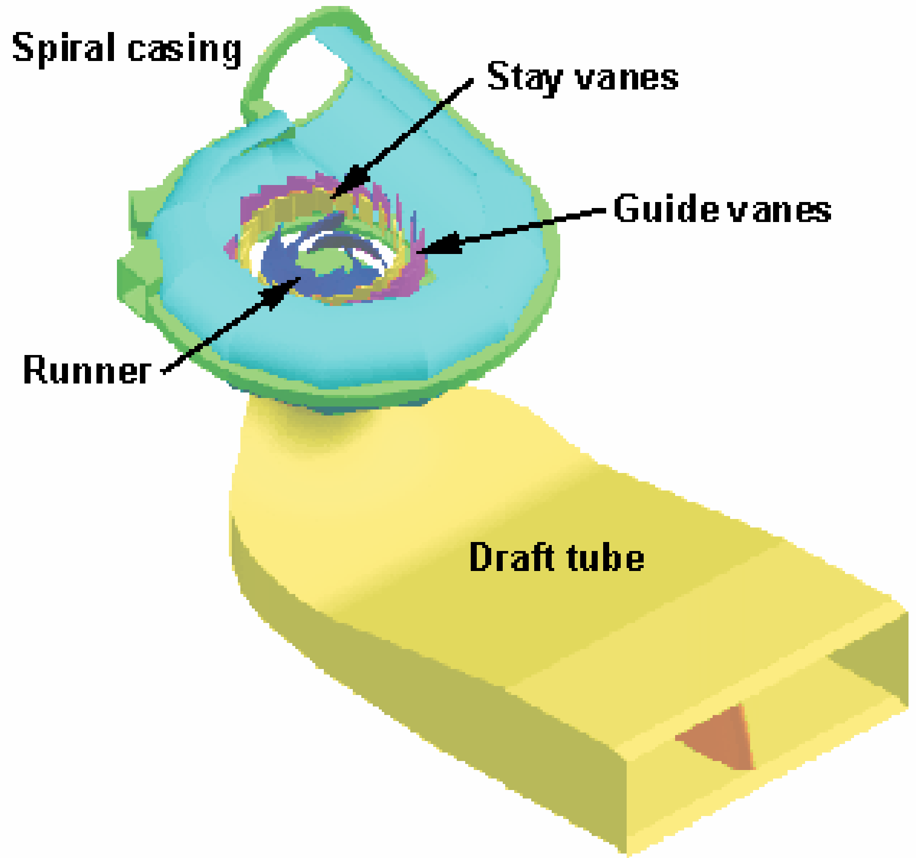
η_m 機械效率（mechanical efficiency）

$$\eta_m = \frac{L}{L_h} = \frac{L}{L+L_m} \quad \frac{\text{水輪機輸出的動力}}{\text{水頭給轉輪之水力動力}}$$

$$L_h = \gamma(Q-q)(H-H_v) \text{〔kgf·m/sec〕}$$

water turbine water wheel

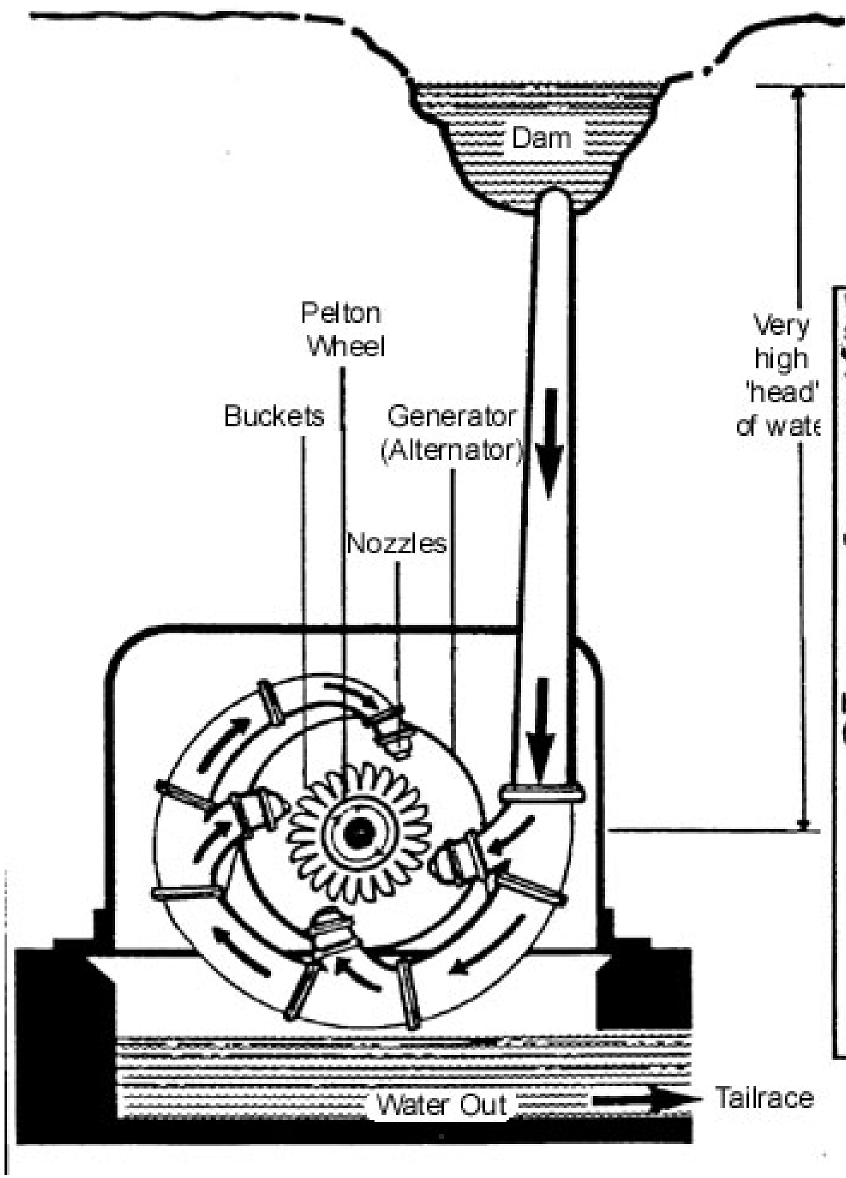
衝動式水輪機(impulse turbine)

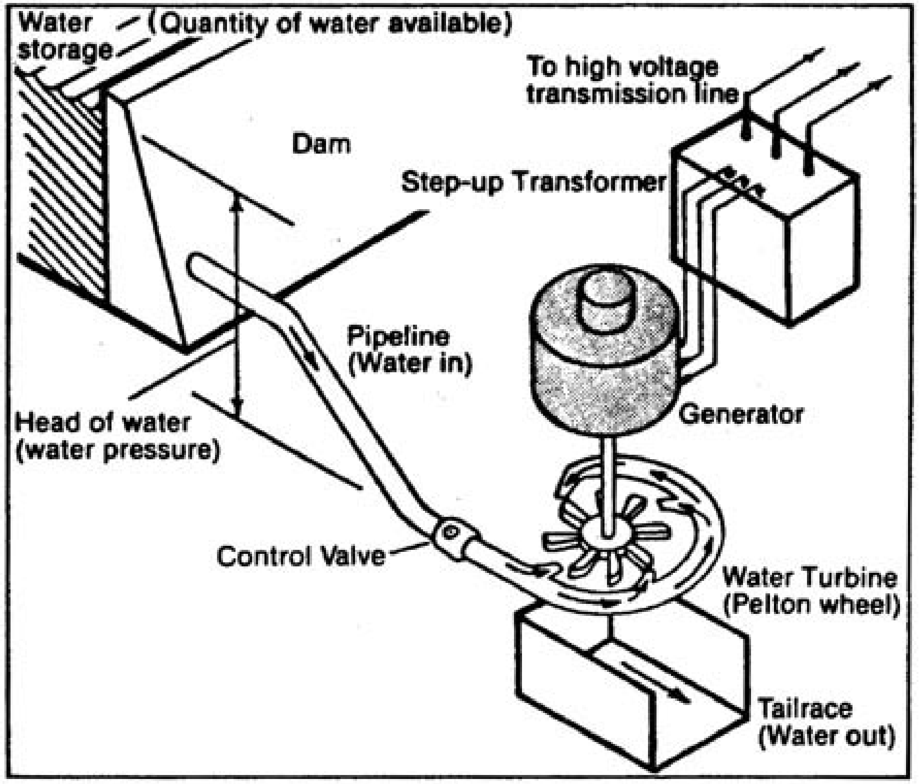


6.3 帕爾登水輪機 Impulse Turbine—Pelton Turbine

帕爾登水輪機主要用於高落差(200m以上),

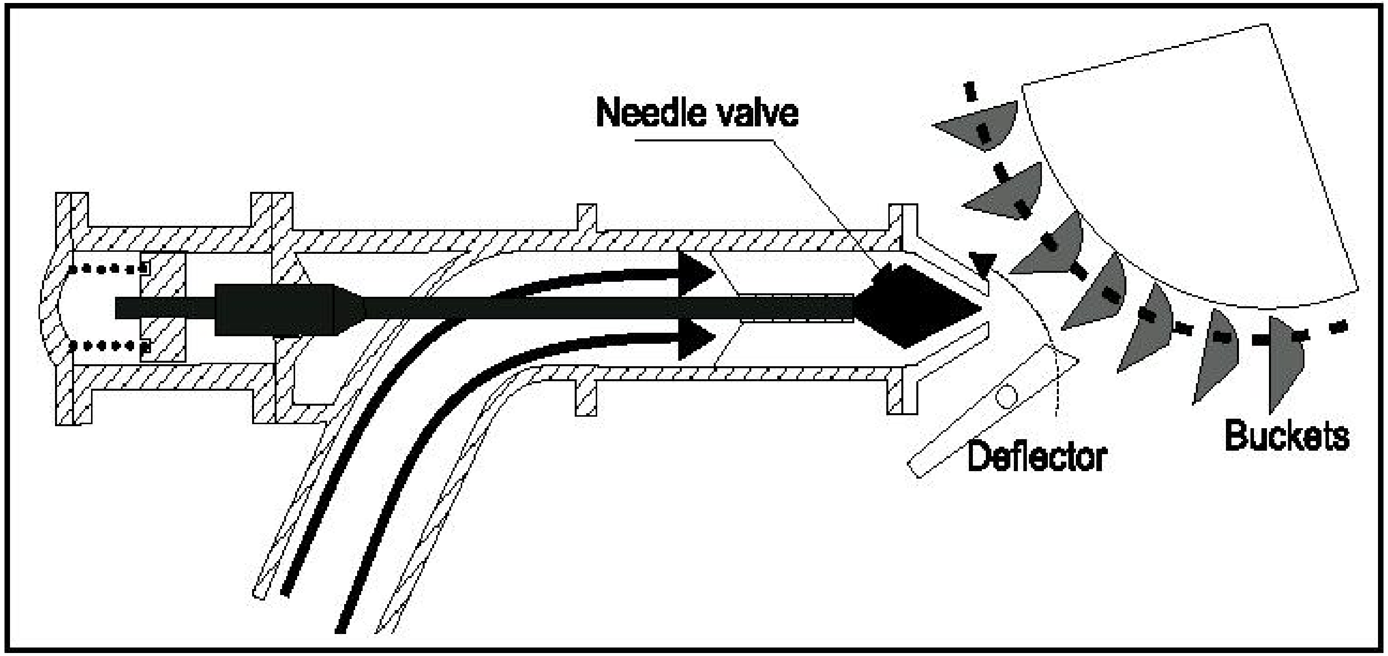
但 150m 以上之中落差 ,在小水量之情況下亦可使用。





The operating mechanism Pelton water turbine

當運作時水由上游水槽（head tank）經水壓管向下流至最下端的噴嘴，在此處即成爲一股射流，而Impulse Turbines 哥安裝之多枚箕斗（bucket）上噴擊，使轉輪高速旋轉。



Single runner two jets

6.3.1 帕爾登水輪機的分類與構造

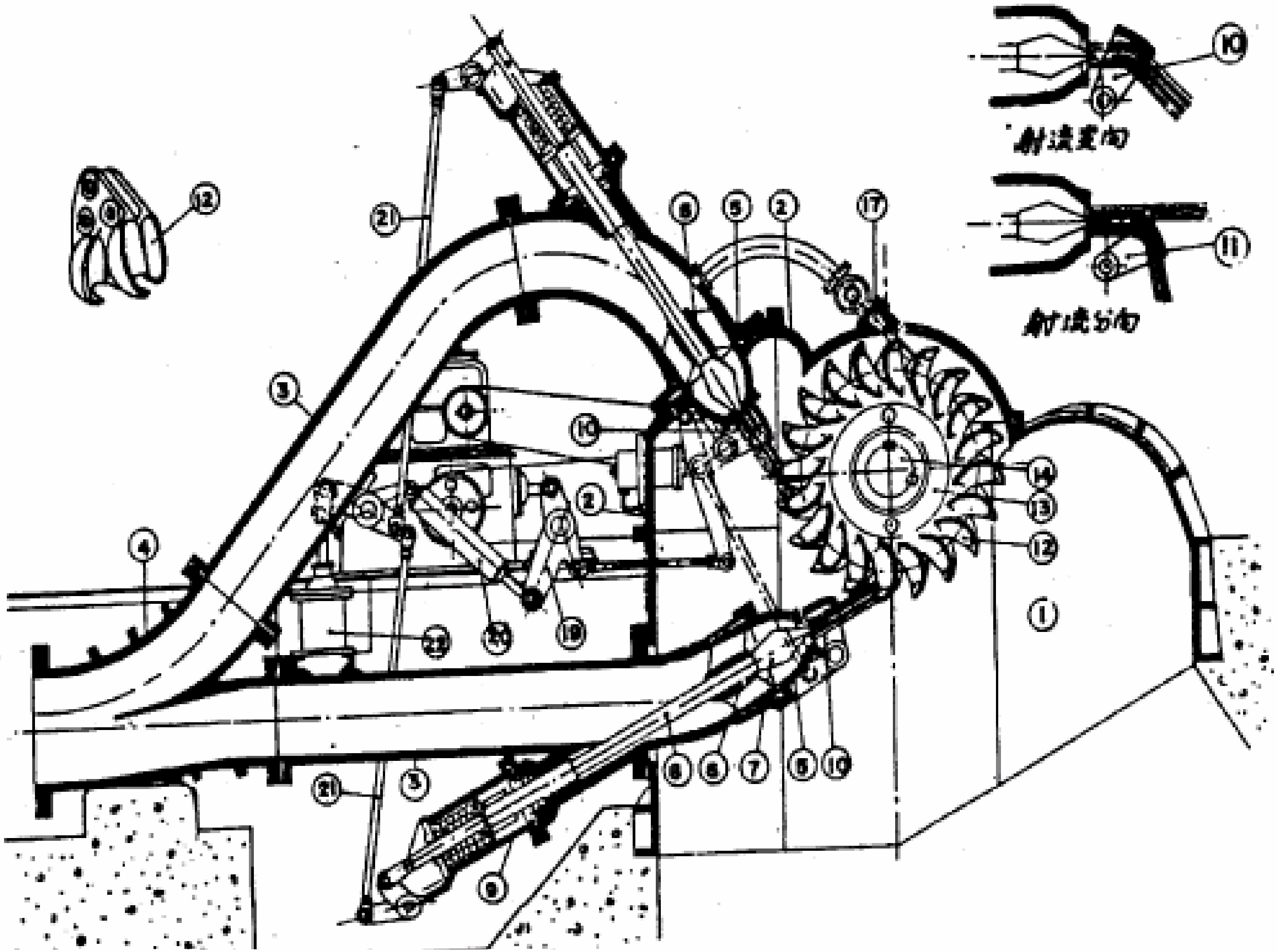


圖 6.3 爲橫軸單輪二射型的整體構造，其主要部份爲轉輪、箕斗(Bucket) 、噴嘴、外殼、偏流板 (reflector)、軸承及調速器等。

6.3.1 帕爾登水輪機的分類與構造

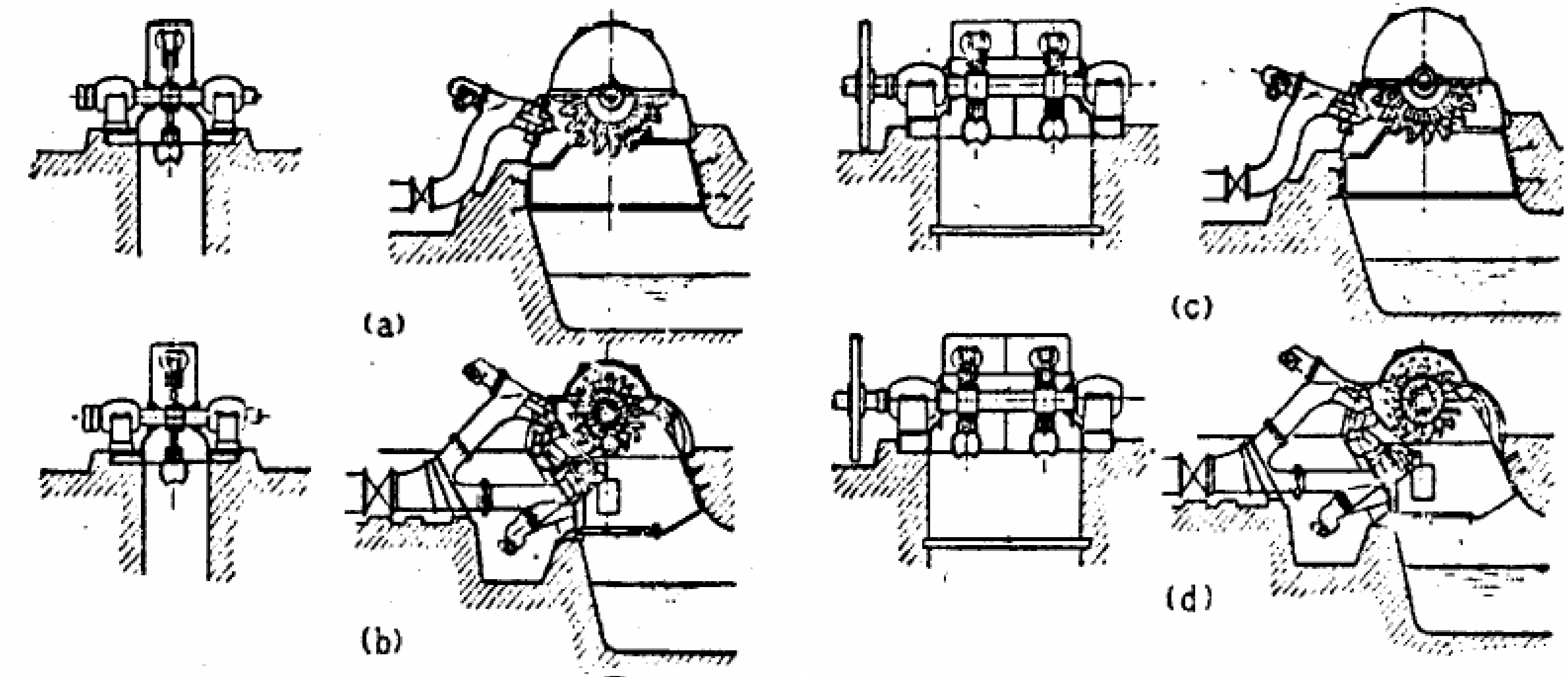
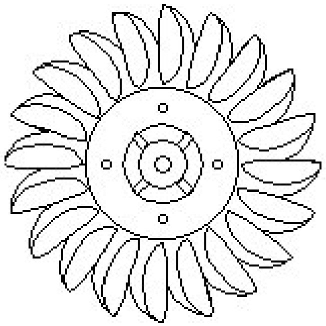
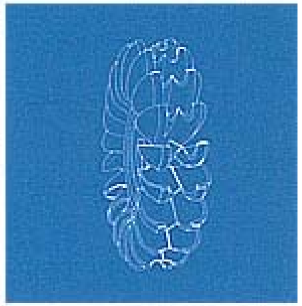


圖 6.2 爲各種形式的帕爾登水輪機、視轉輪、噴嘴 (nozzle) 的數量與配置分爲(a)橫軸單輪單射型 (single nuner single jet) 、(b)橫軸單輪二射型、(c)橫軸二輪二射型、(d) 橫軸二輪四射型等，並依容量大小有立軸式及橫軸式兩種 。

6.3.1 帕爾登水輪機的分類與構造

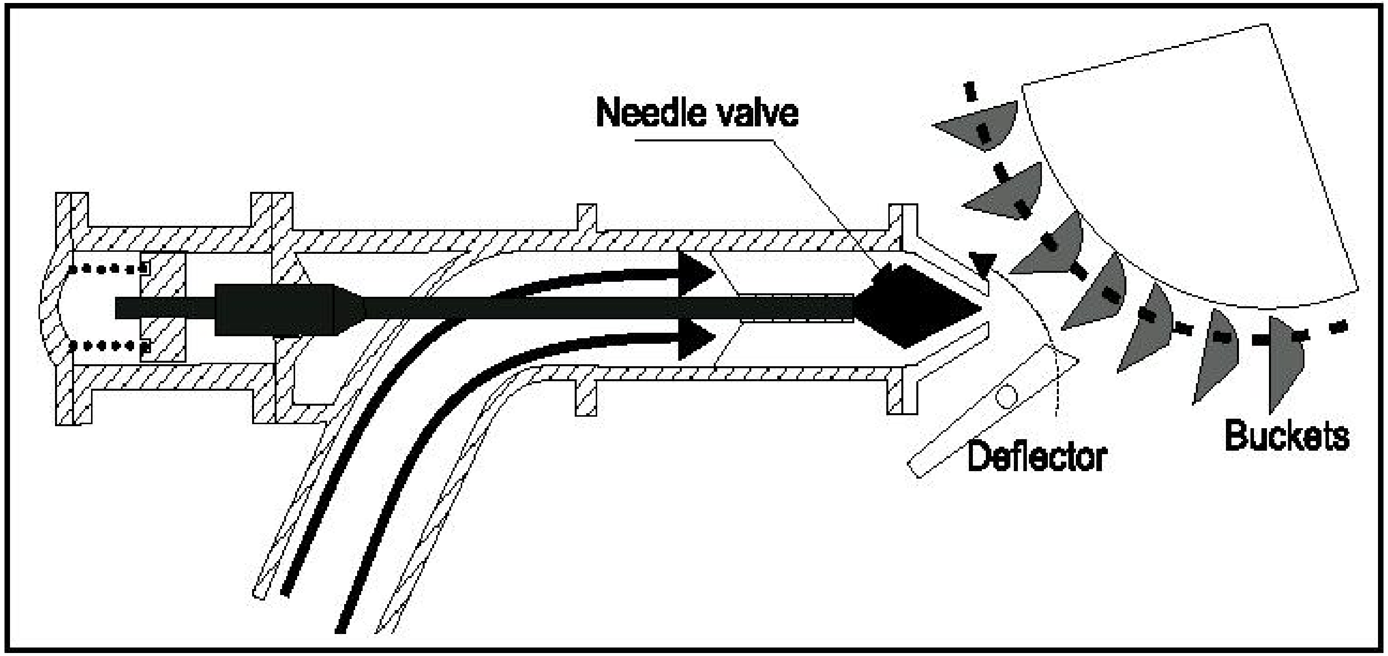
(1)The runner of Pelton 箕斗一般有 15～25 枚





The Pelton wheel is an impulse turbine.

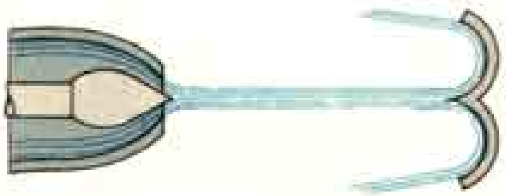
Governor

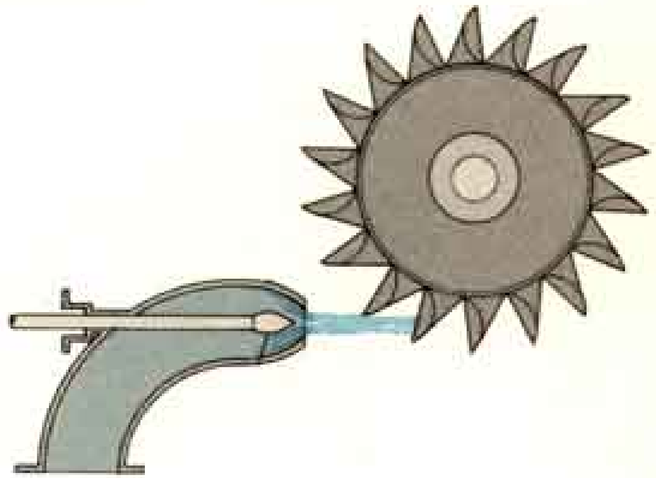


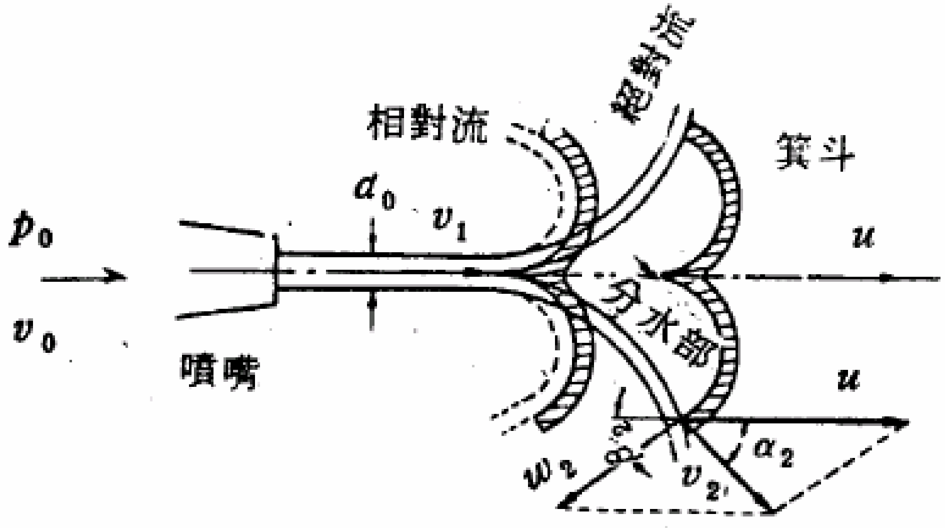
噴嘴中央設有針閥（needle valve），由調速機（governor）中的伺服馬 serve motor）帶動，依負荷的變化可作前後移動以調整水量。

轉流板位於噴嘴與箕斗之間，可在負荷驟降時，用以改變噴嘴方向，因爲要避免水壓管中發生水鎚現象（water hammer）而導致壓力上升，此時噴嘴內之針閥宜徐徐的關閉。

Impulse Turbines







帕爾登水輪機箕斗上水流狀況

$$H=\frac{p_0}{\gamma}+\frac{v_0^{\,2}}{2g}=\frac{v_1^{\,2}}{2g}+\xi_1\frac{v_1^{\,2}}{2g}$$

噴嘴入口處之壓力爲 p_0

噴嘴出口之噴流速度爲 v_1 $\quad v_1=\dfrac{1}{\sqrt{1+\xi_1}}\sqrt{2gH}=C_v\sqrt{2gH}$ 〔m / s〕

噴嘴內部之損失爲 $\xi_1\ v_1^{\,2}/2g$，

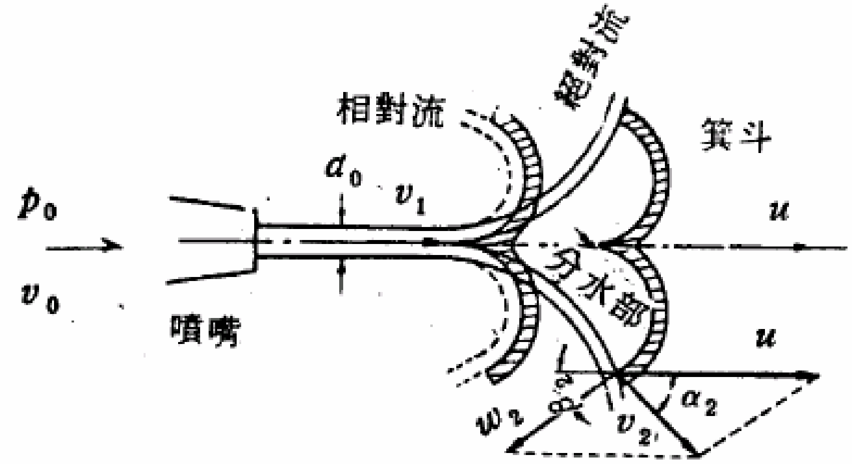
$\xi_1=0.04\sim0.11$

$C_v=0.95\sim0.98$ (coefficient of velocity)

噴嘴效率（nozzle efficiency）η_n

$$\eta_n = \frac{v_1^2/2g}{H} = C_v^2$$

$$= 0.90 \sim 0.95$$



箕斗得到之水力動力L_h爲

$$L_h = u \cdot \{\rho(V_0 - u)\cos\theta[(V_0 - u)A_0] + \rho(V_0 - u)[-(V_0 - u)A_0]\}$$

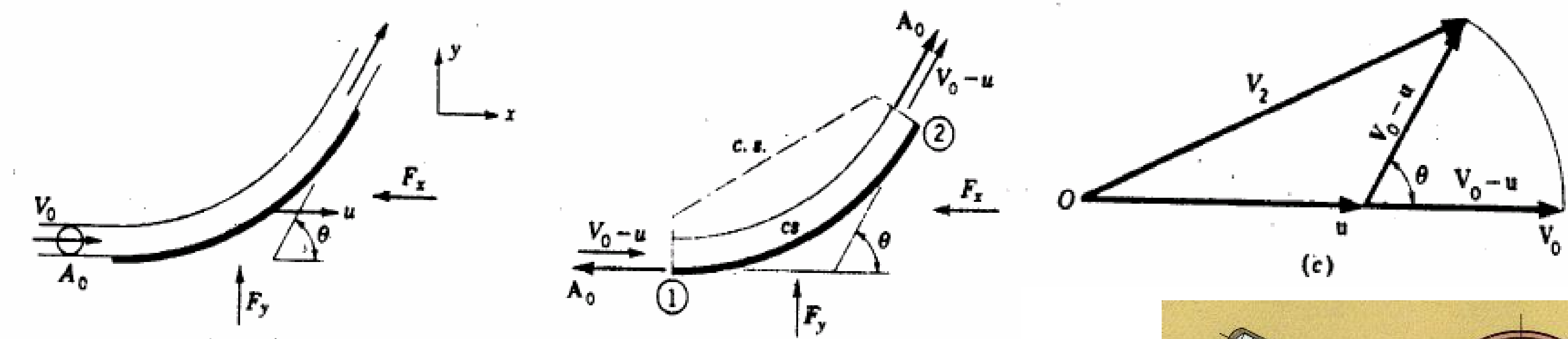
$$L_h = \frac{\gamma Q}{g} u(w_1 + w_2 \cos\beta_2) \text{〔kgf·m/s〕}$$

箕斗出口角爲β_2

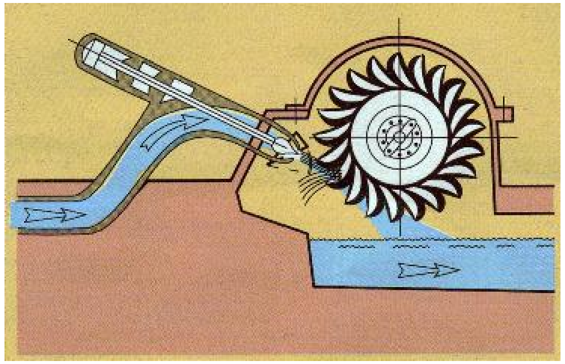
說明 Momentum Equation

$$\sum \vec{F} = \vec{F}_S + \vec{F}_B = \left.\frac{d\vec{P}}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{cv} \vec{V}\rho d\forall + \int_{CS} \vec{V}\rho\vec{V}\cdot d\vec{A}$$

例題 2.12 如圖 2.14 所示，一噴射流打在一可動葉片上，試求葉片所受之力。



解： 設葉片對噴流無摩擦阻力，噴流以對於葉片的相對速度 V_0-u 流入，並以此速度流出。流出的絕對速度 V_2，爲此相對速度與葉片絕對速度 u 的向量和，參照圖(c)。因流動狀況爲穩定流態，應用 (2.37) 式於圖(b)控制體積，得



$$\Sigma F_x = \int_{cs} \rho v_x \mathbf{V}\cdot d\mathbf{A} = -F_x$$
$$= \rho(V_0-u)\cos\theta\,[(V_0-u)A_0] + \rho(V_0-u)[-(V_0-u)A_0]$$

或 $F_x = \rho(V_0-u)^2 A_0(1-\cos\theta)$

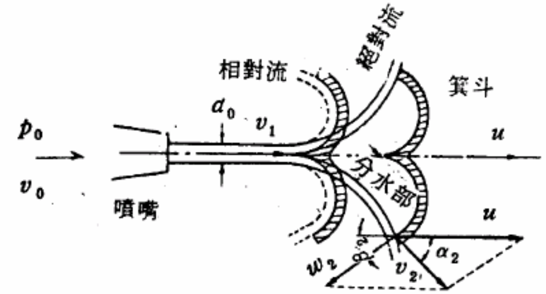
$$L_h = F.u$$
$$= u\cdot\{\rho(V_0-u)\mathrm{Cos}\theta[(V_0-u)A_0] + \rho(V_0-u)[-(V_0-u)A_0]\}$$

$$\Sigma F_y = \int_{cs} \rho v_y \mathbf{V}\cdot d\mathbf{A} = F_y$$
$$= \rho(V_0-u)\sin\theta\,[(V_0-u)A_0]$$

或 $F_y = \rho(V_0-u)^2 A_0 \sin\theta$

$$= \rho(V_0-u)\cos\theta\,[(V_0-u)A_0] + \rho(V_0-u)[-(V_0-u)A_0]$$

箕斗之效率 η_b (bucket efficiency)



$$w_1 = v_1 - u$$

$$\frac{w_1^2}{2g} = \frac{w_2^2}{2g} + \xi_2 \frac{w_2^2}{2g} \qquad w_1^2 = (1+\xi_2)\, w_2^2$$

$$w_2 = \frac{w_1}{\sqrt{1+\xi_2}}$$

水在箕斗中之流動損失爲 $\xi_2 \dfrac{w_2^2}{2g}$

$$L_h = \frac{\gamma Q}{g} u (w_1 + w_2 \cos\beta_2) \quad [\mathrm{kgf \cdot m/s}]$$

$$\boxed{= \frac{\gamma Q}{g} u (v_1 - u)\left(1 + \frac{\cos\beta_2}{\sqrt{1+\xi_2}}\right) \quad [\mathrm{kgf \cdot m/s}]}$$

$$\eta_b = \frac{L_h}{\gamma Q \dfrac{v_1^2}{2g}} = 2\frac{u}{v_1}\left(1 - \frac{u}{v_1}\right)\left(1 + \frac{\cos\beta_2}{\sqrt{1+\xi_2}}\right)$$

$$\frac{d\eta_b}{d\left(\frac{u}{v_1}\right)}=0 \quad \frac{u}{v_1}=\frac{1}{2}$$

$$\eta_{b\max}=\frac{1}{2}\left(1+\frac{\cos\beta_2}{\sqrt{1+\xi_2}}\right)$$

(1) 當噴流速度 v_1 爲葉片速度 u 的二倍時，葉片效率最好

(2) $\beta_2=0$ 時 η_b 爲最大，但實際上爲避免流出箕

(3) 斗後水流衝擊鄰接箕斗之背部，一般 β_2 值取在 $4°\sim15°$ 之間。

$$u=\frac{V_1}{2} \qquad L_{h\max}=\frac{\gamma Q}{g}\frac{V_1^{\,2}}{2}\left(1+\frac{\cos\beta_2}{\sqrt{1+\xi_2}}\right)\ \text{〔kgf·m/s〕}$$

例題 6.4

有一帕爾登水輪機，噴流之直徑為 7.5 cm，噴流之速度為 100 m/sec，箕斗的流出角度為 10°，箕斗在圓周方向之速度與噴流之速度比為 0.47。如水輪機之轉速為 600 rpm，在理想情況下求(a)轉輪之平均直徑(b)產生的動力。

解：(a) 已知 $\dfrac{u}{V_1}=0.47$ 故 $u=0.47\,V_1=0.47\times100=47\,\text{m/sec}$

$$u=r\omega \qquad 47=\left(\frac{D}{2}\right)(2\pi)\left(\frac{600}{60}\right)$$

$$D=1.495\,\text{m}$$

(b) 以 $\xi_2=0$ 代入得產生之動力 L_h

$$L_h=\frac{\gamma Q}{g}u(v_1-u)\left(1+\frac{\cos\beta_2}{\sqrt{1+\xi_2}}\right) \quad [\text{kgf}\cdot\text{m/s}]$$

$$=\frac{\gamma}{g}\left(v_1\frac{\pi d^2}{4}\right)u(v_1-u)(1+\cos\beta_2)/102$$

$$=\frac{1000}{9.8}\times100\times\frac{\pi}{4}\times\left(\frac{7.5}{100}\right)^2\times47\times(100-47)(1+\cos10^\circ)\times\frac{1}{102}$$

$$=2185\,\text{kw}$$

例題 6.5

一帕爾登水輪機，噴流之速度爲 25m/s，流量爲 $1\,m^3/s$，箕斗之圓周速度爲 15m/s，$\beta_2 = 25°$，$\xi_2 = 0.44$，求其水力動力爲若干？

解：

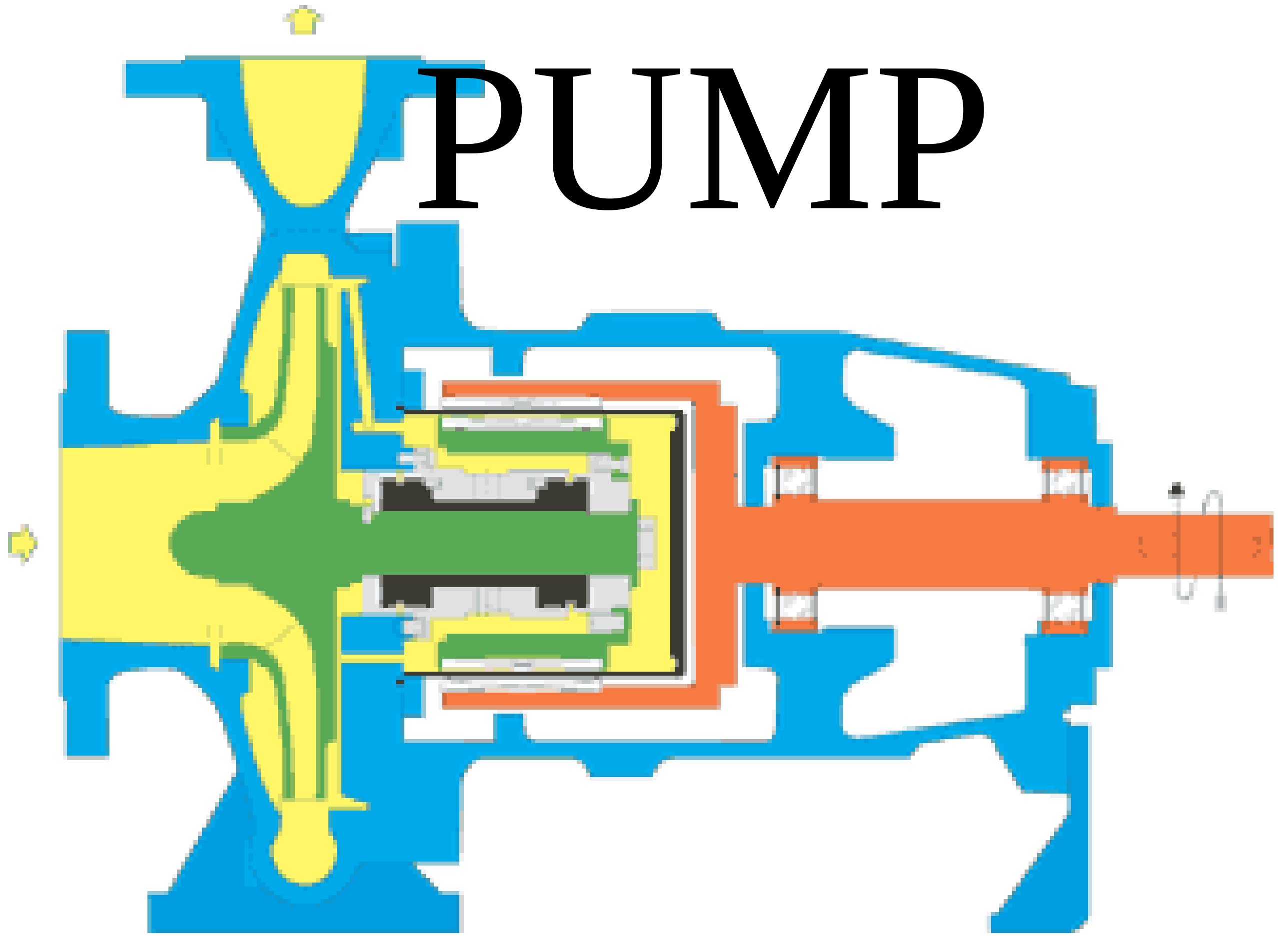
$$L_h = \frac{\gamma Q}{g} u(v_1 - u)\left(1 + \frac{\cos\beta_2}{\sqrt{1+\xi_2}}\right) \quad [\text{kgf}\cdot\text{m/s}]$$

$$= \frac{1000\times 1}{9.8} \times 15 \times (25-15) \times \left[\frac{1+\cos 25°}{\sqrt{1+(0.44)}}\right] \times \frac{1}{102}$$

$$\fallingdotseq 238.4 \text{ kw}$$

Chapter 7

PUMP



至於液體和氣體基本上差異爲：

① 氣體單位體積的重量甚小於液體（如 20°C 的空氣約爲水的 1/830

②氣體具有可壓縮性並在被壓縮或膨脹過程中，伴隨有溫度變化。

泵（pump）係從內燃機或電動機之類的原動機，接受機械能，於吸進液體後施予液體以能量，俾使液體能揚升至較高的地方或壓送至遠方的機械

(1) 依機械運動的形式而分類

a. 作直線運動者　如往復泵〔有活塞式（piston type）、柱塞式（plunger type）〕。

b. 作迴轉運動者　如離心泵（centrifugal pump）、斜流泵(diagonal-flow pump）、軸流泵（axial-flow pump）、齒輪泵（gear pump）、葉片泵（vane pump）等。

c. 不具運動部份者　如再生泵（regenerating pump）、噴流泵（jet pump）、氣泡泵（airlift pump）、水擊泵（water hammering pump）等。

(2) 依作用原理而分類

a. 利用葉輪的迴轉而施予液體壓力者，如離心泵、斜流泵、軸流泵。

b. 利用活塞或柱塞壓出液體者，如往復泵。

(3) 依段數而分類

a. 單段式（single stage）。　　b. 多段式（multistage）。

(4) 依泵軸的方向而分類

a. 水平軸式。

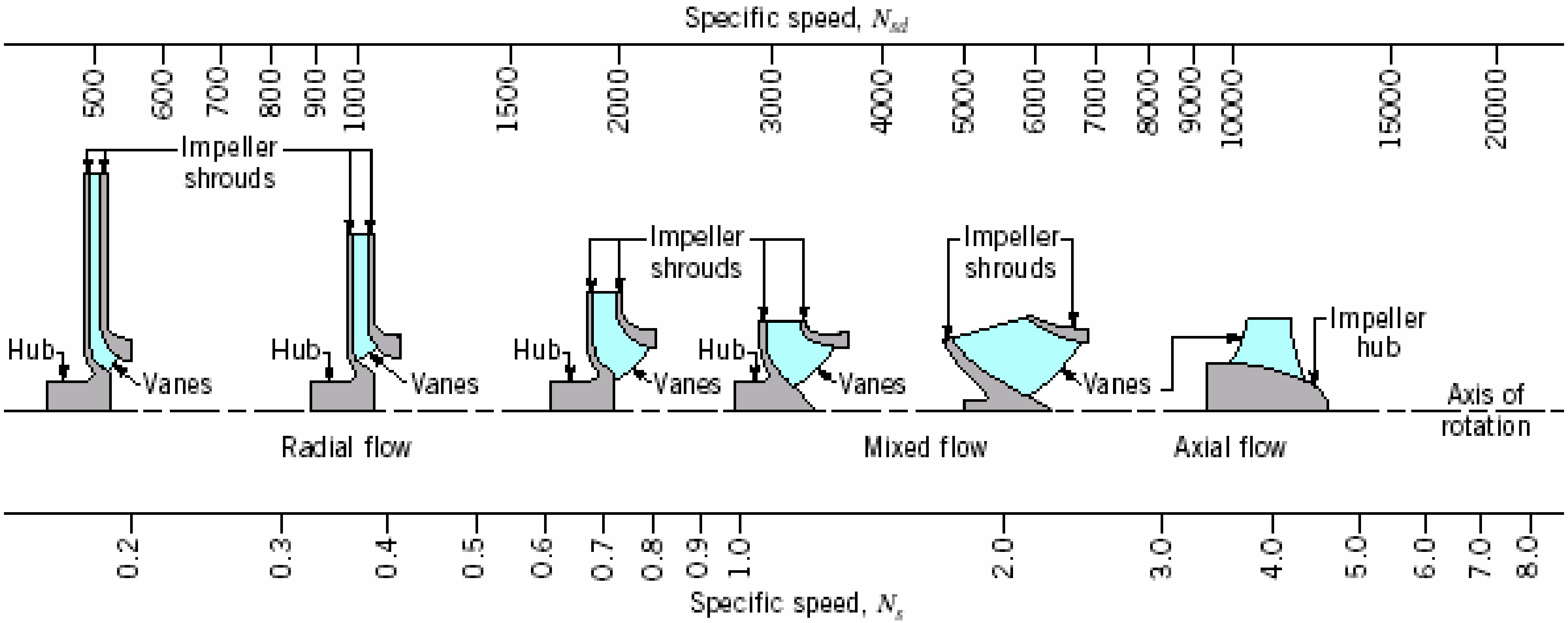
b. 立式軸式。

不過習慣上多採用以下之分類：

a. 輪機形　如離心泵、斜流泵、軸流泵等。

b. 容積形　如往復泵、齒輪泵、葉片泵等。

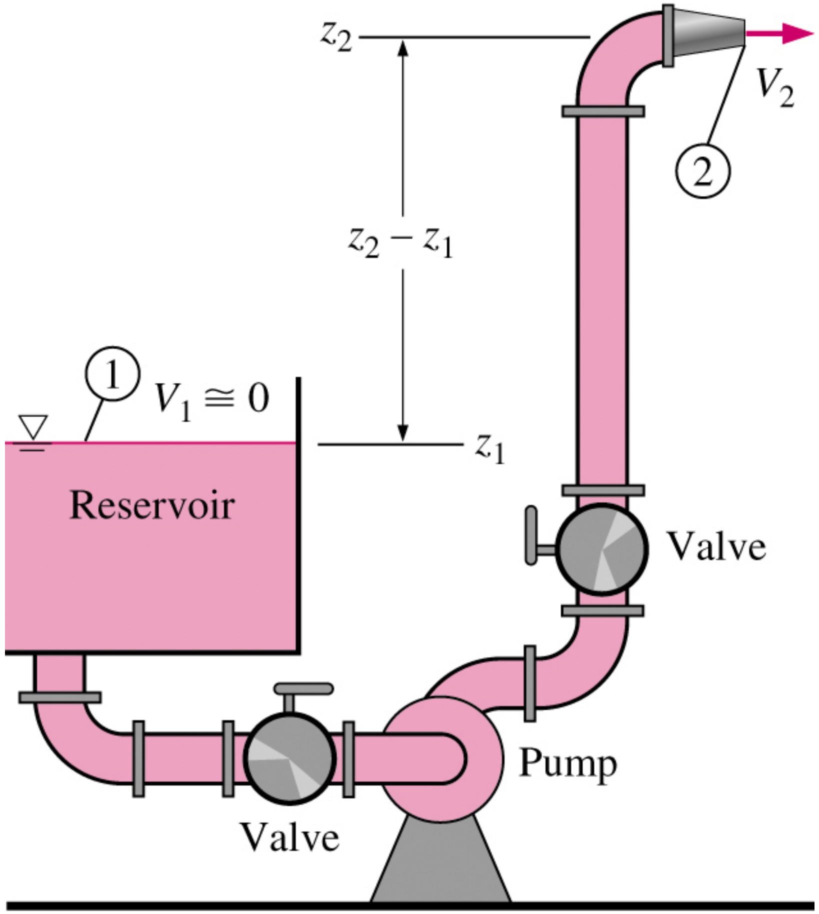
c. 特殊形　如再生泵、噴流泵、氣泡泵、水擊泵等。



■ FIGURE 12.18 **Variation in specific speed with type of pump. (Adapted from Ref. 17, used with permission.)**

Suction Specific Speed

$$S_{sd} = \frac{\omega(\text{rpm})\sqrt{Q(\text{gpm})}}{[\text{NPSH}_R\,(\text{ft})]^{3/4}} \qquad S_s = \frac{\omega\sqrt{Q}}{[g(\text{NPSH}_R)]^{3/4}}$$


z_2
V_2
2
$z_2 - z_1$
1
$V_1 \cong 0$
z_1
Reservoir
Valve
Pump
Valve

7.3 泵的基本要項

吸水面至排水面的高度稱爲實揚程(actual head)以H_a表示

吸水面至泵之中心線的垂直高度稱爲吸入實揚程(actual suction head)以H_s表示

由泵之中心線至排水面的垂直高度稱爲排出實揚程(actual discharge head)以H_d表示

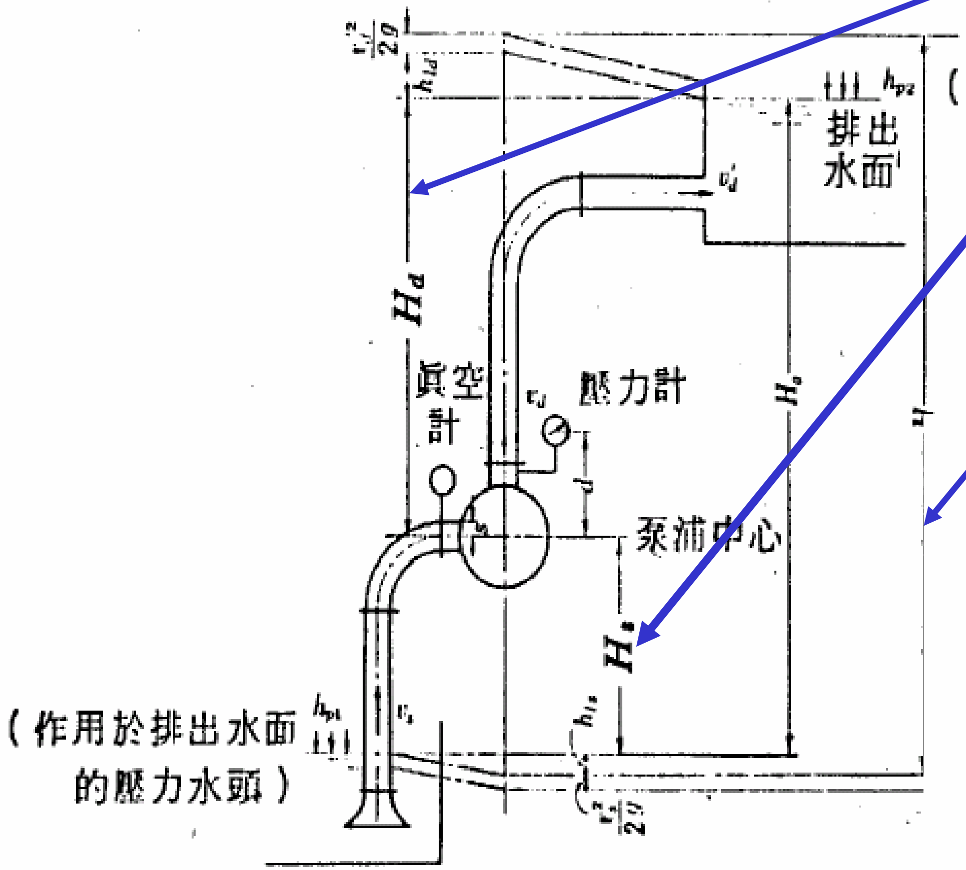
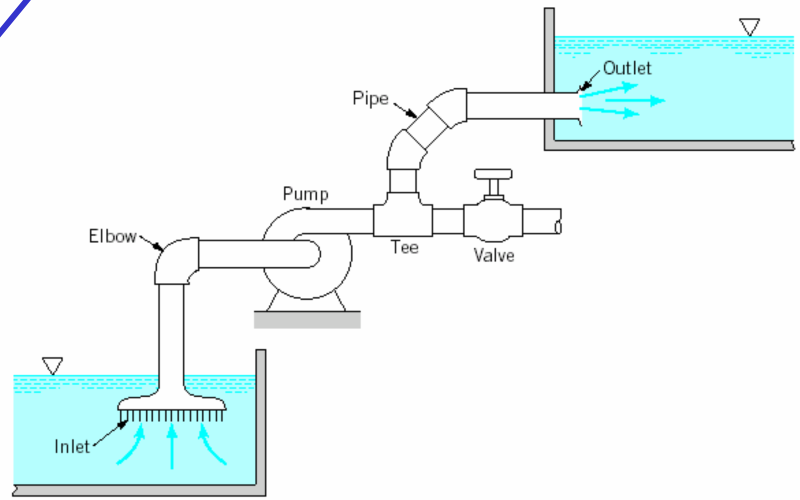


圖7.1 泵的輸水系統

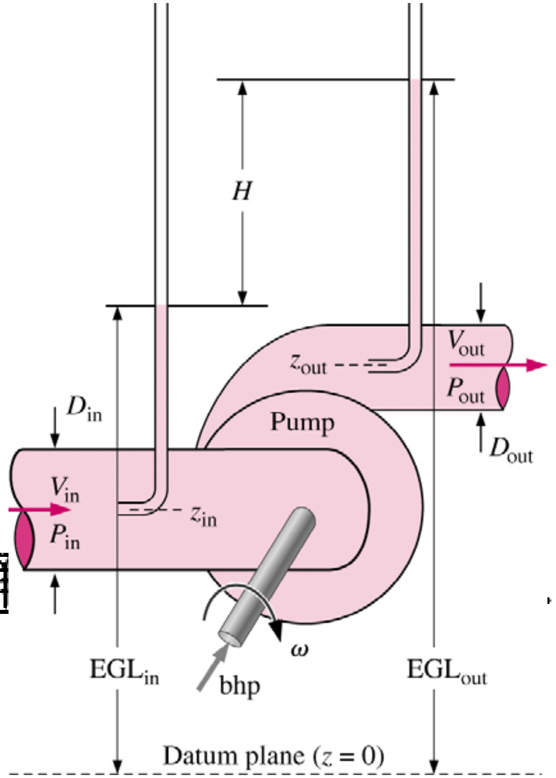


(a) 揚程 (head)

泵的全揚程 單位重量之液體在泵 的入口及出口處所具有的能量差

爲泵供給液體之壓力能、速度能及位能的總和

$$H=(\frac{P_d}{\gamma}+\frac{V_d^2}{2g}+z)-(\frac{P_s}{\gamma}+\frac{V_s^2}{2g}) \quad 〔\mathrm{kgf \cdot m/kgf}〕$$



(b) 流量 (capacity)

在單位時間內泵所輸送之液體體積稱爲流量Q，通 ... n

(c) 動力 (power) 及效率 (efficiency)

$$\boxed{L_w=\frac{\gamma QH}{60} \quad 〔\mathrm{kgf\text{-}m/sec}〕}\boxed{=\frac{\gamma QH}{75\times 60}〔\mathrm{PS}〕=\frac{\gamma QH}{102\times 60}〔\mathrm{kw}〕}$$

泵的流量爲Q（m^3/min） 全揚程爲H(m) 液體的比重量爲γ（kgf/m^3）

泵之全效率簡稱為效率

$$\eta=\frac{L_w}{L}=\frac{\gamma QH}{75\times 60\times L\,[PS]}=\frac{\gamma QH}{102\times 60\times L\,[kw]}$$

原動機(馬達或引擎)加於泵之動力為軸動力L

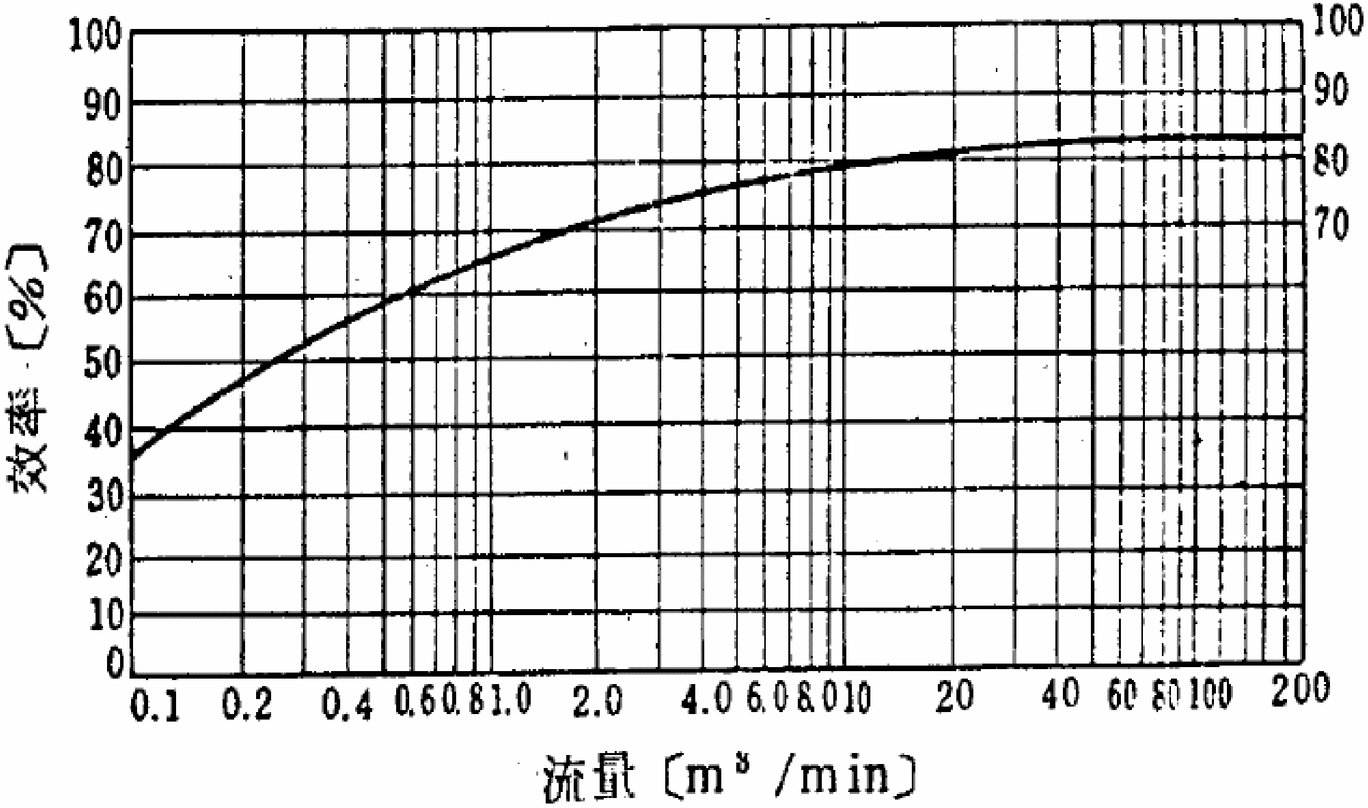


圖7.2 一般用泵之標準效率

7.4.3 離心泵之理論揚程

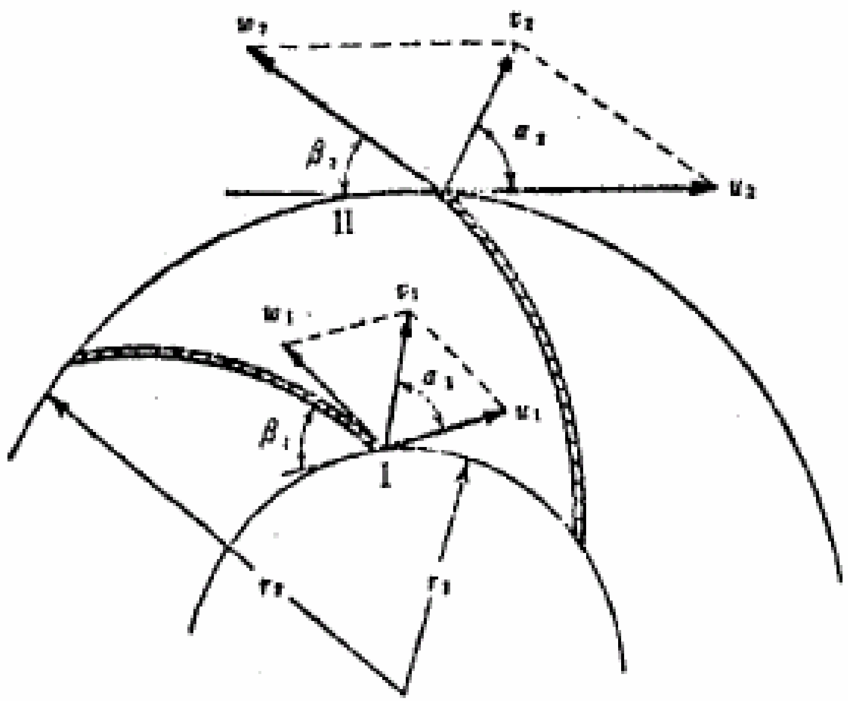


圖 7.13 葉輪的速度向量圖

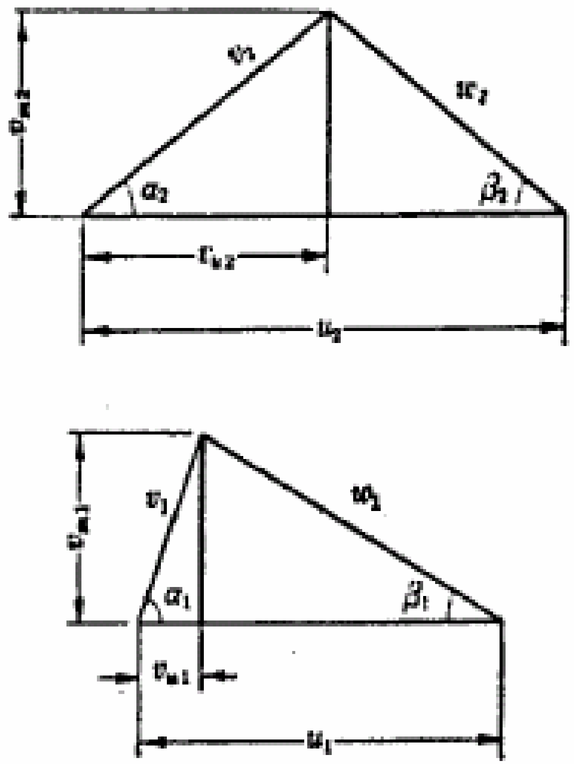
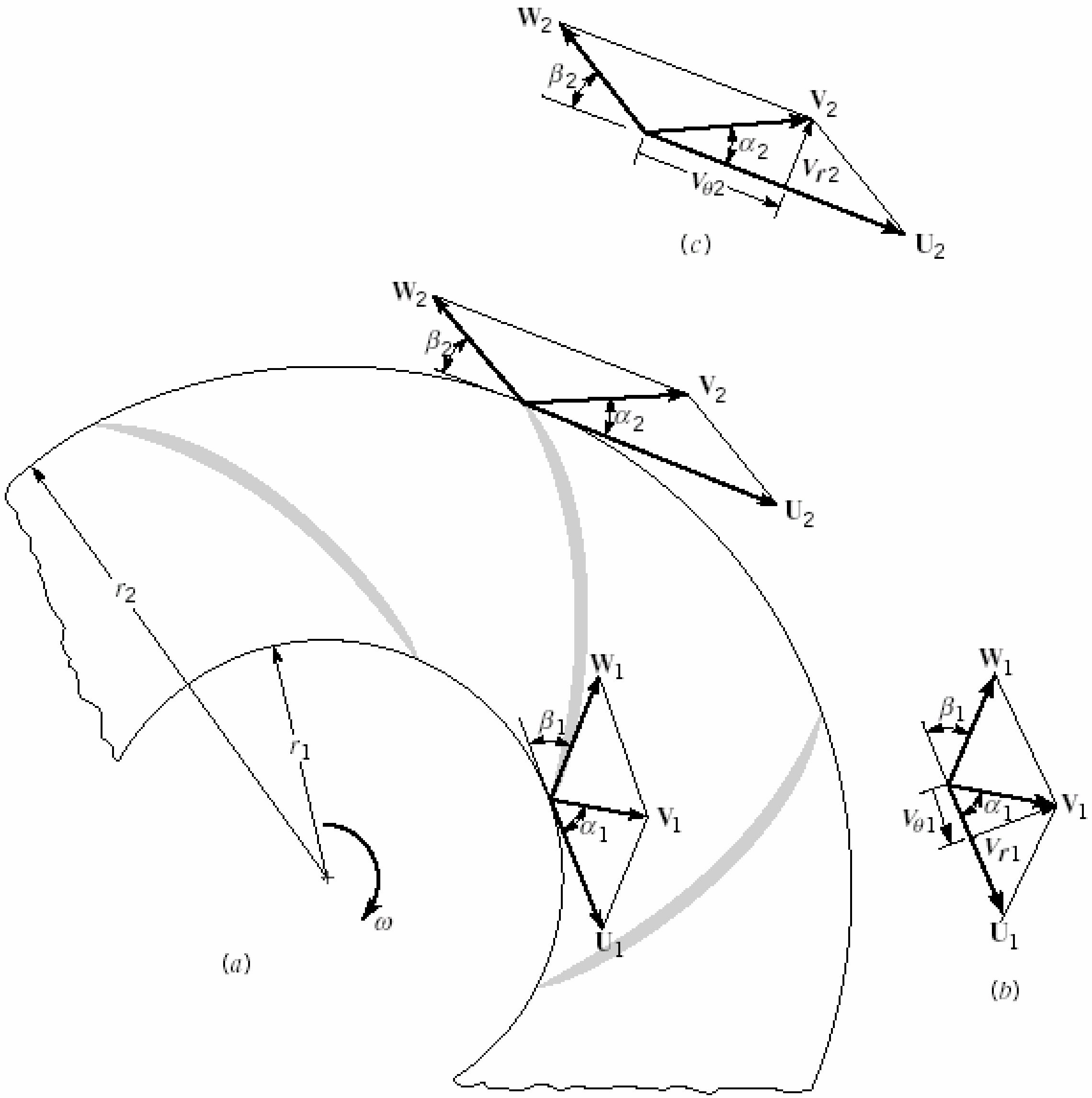


圖 7.14 葉輪入口出口處之速度三角形

葉輪內、外之圓周速度分別爲 $u_1=r_1\omega$ ，$u_2=r_2\omega$

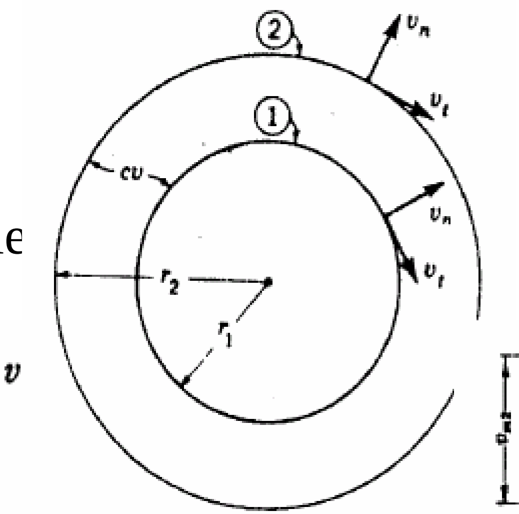
進入葉輪與流出葉輪的絕對 速度分別爲 v_1 ，v_2

水在進入和流出時對葉輪的相對速度各爲 w_1 ，w_2



■ FIGURE 12.8 Velocity diagrams at the inlet and exit of a centrifugal pump impeller.

The Angular Momentum Principle



$$Fr = T_z = \frac{\partial}{\partial t}\int_{cv} \rho r v_t \, dV + \int_{cs} \rho r v$$

式中T_z爲力矩。若流動爲穩定流則$\frac{\partial}{\partial t}\int_{cv} \rho r v_t \, dV = 0$

於是上式簡化爲

$$T_z = \int_{A2} \rho_2 r_2 v_{t2} v_{n2} dA_2 - \int_{A1} \rho_1 r_1 v_{t1} v_{n1} dA_1$$

由連續方程式 $\int_{A1} \rho v_n dA = \int_{A2} \rho v_n dA = \rho Q$

故 $T_z = \rho Q[(rv_t)_2 - (rv_t)_1]$

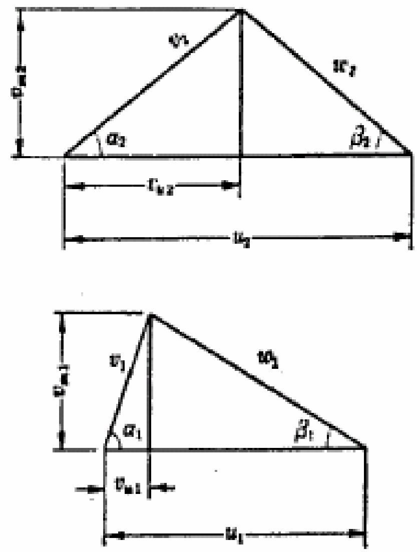


圖7.14 葉輪入口出口處之速度三角形

葉片施於水的扭力T $\boxed{T = \frac{\gamma Q}{g}(r_2 v_2 \cos\alpha_2 - r_1 v_1 \cos\alpha_1) \quad (\text{kgf-m})}$

葉輪施予水之動力 L 之動力 L

$$T=\frac{\gamma Q}{g}\left(r_2v_2\cos\alpha_2-r_1v_1\cos\alpha_1\right)\quad(\text{kgf-m})$$

$$L=\omega\cdot T\quad(\text{kgf-m/s})=\rho Q\omega(r_2V_{\theta2}-r_1V_{\theta1})$$
$$=\rho Q(U_2V_{\theta2}-U_1V_{\theta1})$$
$$=\rho gQh_i$$

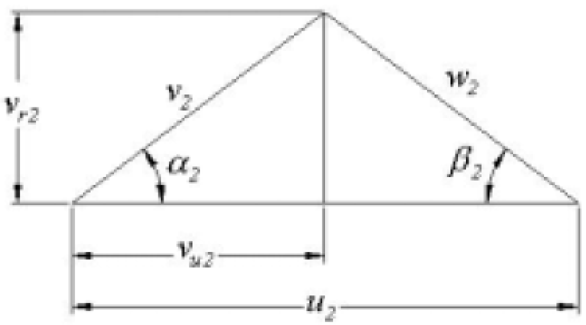
此動力將完全使水高揚，此高揚之水頭稱爲理論揚程（theoretical neaa）

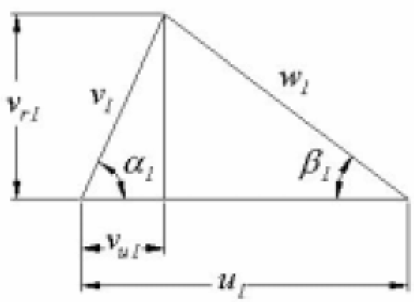
$$H_{th\infty}=\frac{L}{\gamma Q}=\frac{\omega T}{\gamma Q}\ (\text{m})=\frac{\omega}{g}\left(r_2v_2\cos\alpha_2-r_1v_1\cos\alpha_1\right)$$

因 $u_1=r_1\cdot\omega_1$，$u_2=r_2\cdot\omega_2$ $v_2\cos\alpha_2=v_{u2}$，$v_1\cos\alpha_1=v_{u1}$

$$\boxed{H_{th\infty}=\frac{1}{g}\left(u_2v_{u2}-u_1v_{u1}\right)}$$

此式爲泵的理論揚程方程式





$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}$ 和 $\frac{p_2}{\gamma}+\frac{v_2^2}{2g}$ 分別表示進口處與出口處的水頭　式中 h_l 表葉輪中之損失

$$\boxed{H_{th\infty}=\frac{1}{g}(u_2v_{u2}-u_1v_{u1})}=\boxed{\frac{p_2}{\gamma}+\frac{v_2^2}{2g}-\left(\frac{p_1}{\gamma}+\frac{v_1^2}{2g}\right)+h_l}$$

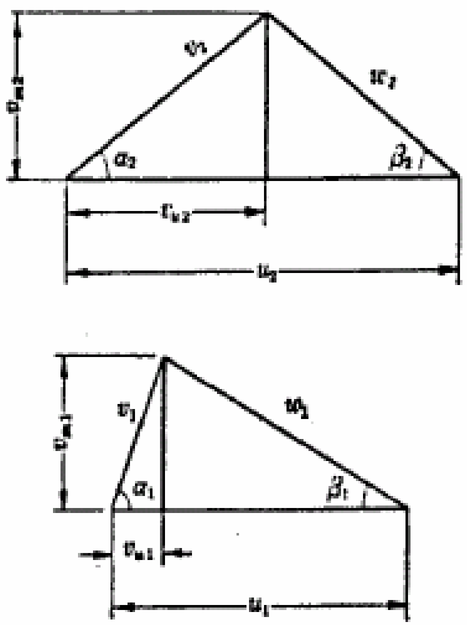


圖 7.14 葉輪入口出口處之速度三角形

$$w_1^2=u_1^2+v_1^2-2u_1v_1\cos\alpha_1$$

$$w_2^2=u_2^2+v_2^2-2u_2v_2\cos\alpha_2$$

$$\frac{p_2}{\gamma}-\frac{p_1}{\gamma}=\frac{u_2^2-u_1^2}{2g}+\frac{w_1^2-w_2^2}{2g}-h_l$$

若水由徑向流入葉輪時 $\alpha_1=90°$，$v_{u1}=0$

$$H_{th\infty}=\frac{1}{g}u_2v_{u2}$$

$$v_{u2} = u_2 - v_{m2} / \tan \beta_2 \qquad \text{流量} \; Q = \pi D_2 b_2 v_{m2}$$

$$H_{th\infty} = \frac{u_2}{g}\left(u_2 - \frac{v_{m2}}{\tan \beta_2}\right) = \frac{u_2^2}{g} - \frac{u_2 v_{m2}}{g \tan \beta_2}$$

$$= \frac{u_2^2}{g} - \frac{u_2 \dfrac{Q}{\pi D_2 b_2}}{g \tan \beta_2} = c_1 n^2 + c_2 n Q$$

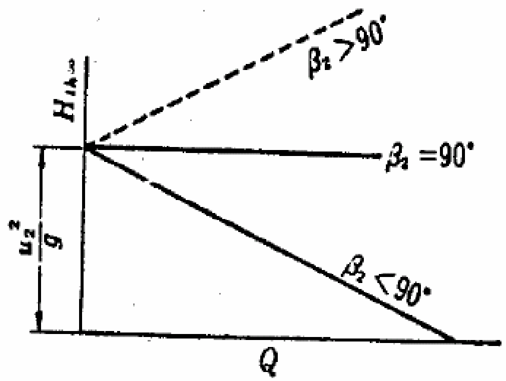


圖 7.15　泵的理論揚程

u_2 正比於 n

一般當 β_2 較大時，流體所得之速度能佔之比率較壓力能佔之比率爲大，而速度太大時流動損失甚大，效率較差，因此爲提高葉輪效率，泵葉之出口角宜採較小角度，其範圍約 10° ～ 35° ，最常用者爲 20° ～ 25° 。

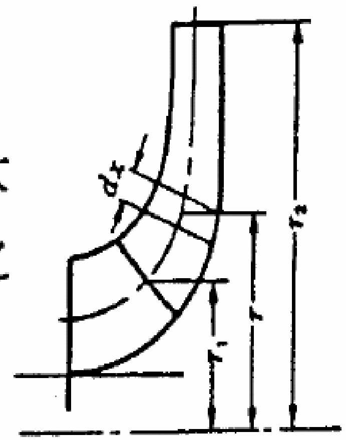


圖 7.16　徑向流動葉輪

實際的理論揚程H_{th}依據Pfleidercr 氏式可以下式表之：

式中ψ爲實驗係數，z爲葉片數目，$S=\frac{r_2}{r_1}r\,dx$（參考圖 7.16 ）爲子午線切面上平面流線對迴轉軸的靜力矩，p爲揚程減少部份。

7.4.4 離心泵的水力損失

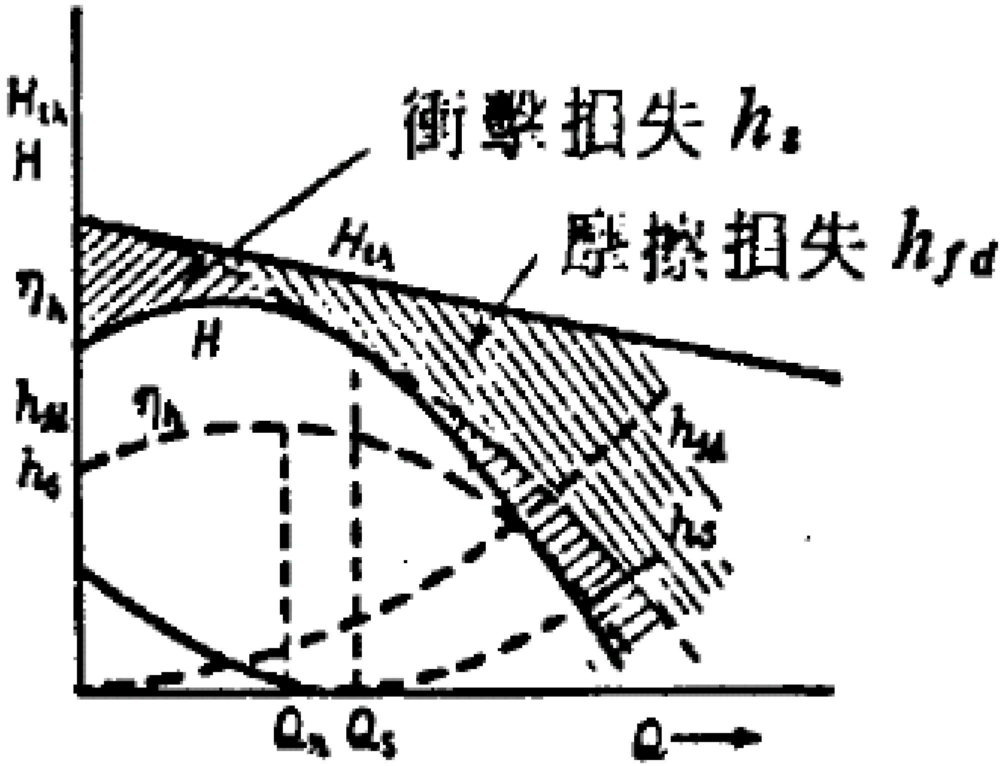


圖7.17 離心泵實際揚程與水力效率曲線

摩擦損失

$$h_f = K_1 Q_1^2$$

衝擊損失 h_s

$$h_s = K_2 (Q - Q_n)^2$$

7.4.6 離心泵之特性曲線

泵在一定的轉速N及吸入揚程H_s下，流量Q與全揚程H，動力L及效率η之變化有一定關係，通常以流量爲橫座標，其餘諸項爲縱座標，用曲線表示其間之變化關係，此種關係曲線稱爲泵之**特性曲線**(characteristic curve)。

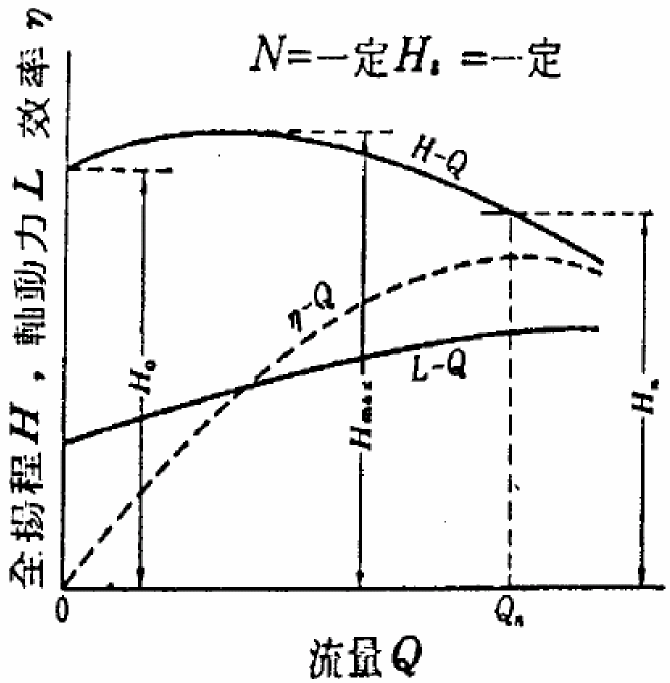


圖7.18 離心泵之特性曲線

在效率最高點之揚程H_n稱爲**正規揚程**(normal head，

其對應之流量Q_n爲正規流量(normal capacity)

流量爲零之揚程稱爲關閉揚程(shut-off head)

7.4.7 離心泵的運轉

(a) 單獨運轉　於圖 7.21 中，H_l' 曲線稱爲管路阻力曲線(pipe line resistance curve)，此曲線代表輸水系統中配管及閥門等之損失水頭，H_a 爲泵的實揚程時，將 H_l' 曲線移昇於 H_a 之上得 H_a+H_l' 之曲線，此曲線與泵的特性曲線 H-Q 之交點 A 即爲泵之操作點，

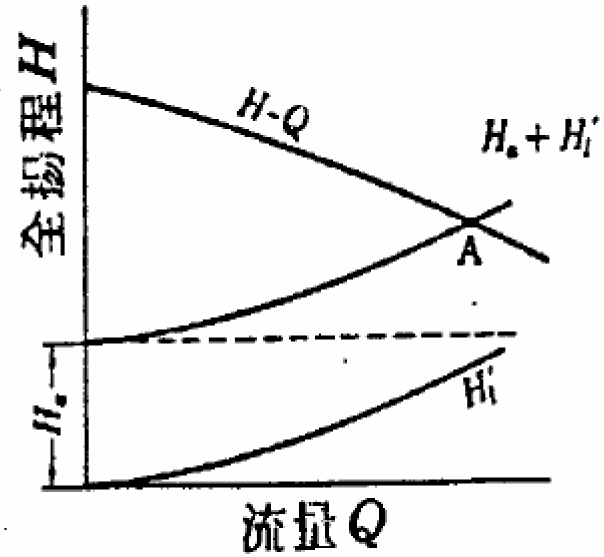


圖7.21　泵的特性曲線與管路阻力曲線

(b) 串聯運轉 (series operation)

於流量與揚程都要求能適應大幅度的變動選擇時，管路系統宜使用兩部或兩部以上的泵。

(1) 特性相同的兩泵串聯：

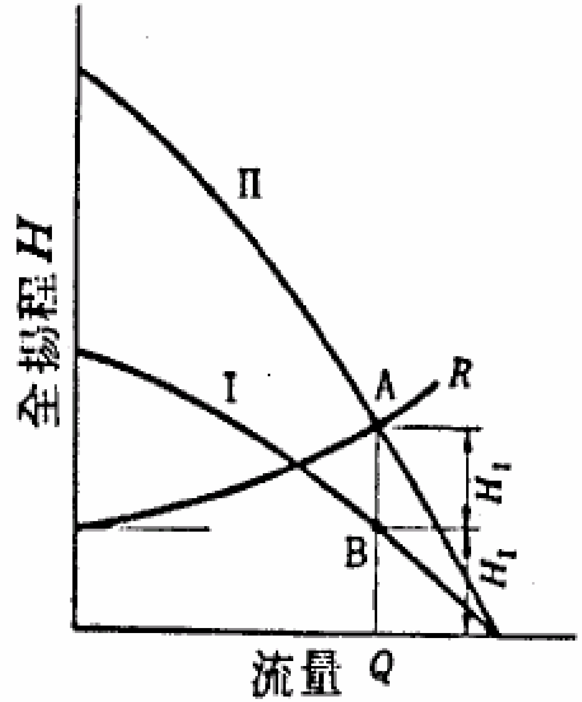


圖7.23 特性相同的兩泵串聯運轉之合成特性曲線

(2) 特性不同的兩泵串聯:

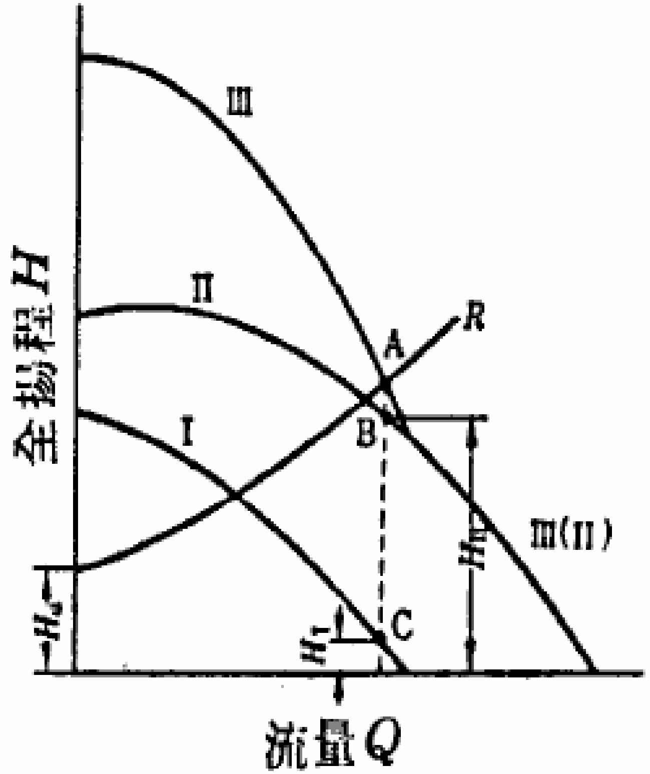


圖7.24 特性不同的兩泵串聯運轉之合成特性曲線

(c) 並聯運轉(parallel operation)

當管路系統中流量變化較大而一部泵之容量不足以應付時宜採用兩泵或兩個以上之泵並聯組合。

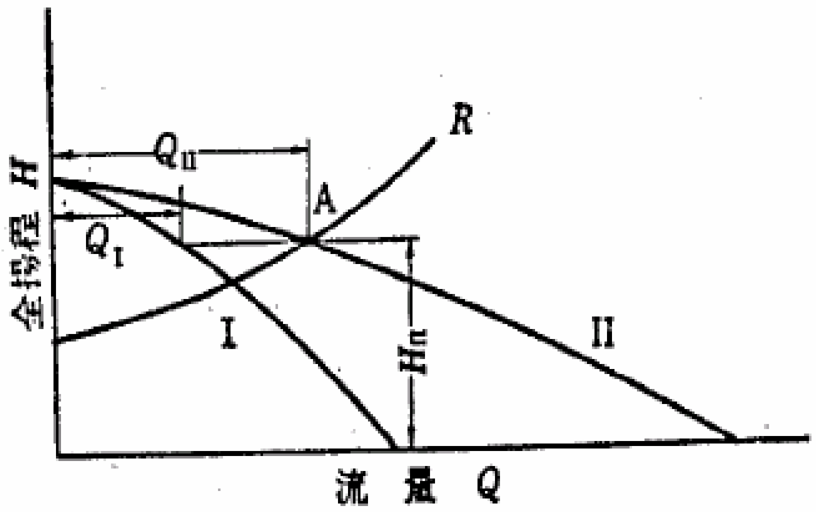


圖7.25 特性相同之兩泵並聯運轉之合成特性曲線

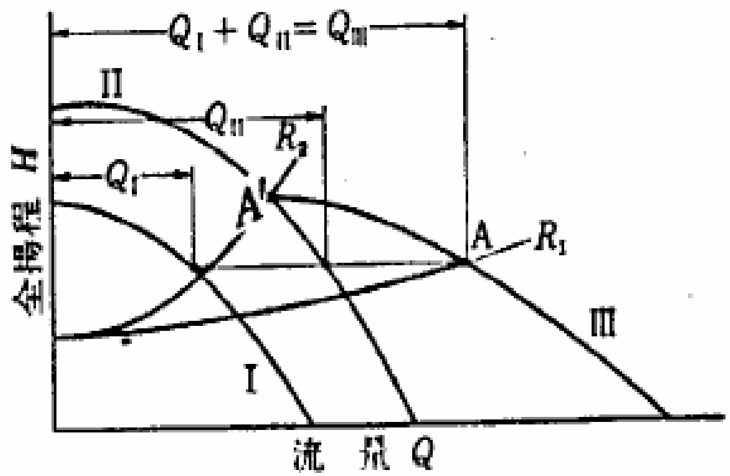
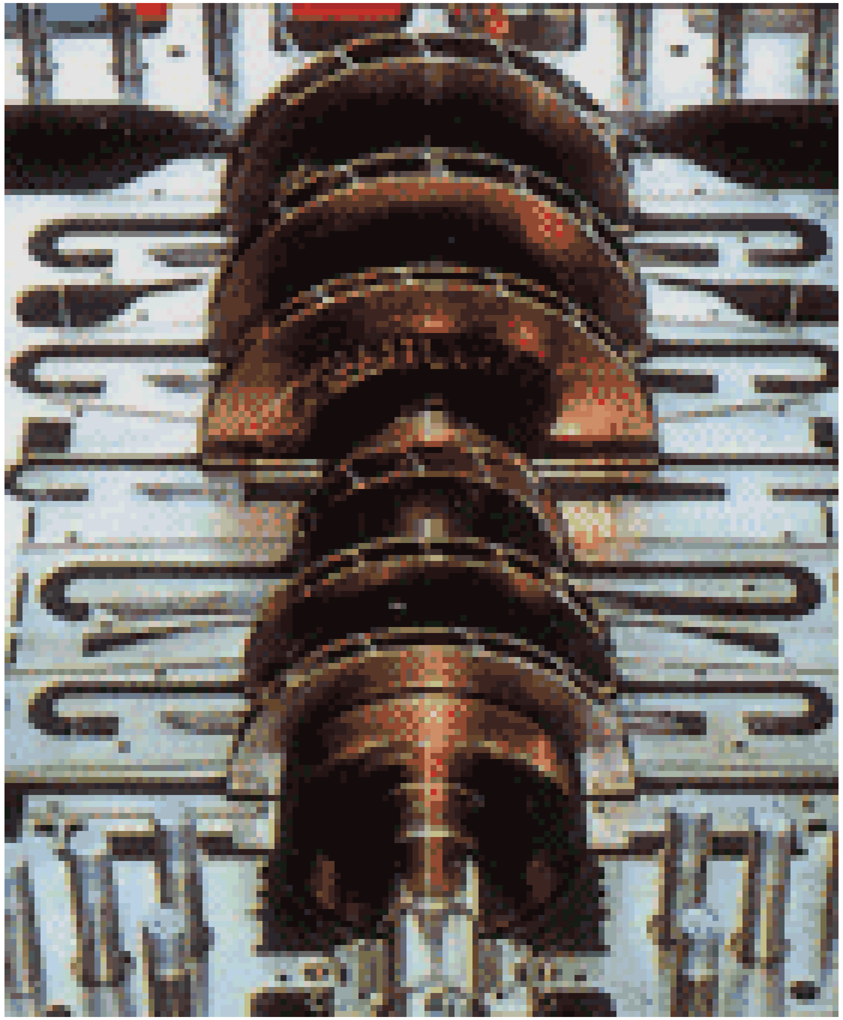


圖7.26 特性不同之兩泵，並聯運轉之特性曲線

第 九 章

熱力學概論

空氣機械所需具備的基礎熱力學知識



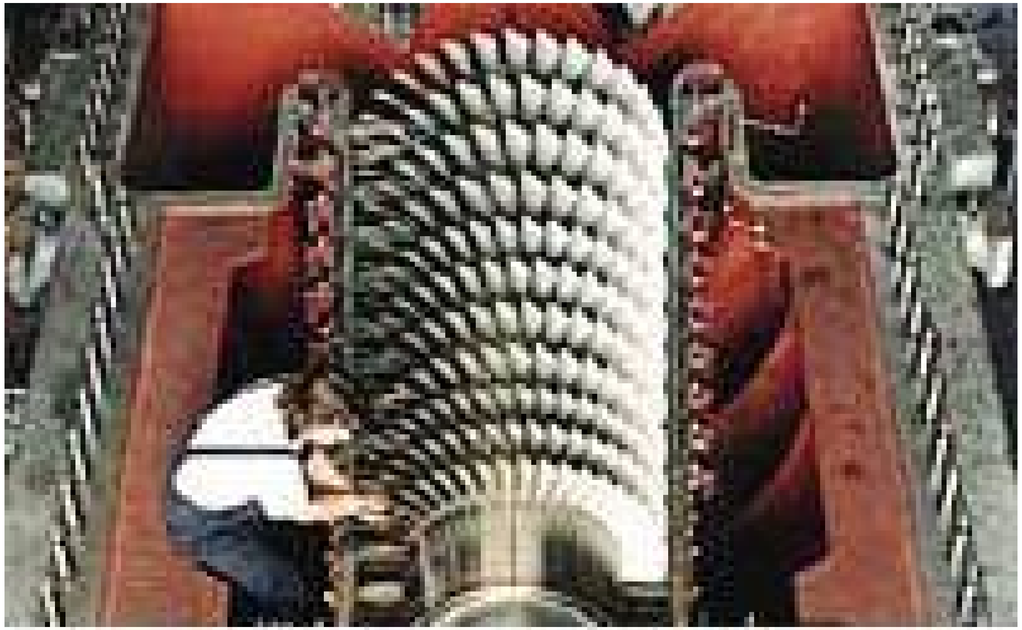
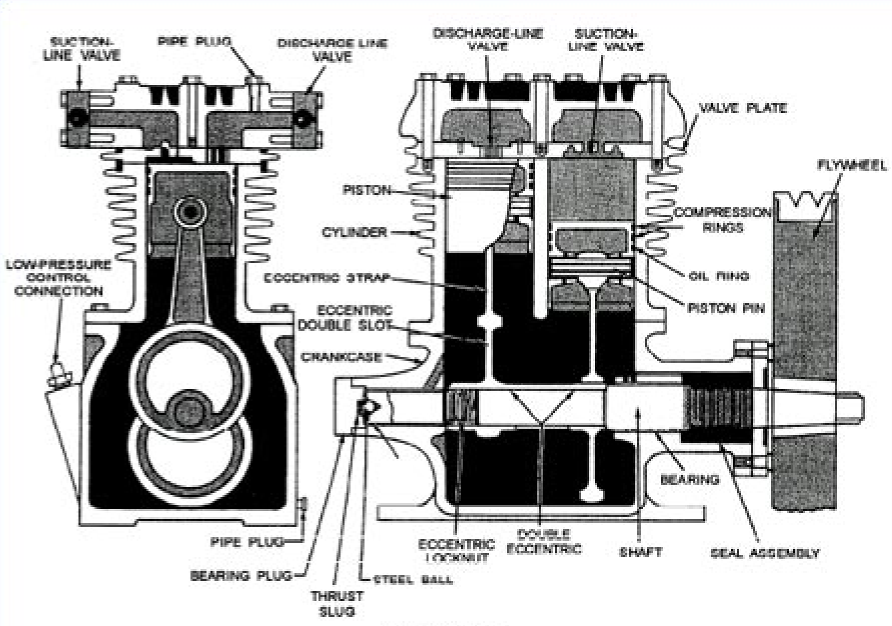
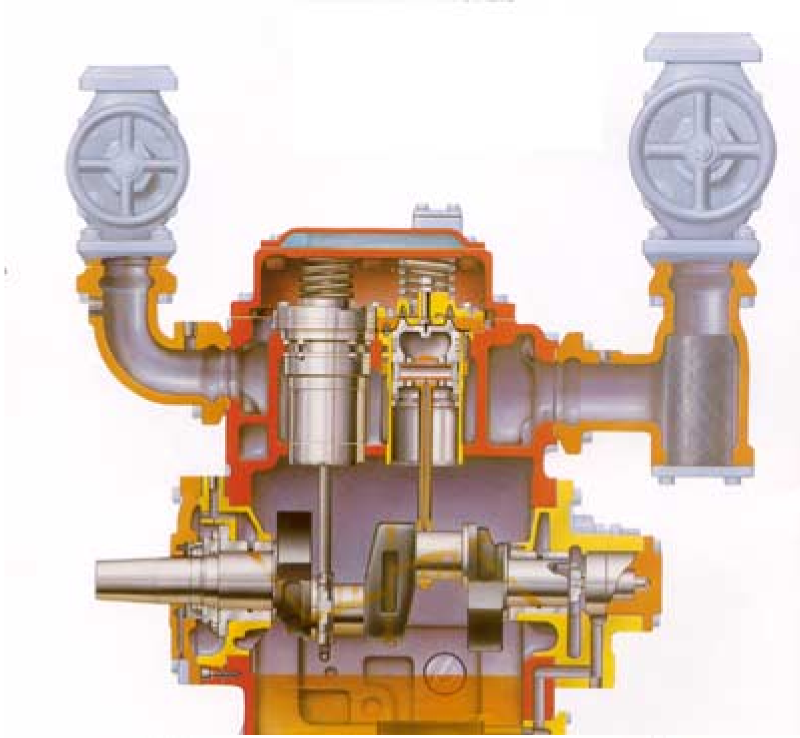


Figure 2


SUCTION-LINE VALVE
PIPE PLUG
DISCHARGE LINE VALVE
DISCHARGE-LINE VALVE
SUCTION-LINE VALVE
VALVE PLATE
FLYWHEEL
PISTON
CYLINDER
COMPRESSION RINGS
LOW-PRESSURE CONTROL CONNECTION
ECCENTRIC STRAP
OIL RING
PISTON PIN
ECCENTRIC DOUBLE SLOT
CRANKCASE
BEARING
PIPE PLUG
ECCENTRIC LOCKNUT
DOUBLE ECCENTRIC
SHAFT
SEAL ASSEMBLY
BEARING PLUG
STEEL BALL
THRUST SLUG
FIGURE 1



9.1 空氣與各種氣體的物理性質

9.1.1 標準大氣

設 t_0，p_0 及 ρ_0 分別表海平面上之溫度、壓力及密度 t、p 及 ρ 則分別爲距離海平面 z(km) 處之溫度、壓力及密度

$z \leq 11$ (km) (即對流層)

$$t = t_0 - 6.5z \quad (°C)$$

$$p = p_0(1-0.02257z)^{5.256} \quad (mmHg)$$

$$\rho = \rho_0(1-0.02257z)^{4.256} \quad (kgf \cdot s^2/m^4)$$

在 $t_0 = 15°C$ 時，$p_0 = 760\ mmHg = 10332.8\ kgf/m^2$

$$\rho_0 = 0.125\ kgf \cdot s^2/m^4$$

在標準狀態（0°C，760mmHg，$g=980.665m^2/s$）下乾燥空氣的成份

表9.2 乾燥空氣的標準成份

	氮（N_2）	氧（O_2）	氬（Ar）	二氧化碳（CO_2）
體積組成（%）	78.09	20.95	0.93	0.03
重量組成（%）	75.53	23.14	1.28	0.05

表9.1 空氣與氣體的物理性質

氣體 名稱	氣體 分子式	分子量 $\frac{\mathrm{kgf\cdot sec^2}}{\mathrm{m}}$	氣體常數 R $\frac{\mathrm{kgf\cdot m}}{\mathrm{kgf\cdot °K}}$	比重量 γ kgf /m³ 760 mmHg 0°C	比熱 kcal / kgf°C c_p 15°C	比熱 kcal / kgf°C c_v 15°C	$c_p/c_v=k$
乾空氣	—	28.968	29.27	1.293	0.241	0.172	1.401
氧	O_2	32.000	26.50	1.429	0.218	0.156	1.400
氮	N_2	28.020	30.26	1.251	0.249	0.178	1.401
氫	H_2	2.016	420.6	0.0899	3.408	2.42	1.407
一氧化硫	CO	28.000	30.29	1.250	0.248	0.177	1.404
二氧化硫	CO_2	44.000	19.27	1.964	16°C 0.200	16°C 0.153	1.302
二氧化硫	SO_2	64.070	13.24	2.857	0.152	0.109	1.39
氨	NH_2	17.034	49.78	0.760	14°C 0.514	14°C 0.393	1.309
乙炔	C_2H_2	26.016	32.60	1.161	0.383	0.304	1.26
甲烷	CH_4	16.032	52.89	0.715	0.528	0.403	1.31
乙烷	C_2H_8	30.07	28.00	1.356	0.382	0.313	1.22
丙烷	C_2H_8	44.09	19.4	1.970			
氯氣	Cl_2	71.00	11.55	3.167	0.115	0.085	1.36
水蒸氣	H_2O	18.016	47.07	100°C 0.598	100°C 0.490	100°C 0.368	1.33
NH_3 合成樹脂 0°C, 1 ata	—	—	99.55	0.3679	0.8055	0.578	1.406

9.1.2 空氣的比重量與濬度

- 在溫度 0°C，壓力 760mmHg．含 0.04 %二氧化碳
  乾燥空氣的比重量 $\gamma = 1.2931\ \text{kgf}/\text{m}^3$

現乾燥空氣在壓力爲 p (mmHg)，溫度爲 t°C時之比重量

$$\boxed{\gamma = 1.2931 \times \frac{273}{273+t} \times \frac{p}{760} \ [\text{kgf}/\text{m}^3]}$$

$$\rho = \frac{P}{RT} \Rightarrow \gamma = \rho g = \frac{Pg}{RT}$$

$$\frac{\gamma_2}{\gamma_1} = \frac{p_2 T_1}{p_1 T_2} \Rightarrow \gamma_2 = \gamma_1 \frac{p_2 T_1}{p_1 T_2}$$

$$\boxed{\mu = 1.758 \times 10^{-6} \times \frac{380}{380+t} \times \left(\frac{273+t}{273}\right)^{3/2} \ [\text{kgf-s}/\text{m}^2]}$$

$$\nu = \frac{\mu}{\rho}$$

- 在 150°C，760mmHg 下大氣之

$\mu = 1.833 \times 10^{-6}\ \text{kgf-s}/\text{m}^2$ $\nu = 1.466 \times 10^{-5}\ \text{m}^2/\text{s}$

9.1.3 濕空氣之濕度與比重量

- 絕對濕度(absolute humidity) ω 或
  濕度比(humidity ratio)

$$\boxed{\omega = \frac{w_v}{w_a}}$$

濕空氣中水汽的質量 w_v
乾空氣的質量 w_a

- 相對濕度(relative humidity)

$$\boxed{\phi = \frac{\gamma_w}{\gamma_s} \times 100 \doteqdot \frac{p_w}{p_s} \times 100\ (\%)}$$

γ_w (kgf/m^3) 空氣中所含水蒸汽重量
γ_s (kgf/m^3) 同溫度下飽和水蒸汽的比重量
p_w 濕空氣中水蒸汽的部份壓力(partial pressure)

- 濕空氣中空氣的部份壓力 $p_a = p - p_w = p - \phi p_s$

$$\phi = \frac{\gamma_w}{\gamma_s} \times 100 \doteqdot \frac{p_w}{p_s} \times 100\ (\%)$$

- 濕空氣中所含空氣的比重量

$$\rho_a = \frac{P_a}{RT_a} \Rightarrow \gamma_a = \rho_a g = \frac{P_a g}{RT_a}$$

$$\boxed{\gamma_a = \frac{p - \phi p_s}{R_a T} \quad [\mathrm{kgf/m^3}]}$$

R_a 乾空氣的氣體常數

- 濕空氣中水蒸汽的比重量

$$\boxed{\gamma_w = \frac{p_w}{R_s T} = \frac{\phi p_s}{R_s T} \quad [\mathrm{kgf/m^3}]}$$

R_s 水蒸汽的汽體常數

- 濕空氣的比重量

$$\gamma = \gamma_a + \gamma_w = \frac{p - \phi p_s}{R_a T} + \frac{\phi p_s}{R_s T} = \frac{p - \phi p_s (1 - R_a / R_s)}{R_a T}$$

$$\gamma=\gamma_a+\gamma_w=\frac{p-\phi p_s}{R_aT}+\frac{\phi p_s}{R_sT}=\frac{p-\phi p_s(1-R_a/R_s)}{R_aT}$$

If $R_a=29.27$, $R_s=47.07$ $\boxed{\gamma=\frac{p-0.378\phi p_s}{R_aT}}$

- 若某溫度 t°C 時 相對濕度 爲ϕ 的濕空氣比重量

$$\boxed{\gamma=1.2931\times\frac{273}{273+t}\times\frac{H-0.378\phi F}{760}}$$

$$\because \gamma_2=\gamma_1\frac{p_2T_1}{p_1T_2}$$

水蒸汽飽和蒸汽壓 p_s (kgf / cm²) 爲 F (mmHg)

濕空氣的壓力 p (kgf / cm²) 爲 H (mmHg)

濕度計上讀得的乾球與濕球的溫度（分別爲 t°C 與 t_f°C）

對應各該溫度下水蒸汽的壓力 p_s 及 p_{sf}

氣壓計上濕空氣的壓力 H（mmHg）

則相對濕度 ϕ

$$\boxed{\phi = \frac{\gamma_w}{\gamma_s} \times 100 \doteqdot \frac{p_w}{p_s} \times 100 \ (\%)}$$

$$\phi = \frac{p_{sf} - 0.00066H(t - t_f)}{p_s}$$

例題 9.1

試求在溫度 20°C，壓力 750mmHg 下，乾空氣與濕度 75 % 濕空氣的比重量。

解：

由 (9.1) 式，乾空氣的比重量為

$$\gamma = 1.2931 \times \frac{273}{273+20} \times \frac{750}{760} = 1.189 \text{ kgf/m}^3$$

因濕空氣在 20°C 時飽和蒸汽壓為

$$F = 0.024 \text{ kgf/cm}^2 \text{ abs} = \frac{0.024}{1.03328} \times 760 = 17.64 \text{ mmHg}$$

應用 (9.10) 式，相對濕度 $\phi = 0.75$ 之濕空氣比重量為

$$\gamma = 1.2931 \times \frac{273}{273+20} \times \frac{750 - 0.378 \times 0.75 \times 17.64}{760}$$

$$= 1.181 \text{ kgf/m}^3$$

9.2 熱力學第一定律

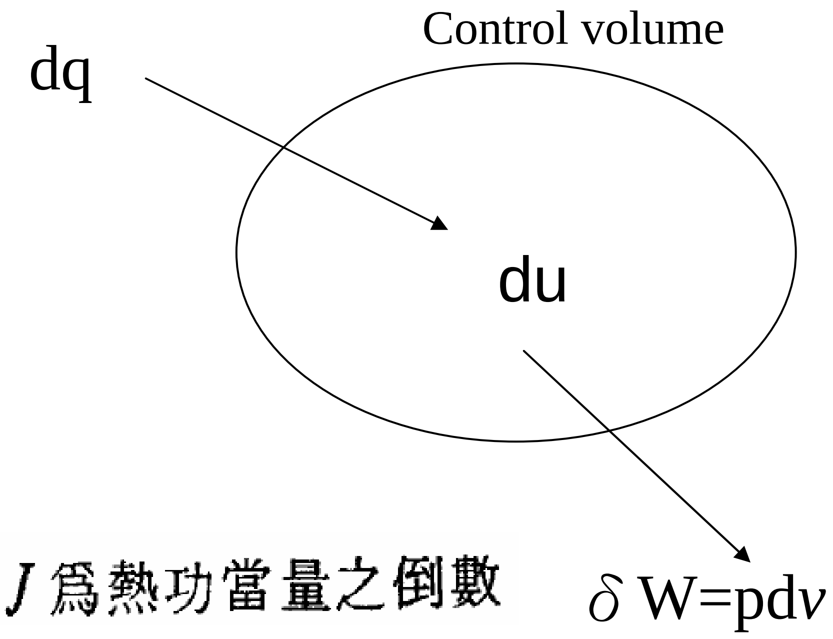
Joule Law

$$\boxed{W = JQ \, (\mathrm{kgf\text{-}m})}$$

熱力學第一定律

$$\boxed{dq = du + A\,p\,dv}$$

$A = 1/J$ 爲熱功當量之倒數



$$\boxed{h = u + A\,pv}$$

$$dh = du + A\,p\,dv + A\,v\,dp$$
$$= dq - A\,p\,dv + A\,p\,dv + A\,v\,dp$$
$$= dq + A\,v\,dp$$

9.4 比　熱

$$\boxed{C=\frac{dq}{dT}\ \text{kcal / kgf °C}}$$

定壓比熱爲壓力保持不變下，使單位重量之氣體溫度升高 1°C 所需之熱量

$$\boxed{C_p=\left(\frac{\partial q}{\partial T}\right)_p}=\left(\frac{\partial h-Av\partial p}{\partial T}\right)_p=\left(\frac{\partial h}{\partial T}\right)_p$$

$$C_p=\frac{dh}{dT}$$　　因焓 h 不隨壓力而變

$$\boxed{h=C_pT}$$

定容比熱為體積保持不變下，使單位重量之氣體溫度升高 1°C 所需之熱量

$$\boxed{C_v = \left(\frac{\partial q}{\partial T}\right)_v} = \left(\frac{\partial u + A p \partial v}{\partial T}\right)_v = \left(\frac{\partial u}{\partial T}\right)_v$$

$$C_v = \frac{du}{dT}$$

因內能 u 不隨比容而變

$$\boxed{u = C_v T}$$

$$h = u + Apv = u + ART$$

$$C_pT = C_v dT + ART$$

$$\boxed{C_p - C_v = AR}$$

$$k = \frac{C_p}{C_v} \qquad C_p - \frac{C_p}{k} = AR$$

$$\boxed{C_p = \frac{k}{k-1} AR}$$

$$kC_v - C_v = AR$$

$$\boxed{C_v = \frac{AR}{k-1}}$$

9.5 氣體的狀態變化

9.5.1 等溫壓縮或膨脹

T=const

Ideal gas $Pv=RT$ $p_1 v_1 = p_2 v_2$

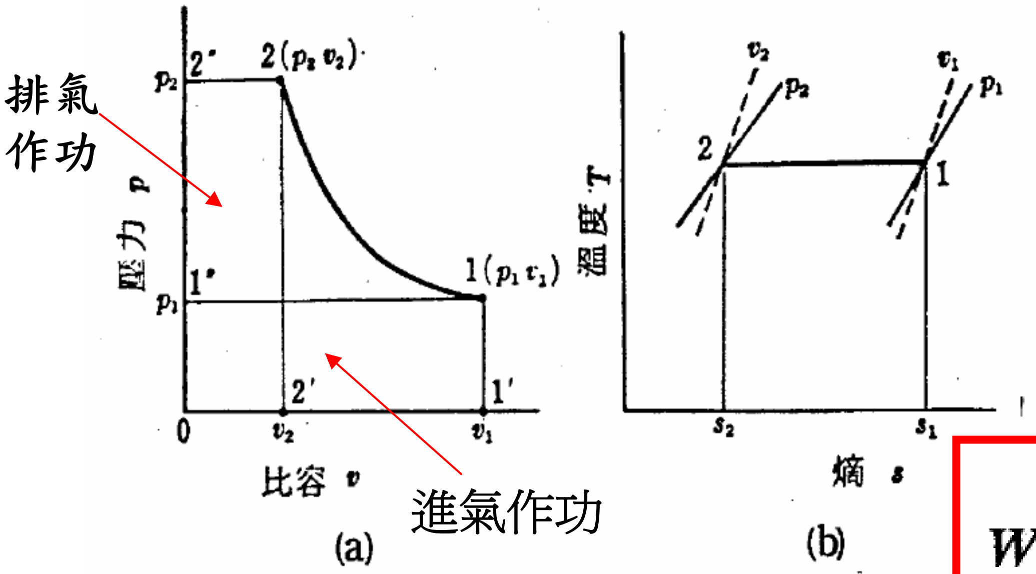
$$W = -\int_1^2 p\,dv$$

$$= -RT\int_1^2 \frac{dv}{v}$$

$$= -RT \ln v \Big|_{v_1}^{v_2}$$

$$= RT \ln \frac{v_1}{v_2}$$

等溫壓縮 $v_1 / v_2 = p_2 / p_1$



(a) (b)

圖9.1 等溫壓縮的 $p-v$ 曲線與 $T-S$ 曲線

壓縮氣體所作之功

$$\boxed{W = p_1 v_1 \ln \frac{p_2}{p_1} = p_2 v_2 \ln\left(\frac{p_2}{p_1}\right)}$$

$$= [\text{面積 } 1-2-2'-1']$$

壓縮機之淨做功 W_{is}

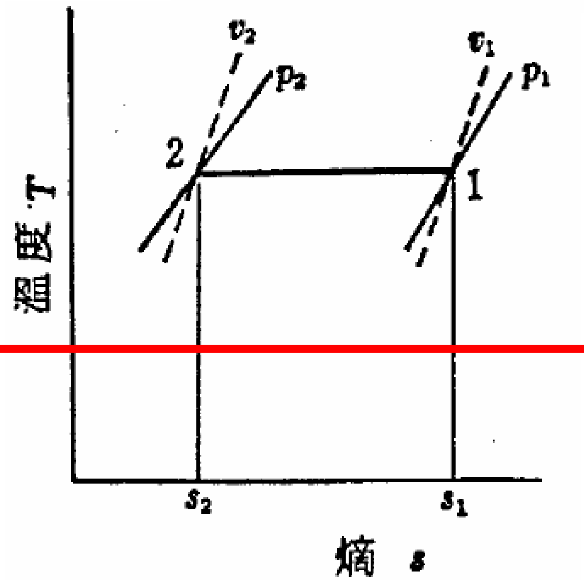
$$\boxed{p_1 v_1 = p_2 v_2}$$

$$\boxed{W_{is} = p_1 v_1 \ln\left(\frac{p_2}{p_1}\right) + \cancel{p_2 v_2} - \cancel{p_1 v_1}}$$

$$W_{is} = p_1 v_1 \ln\left(\frac{p_2}{p_1}\right) \quad [\mathrm{kgf\text{-}m/kgf}]$$

$$= GRT \ln\left(\frac{p_2}{p_1}\right) \quad [\mathrm{kgf\text{-}m}]$$

熵 (entropy) 之變化量 $S_2 - S_1 = \int \frac{dq}{T}$



等溫過程對外傳出之熱量 $\boxed{q = T(S_2 - S_1)}$

例題 9.2：

將 2 kgf 的乾燥空氣在 20°C 下等溫壓縮，壓力從 1 kgf / cm² 增至 5 kgf / cm²，試求壓縮功。但空氣之氣體常數 $R = 29.27$〔kgf-m/kgf·°K〕。

解：

依題意 $p_1 = 1$ kgf / cm²，$p_2 = 5$ kgf / cm²，$T = 273 + 20 = 293$°K

$R = 29.27$ kgf-m/kgf°K，$G = 2$ kgf

由 (9.31b)，

$$\boxed{W_{is} = GRT \ln\left(\frac{p_2}{p_1}\right) \quad \text{〔kgf-m〕}}$$

$$W_{is} = 2 \times 29.27 \times 293 \times \ln\left(\frac{5}{1}\right) = 2.76 \times 10^4 \text{ kgf-m}$$

9.5.2
絕熱壓縮或膨脹過程 (adiabatic compressive or expansive process)

$\boxed{dq=0}$ 熱力學第一定律

$$dq = du + A\,p\,dv = 0$$

$$C_v\,dT + A\,p\,dv = 0 \quad \text{或} \quad C_v\,dT = -A\,p\,dv$$

$$dh = du + A\,p\,dv + A\,v\,dp$$

$$A\,v\,dp + A\,p\,dv = dh - du = C_p\,dT - C_v\,dT = (k-1)C_v\,dT$$

$$= (1-k)A\,p\,dv$$

$$k\,p\,dv + v\,dp = 0$$

$$\boxed{pv^k = \text{常數}}$$

絶熱壓縮或膨脹 $\boxed{dq=0}$

$$\boxed{p_1 v_1{}^k = p_2 v_2{}^k = \text{常數}}$$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^k \quad \text{或} \quad \frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{1/k}}$$

$$\frac{p_2}{p_1}\frac{v_2}{v_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}}$$

$$dq = C_v\,dT + Ap\,dv = C_v\,dT + ART\,dv/v$$
$$= C_v\,dT - ART\,dp/p$$

兩邊除以T，並積分之得

$$S_2 - S_1 = C_v \ln\frac{T_1}{T_2} + AR\ln\frac{v_1}{v_2} = C_p \ln\frac{T_1}{T_2} - AR\ln\frac{p_1}{p_2}$$
$$= C_v \ln\frac{p_1 v_1{}^k}{p_2 v_2{}^k} = 0 \qquad \boxed{pv^k = \text{常數}}$$

絕熱壓縮所作之功W_{ad}

$$W_{ad} = \int_1^2 -p\,dv = -C\int_{v_1}^{v_2}\frac{dv}{v^k}$$
$$= -\frac{C}{1-k}\left(v_2{}^{1-k} - v_1{}^{1-k}\right)$$
$$= \frac{1}{k-1}\left(p_2 v_2 - p_1 v_1\right)$$

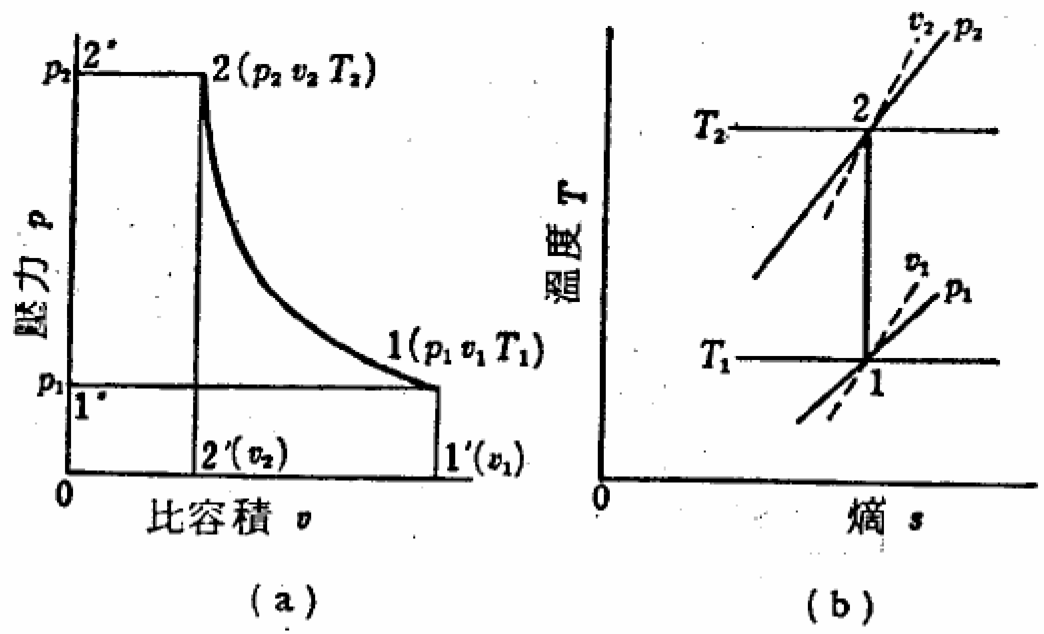


圖 9.2 絕熱壓縮 $p-v$ 曲線與 $T-S$ 曲線

$$W_{ad}=\frac{1}{k-1}(p_2v_2-p_1v_1) = \text{面積 } 1'-1-2-2'$$

$$=\frac{1}{k-1}p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right] \quad [\text{kgf-m/kgf}]$$

$$W_{ad}=\frac{1}{k-1}(p_2v_2-p_1v_1)+p_2v_2-p_1v_1 = \frac{k}{k-1}(p_2v_2-p_1v_1)$$

$$W_{ad}=\frac{1}{k-1}(p_2v_2-p_1v_1)+p_2v_2-p_1v_1=\frac{k}{k-1}(p_2v_2-p_1v_1)$$

或

$$W_{ad}=\frac{1}{k-1}p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right]+p_2v_2-p_1v_1$$

$$=\frac{1}{k-1}p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right]+p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right]$$

$$=\frac{k}{k-1}p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right]\quad[\mathrm{kgf\text{-}m/kgf}]$$

5.3 多變壓縮或膨脹（polytropic compression or expansion）

$$\boxed{pv^n = 常數} \quad Tv^{n-1} = 常數$$

n 爲多變指數（polytropic exponent），$n \neq k \neq 1$

多變壓縮功 W_{pol}

$$W_{pol} = \frac{1}{n-1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/n} - 1 \right] \text{〔kgf-m/kgf〕}$$

含有進氣、排氣的壓縮機中，其功爲

$$\boxed{W_{pol} = \frac{n}{n-1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/n} - 1 \right] \text{〔kgf-m/kgf〕}}$$

等温壓縮或膨脹所作之功W_{ad}

$$W_{is} = p_1 v_1 \ln\left(\frac{p_2}{p_1}\right)$$

絕熱壓縮所作之功W_{ad}

$$W_{ad} = \frac{k}{k-1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1 \right] \quad [\mathrm{kgf\text{-}m/kgf}]$$

多變壓縮或膨脹所作之功W_{ad}

$$W_{pol} = \frac{n}{n-1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1 \right] [\mathrm{kgf\text{-}m/kgf}]$$

例題 9.3

20°C 的乾燥空氣從壓力為 1 kgf / cm² 經多變壓縮至 5 kgf / cm²，設 $n=1.3$ ，求壓縮後氣體的溫度與壓縮功。

解：

由 (9.38) 式

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

將 $p_1 = 1$ kgf / cm² ，$p_2 = 5$ kgf / cm² ，$T_1 = 273 + 20 = 293$°K，$n = 1.3$

代入得 $T_2 = 293 \times 50^{0.3/1.0} = 293 \times 5^{0.231} = 425\ \text{K} = 152$°C

由 (9.45) 式，多變壓縮功 W_{pol} 為

$$W_{pol} = \frac{1}{n-1} RT_1 \left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1 \right]$$

$$= \frac{1}{1.3-1} \times 29.27 \times (273+20) \left[\left(\frac{5}{1}\right)^{(1.3-1)/1.3} - 1 \right]$$

$$= 12857.7\ \text{kgf-m/kgf}$$

9.7 靜温、動温和全温

現設氣體在絕熱過程中運動，於速度爲 v，溫度爲 T 時被截止流動而變成速度爲零，溫度爲 T_t

$$\frac{k}{k-1}RT_1+\frac{1}{2}V_1^{\ 2}=\frac{k}{k-1}RT_2+\frac{1}{2}V_2^{\ 2}$$

$$\Rightarrow \quad T_1+\frac{k-1}{kR}\frac{V_1^{\ 2}}{2g}=T_2+\frac{k-1}{kR}\frac{V_2^{\ 2}}{2g}$$

$$\Rightarrow \quad \boxed{T+\frac{k-1}{kR}\frac{V^2}{2g}=T_t}$$

靜溫 (static temperature) + 動溫 (dynamic temperature) = 全溫 (total temperature)

可壓縮性流體 = 停滯點溫度 (stagnation point temperature)

9.8 壓力、溫度、密度與馬赫數之關係

流體中壓力波傳播的速度稱爲音速 C

$$\boxed{C=\sqrt{kgRT}}$$

k 爲比熱比　R 爲氣體常數

g 爲重力加速度　T 爲靜溫

馬赫數(Mack number)

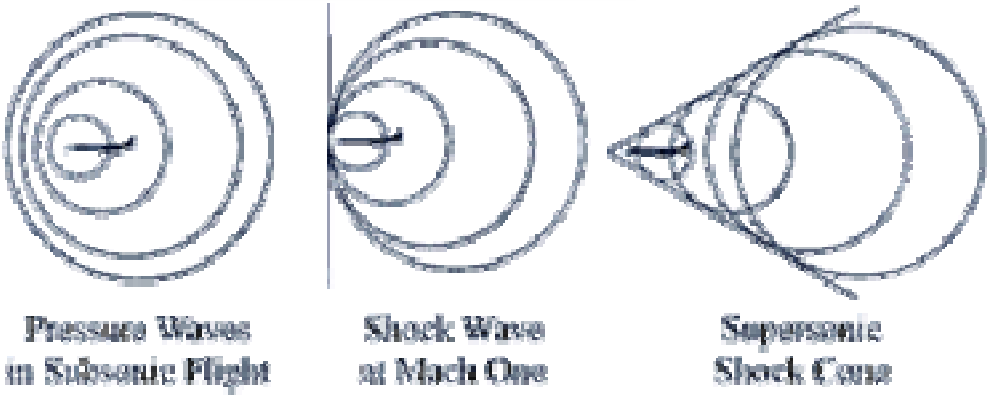
$$\boxed{N_M=\frac{V}{C}=\frac{V}{\sqrt{kgRT}}}$$

$N_M<1$ 之流動稱爲次音速流

$N_M=1$ 時爲音速流

$N_M>1$ 時爲超音速

流體流動之速度 V



在可壓縮性流體裏，設流速爲零時之溫度（全溫）爲T_t，壓力（全壓）爲P_t，密度爲ρ_t之流動，經絕熱變化而變爲流速V，溫度（靜溫）T，壓力（靜壓）P，密度ρ，馬赫數爲N_M

由 $T_t = T + \frac{1}{R}\frac{k-1}{k}\frac{V^2}{2g} = 常數$ $\quad V = N_M\sqrt{kgRT}$

$$\boxed{T = \frac{T_t}{1+\frac{k-1}{2}N_M{}^2}} \qquad \frac{T}{T_t} = 1 + \frac{k-1}{2}N_M{}^2$$

對無滯性氣體 $\mu = 0$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad \frac{T}{T_t} = \left(\frac{P}{P_t}\right)^{(k-1)/k} \text{及} \frac{T}{T_t} = \left(\frac{\rho}{\rho_t}\right)^{k-1}$$

$$\boxed{P = \frac{P_t}{\left(1+\frac{k-1}{2}N_M^2\right)^{k/k-1}}} \qquad \boxed{\rho = \frac{\rho_t}{\left(1+\frac{k-1}{2}N_M^2\right)^{1/k-1}}}$$

例題 9.5

求空氣溫度（靜溫）爲 20°C ，風速爲 400m/ s 時之馬赫數？

解：

$$馬赫數 N_M = \frac{V}{\sqrt{kgRT}}$$

$$= \frac{400}{\sqrt{1.4 \times 9.81 \times 29.27 \times (273 + 20)}}$$

$$= 1.165$$

例題 9.6

有一空氣槽之內部壓力為 3.5 kgf / cm² abs ，溫度 34°C，空氣經一噴嘴噴至大氣中，設噴嘴喉部之馬赫數為 1 ，試求喉部處之空氣溫度、靜壓、密度及流速。

解：

$$T=\frac{T_t}{1+\frac{k-1}{2}N_M^2}=\frac{273+34}{1+\frac{1.4-1}{2}(1)^2}=255.83\,\text{K}=-17.17°\text{C}$$

$$P=\frac{p_t}{\left(1+\frac{k-1}{2}N_M^2\right)^{k/(k-1)}}=\frac{3.5}{\left[1+\frac{1.4-1}{2}(1)^2\right]^{1.4/(1.4-1)}}$$

$$=1.85\ \text{kgf}/\text{cm}^2\ \text{abs}$$

$$\rho = \frac{\rho_t}{\left(1+\frac{k-1}{2}N_M^2\right)^{1/k-1}} = \frac{P_t/RT_t}{\left(1+\frac{k-1}{2}N_M^2\right)^{1/k-1}}$$

$$= \frac{\dfrac{3.5\times 10000}{29.27\times(273+34)}}{\left[1+\dfrac{1.4-1}{2}\times(1)^2\right]^{1/(1.4-1)}} = 2.46\ \mathrm{kgf\cdot sec^2/m^4}$$

$$V_1 = N_{M1}\sqrt{kgRT}$$

$$= 1\times\sqrt{1.4\times 9.81\times 29.27\times(255.83)}$$

$$= 320.7\ \mathrm{m/s}$$

第十章

空氣機械(I)

Air Machinery (I)

10.1.1 分 類

日本機械學會

- 壓力在 1 kgf / cm² 以上者(P >1kgf/cm²=10000mmAg =10mAg) 屬於 壓縮機
- 送風機中壓力在 1000 mm Aq 以上，10 m Aq 以內(1kgf/cm²>P>0.1kgf/cm²) 則稱爲鼓風機 (blower)
- 壓力在 1000mmAq 以內者(0.1kgf/cm² >P) 又稱爲扇風機 (fan)

$1mmAq=1kgf/m^2$

$1kgf/cm^2=10000kgk/m^2=10000mmAg=10mAg$

$0.1kgf/cm^2=1000mmAq$

$1000mmAq=0.1kgf/cm^2$

$P >1kgf/cm^2=10000mmAg$ =10mAg 壓縮機(Compressor)

10mAg>P>1000mmAq or $1kgf/cm^2>P>0.1kgf/cm^2$ 鼓風機 (Blower)

$0.1kgf/cm^2 >P$ 風扇 (Fan)

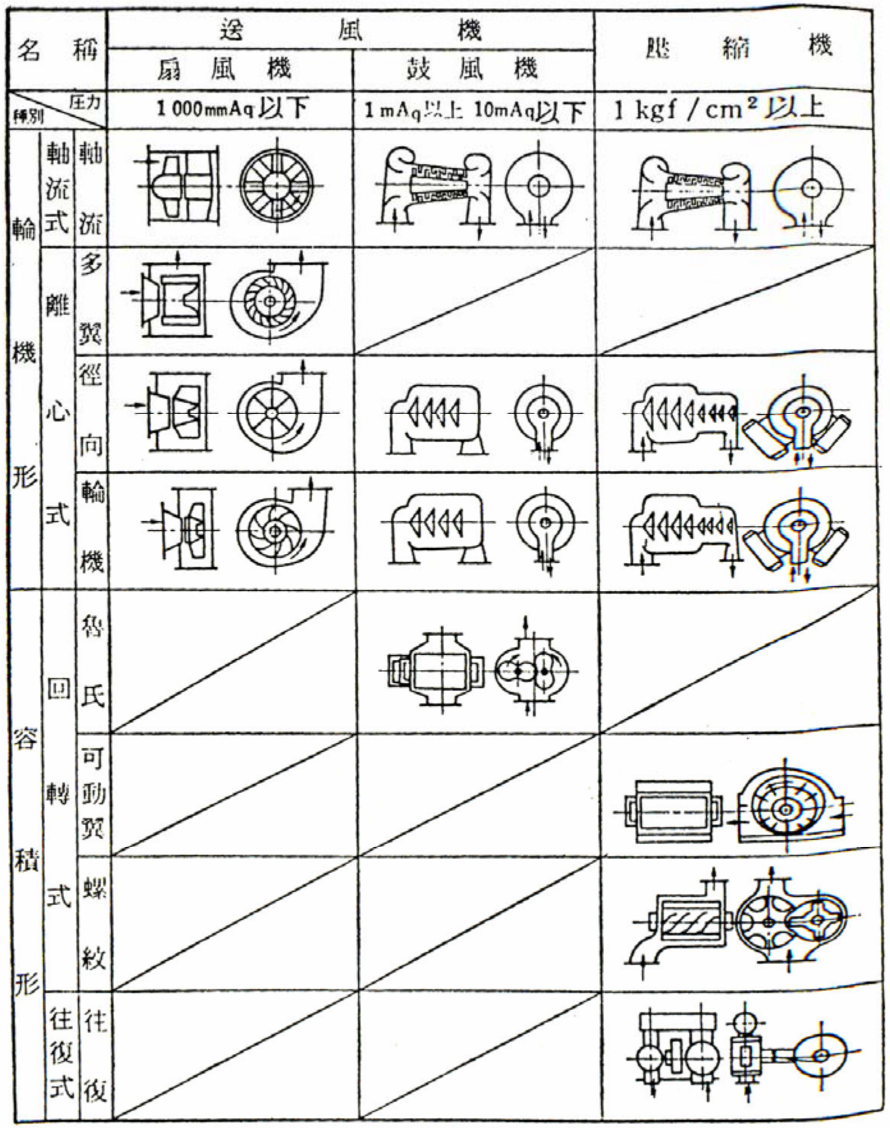


圖 10.2 送風機與壓縮機的適用領域

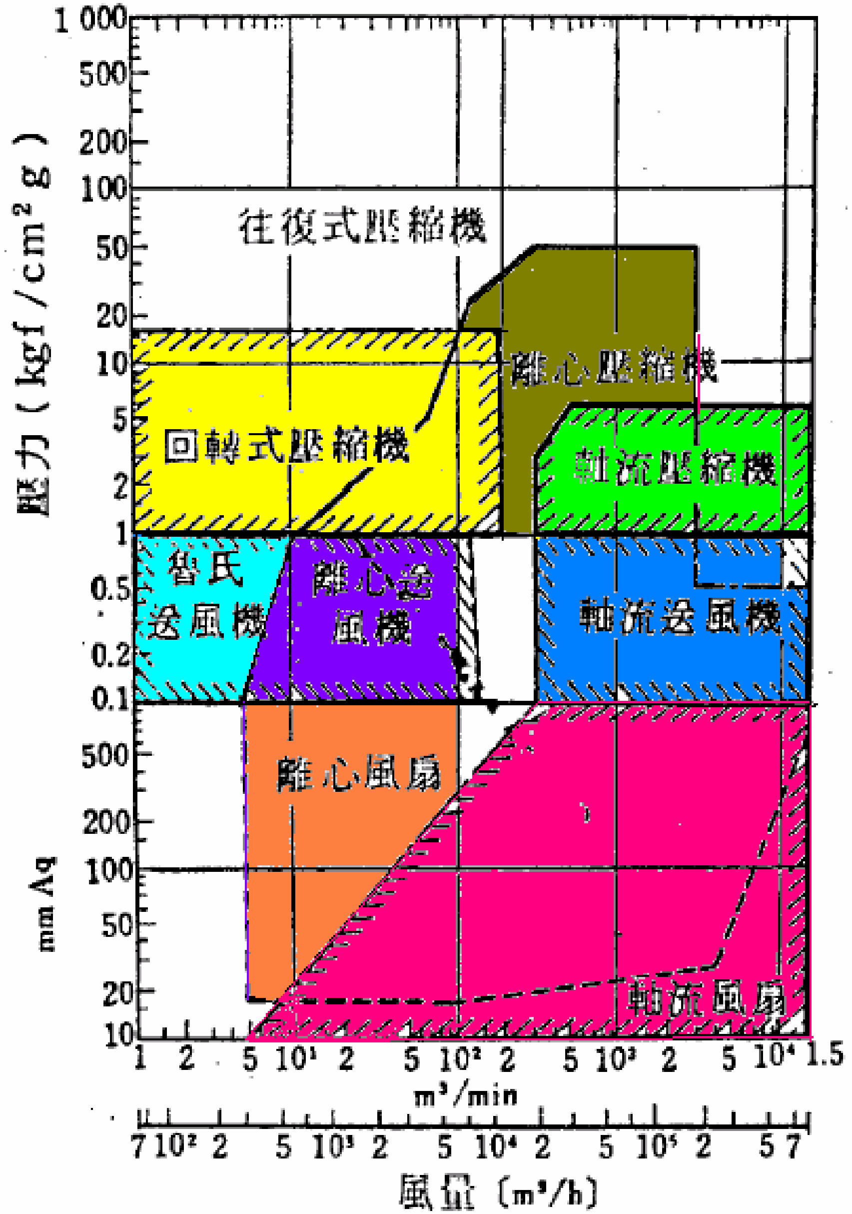


圖 10.2 送風機與壓縮機的適用領域

10.1.2 風量、風壓、動力效率與溫度上升

(a) 風量：爲每單位時間內的排氣量 kgf /min m³ /min 或 m³ / hr

進氣以標準狀態 (大氣壓力 760mmHg ，溫度 20°C ，相對濕度 75 % ，比重量 1.2kgf /m³

(b) 壓力：常用單位有 mmAq 及 mmHg 於高壓力時則用 kgf / cm² (

若爲絕對壓力則加註 abs ，錶壓力加註 g)

在壓縮機的壓力表示上，用出氣與進氣的壓力比 (pressure ratio) 作代表

$1kgf/m^2 = 1mmAq$

$1kgf/cm^2=10000kgf/m^2=10000mmAg=10mAg$

送風機的壓力表示

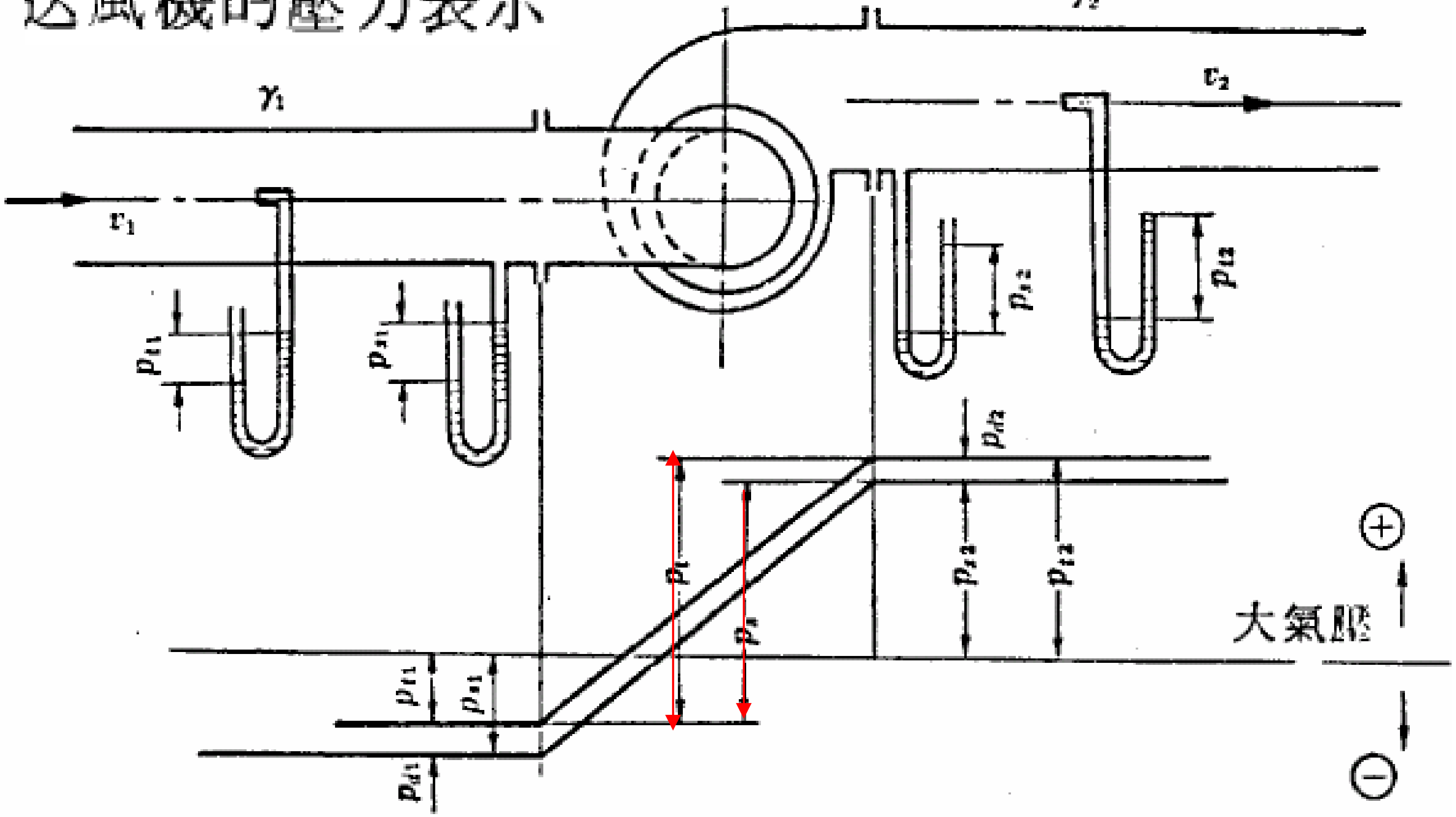


圖 10.3 送風機的壓力上升說明

送風機全壓 $p_t = p_{t2} - p_{t1} = (p_{s2} - p_{s1}) + (p_{d2} - p_{d1})$

送風機靜壓 $\boxed{p_s = p_t - p_{d2}} = (p_{s2} - p_{s1}) - p_{d1}$

送風機之靜壓又稱為風壓

(c) 動力與效率

- 因在壓力比小於 1.03（或風速小於 100m/s）時，氣體可看成不可壓縮性
- 理論全壓空氣動力 L_t

$$\boxed{L_t = \frac{Q_1 p_t}{60 \times 102} = \frac{Q_1}{6120}\{(p_{s2} - p_{s1}) + (p_{d2} - p_{d1})\} \quad (\text{kw})}$$

- 理論靜壓空氣動力 L_s

$$\boxed{L_s = \frac{Q_1 p_s}{60 \times 102} = \frac{Q_1}{6120}\{(p_{s2} - p_{s1}) - p_{d1}\} \quad (\text{kw})}$$

Q_1 為進氣處的風量（m^3/min）

壓力 p 以 mmAq（= kgf/m^2）

1kw=101.97kgf-m/s
≒102kgf-m/s

- 若壓力比在 1.03 以上時 上二式可做適當修正使用，即

  壓力比介於1.07至1.03之間,

$$L_t = \frac{Q_1}{6120}\{(p_{s2}-p_{s1})\times(1-\frac{p_{s2}-p_{s1}}{2kp_{s1}})+(p_{d2}-p_{d1})(\mathrm{kw})$$

$$L_s = \frac{Q_2}{6120}\{(p_{s2}-p_{s1})\times(1-\frac{p_{s2}-p_{s1}}{2kp_{s1}})-p_{d1}\}(\mathrm{kw})$$

k 爲比熱比 (specific heat ratio)

全壓效率 η_t $\boxed{\eta_t = \frac{L_t}{L}\times 100\%}$

靜壓效率 η_s $\boxed{\eta_s = \frac{L_s}{L}\times 100\%}$

實際的軸動力爲 L (kw)

●● 壓力比在 1.07 以上者，氣體的密度與溫度變化都較大（可壓縮）

(1) 理論絕熱動力與全絕熱效率

$W = JQ$〔kgf-m〕

一般無冷卻作用的鼓風機或壓縮機其壓縮功用絕熱理論計算

$v=Q/t=Q/60$〔m3/s〕

$Q=vt$

1kw=101.97kgf-m/s

≒102kgf-m/s

● $$\boxed{W_{ad} = \frac{k}{k-1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1 \right]}$$

若進氣量爲 Q_1 (m³/min) 時則理論絕熱動力 L_{ad} 爲

$L=W/t$

● $$L_{ad} = W/t$$

$$= \frac{k}{k-1} \frac{P_1 Q_1}{6120} \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1 \right]$$

$$= \frac{k}{k-1} \frac{GRT_1}{6120} \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1 \right] \quad \text{(kw)}$$

p_1，p_2 分別爲進排氣壓力

G 爲氣體的重量進氣量 [kgf/min]，即 $G = \gamma_1 Q$，R 爲氣體常數 [kgf-m/kgf-°k]

全絕熱效率 (overall adiabatic efficiency) η_{tad}

● $$\eta_{tad} = \frac{L_{ad}}{L}$$ 約爲 0.60 ～ 0.85

$$\boxed{W_{pol} > W_{ad} > W_{is}}$$

(2) 理論等溫動力與全等溫效率

有冷卻作用且壓縮過程中保持和進氣溫度相同者，以等溫壓縮理論計算

單位重量的氣體等溫壓縮時所作功

$$W_{is} = p_1 v_1 \ln\left(\frac{p_2}{p_1}\right) \quad \text{〔kgf-m/kgf〕}$$

$$\boxed{L_{is} = \frac{p_1 Q_1}{6120} \ln\left(\frac{p_2}{p_1}\right) = \frac{GRT_1}{6120} \ln\frac{p_2}{p_1} \ (\text{kw})}$$

全等溫效率 (over all isothermal efficiency)

$$\eta_{tis} = \frac{L_{is}}{L} = 0.50 \sim 0.70$$

$$\boxed{W_{pol} > W_{ad} > W_{is}}$$

(3) 理論多變動力 L_{pol} (Polytropic power)

- $$W_{pol}=\frac{n}{n-1}p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n}-1\right]\ [\text{kgf-m/kgf}]$$
- $$L_{pol}=\frac{n}{n-1}\frac{p_1Q_1}{6120}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n}-1\right]\ [\text{kw}]$$

n 爲多變指數

- 內動力 L_i 氣體經過壓縮機時獲得之全部能量

$L_i=L-L_m$=軸動力−機械損失動力

$$L_i=\frac{k}{k-1}\frac{p_1Q_1}{6120}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n}-1\right]\ (\text{kw})$$

- 等溫效率 η_{is} (isothermal efficiency)

$$\eta_{is} = \frac{L_{is}}{L_i} = \frac{\ln(p_2/p_1)}{(\frac{k}{k-1})[(\frac{p_2}{p_1})^{n-1/n} - 1]}$$

- 絕熱效率 η_{ad} (adiabatic efficiency)

$$\eta_{ad} = \frac{L_{ad}}{L_i} = \frac{(\frac{p_2}{p_1})^{(k-1)/k} - 1}{(\frac{p_2}{p_1})^{(n-1)/n} - 1}$$

$$\eta_{ad} = \frac{(\frac{p_2}{p_1})^{\frac{k-1}{k}} - 1}{(\frac{p_2}{p_1})^{\frac{1}{\eta_{pol}} \cdot \frac{k-1}{k}} - 1}$$

空氣機械有內部的摩擦、衝突等流體損失、漏洩損失，以及填封和軸承等摩擦的機械損失

全效率(overall efficiency)

$$\eta = \eta_h \eta_v \eta_m = \frac{\text{理論動力}}{\text{原動機之軸動力}}$$

流體效率 η_h

容積效率 η_v

機械效率 η_m

例題 10.2: 有一壓縮機，進氣量爲 2 m³ /min ，壓力爲 1.12 kgf / cm² abs，今將此空氣壓縮至 4.8 kgf / cm² abs 之壓力時所須之軸動力爲 12.5 PS，試求此時之絕熱動力及全絕熱效率各爲若干？

解：

1PS=75kgf-m/s

$$L_{ad}=\frac{k}{k-1}\ \frac{p_1Q_1}{75\times 60}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right]$$

$$=\frac{1.4}{1.4-1}\ \frac{1.12\times 10^4\times 2}{4500}\left[\left(\frac{4.8}{1.12}\right)^{(1.4-1)/1.4}-1\right]$$

$$=9.0\,\mathrm{PS}$$

$$\eta_{tad}=\frac{L_{ad}}{L}\times 100\,\%=\frac{9.0}{12.5}\times 100\,\%=72\,\%$$

(d) 溫度上升

若進氣溫度爲T_1

$$p_1 v_1^k = p_2 v_2^k = \text{常數}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

- 絕熱過程的理論排氣溫度爲$T_{2th} = T_1 (p_2 / p_1)^{(k-1)/k}$

$$PV^n = \text{constant}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n} = \left(\frac{V_1}{V_2}\right)^{(n-1)}$$

- 實際排氣溫度爲$T_2 = T_1 (p_2 / p_1)^{(n-1)/n}$
- 若設絕熱溫度效率 (adiabatic thermal efficiency)

$$\eta_{ad} = \frac{T_{2th} - T_1}{T_2 - T_1}$$

- 實際排氣溫度

$$T_2 = T_1 + \frac{T_{2th} - T_1}{\eta_{ad}} = T_1 + \frac{T_1 \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1 \right]}{\eta_{ad}}$$

$$= T_1 \left[1 + \frac{\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1}{\eta_{ad}} \right]$$

例題10.5

今有一輪機鼓風機（turbo blower），進氣溫度爲 20°C，壓力爲 1.033 kgf / cm² abs，排氣壓力爲 3000 mm Aq，絕熱效率爲 75% 時，求排氣的溫度。

解：

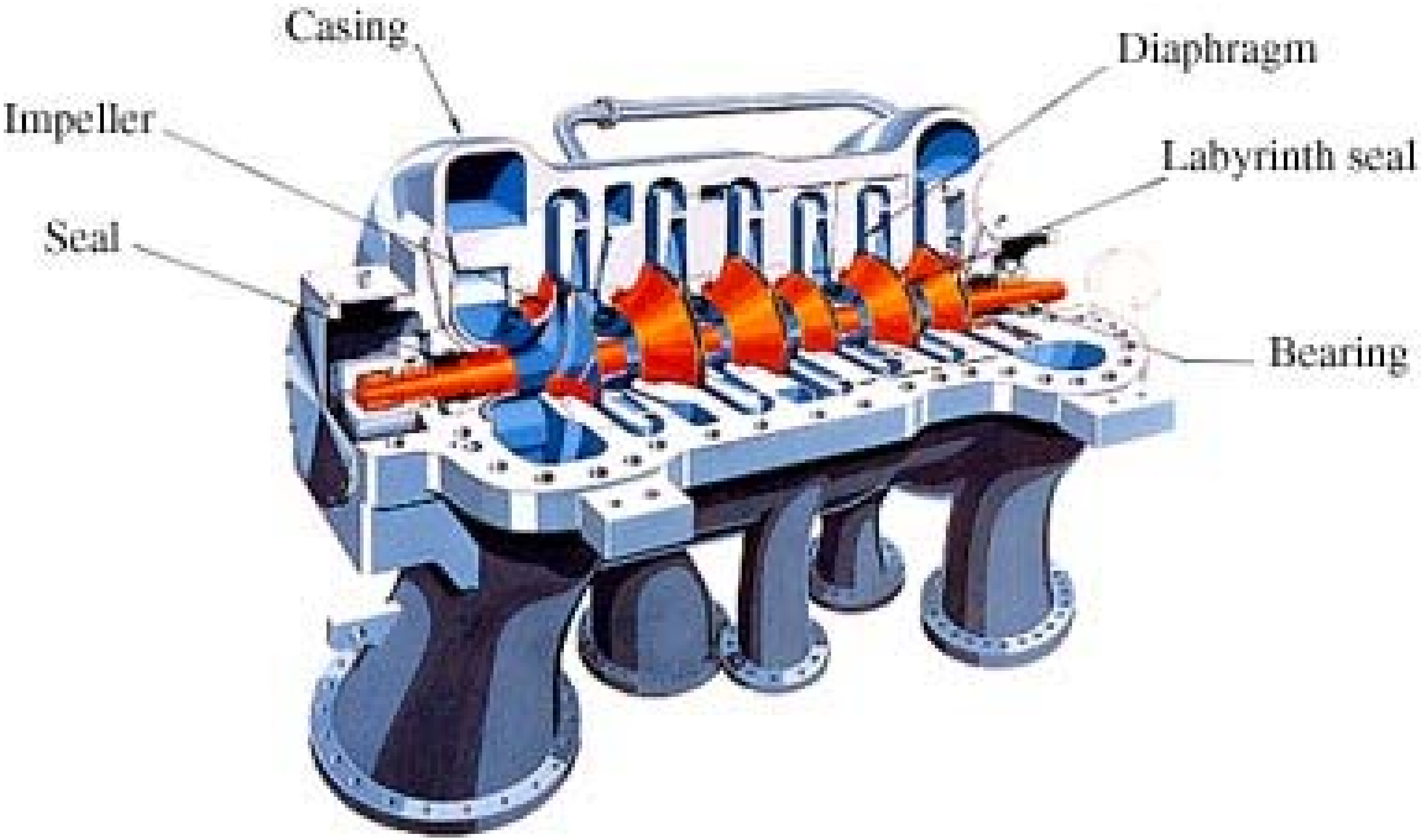
$$\frac{p_2}{p_1}=\frac{(1.033\times10^4+3000)}{1.033\times10^4}=1.29$$

$$T_2=T_1\left[1+\frac{\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1}{\eta_{ad}}\right]$$

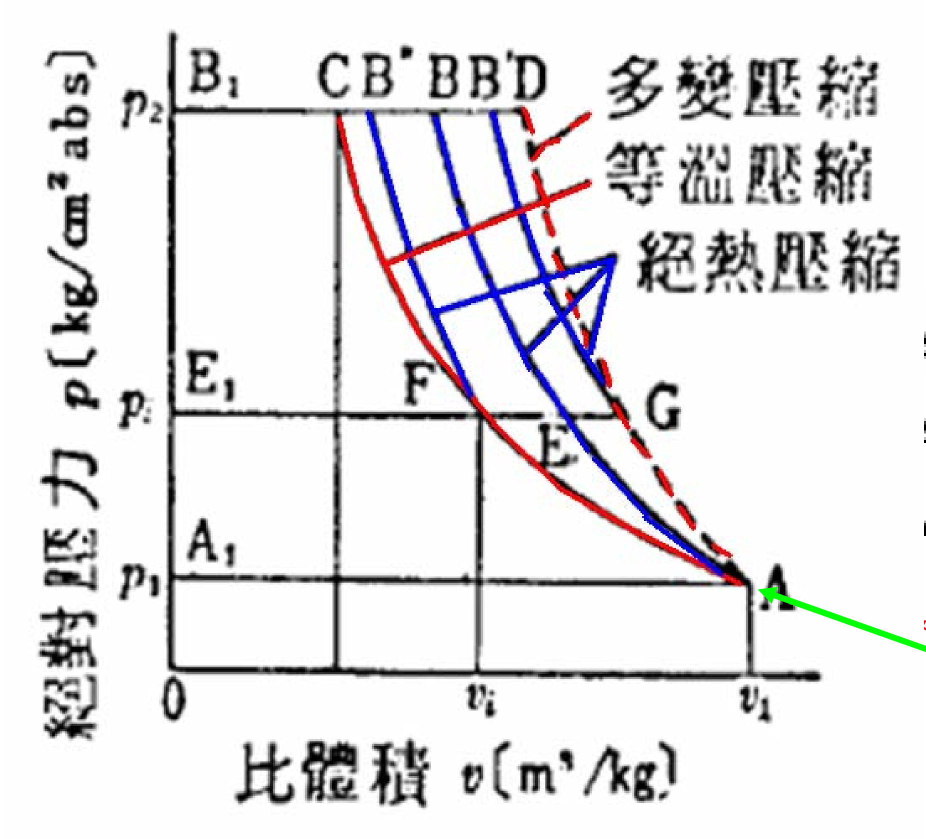
$$\text{排氣溫度}\ T_2=(273+20)\left[1+\frac{1.29^{(1.4-1)/1.4}-1}{0.75}\right]$$

$$=322.5\text{K}\quad=49.5°\text{C}$$

10.1.3 多段壓縮



多段壓縮機的 $p-v$ 圖



$$PV^n = \text{constant}$$

n=n: polytropic

n=1: Isothermal

n=k: Isentropic

AFC 爲等溫壓縮過程

AEB 爲絕熱壓縮過程

AGD 爲 $n>k$ 時的多變壓縮過程

起始點相同時壓縮過程

$$W_{pol} > W_{ad} > W_{is}$$

- 若活塞速度快且氣缸冷卻效果良好，則壓縮過程介於等溫與絕熱之間，即 $1 < n < k$
- 在離心或軸流式壓縮機中，因氣體被急速壓縮，未能充分冷卻，加上機內的各種損耗都轉變成熱，致氣體溫度上升，一般爲 $n > k$

令 f_∞爲多變和絕熱揚程之差與絕熱揚程之比

$\boxed{W_{pol} > W_{ad} > W_{is}}$

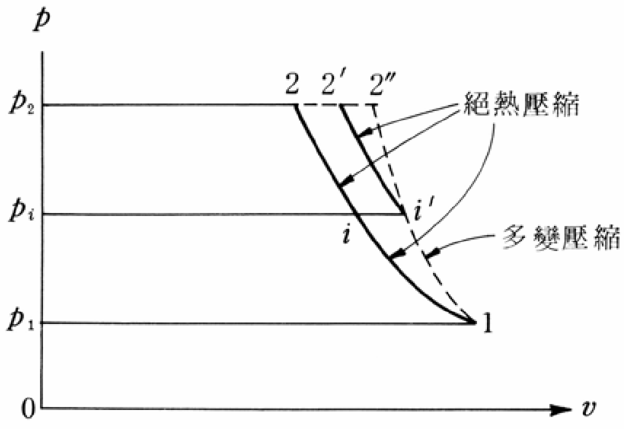
$\boxed{W_{pol} > W_{ad} > W_{is}}$ ● 若壓縮過程之段數爲無限多時，其情況等於多變壓縮過程

令 $$f_\infty = \frac{W_{pol} - W_{ad}}{W_{ad}} = \frac{W_{pol}}{W_{ad}} - 1 = \frac{\eta_{pol}}{\eta_{ad}} - 1, \quad \eta_{pol} = \frac{L_{pol}}{L_i} = \frac{n}{n-1}\frac{k-1}{k}$$

$$\Rightarrow \frac{n}{n-1} = \frac{1}{\eta_{pol}}\frac{k-1}{k}$$

$$f_\infty = \frac{\eta_{pol}}{\eta_{ad}} - 1 = \frac{\frac{n}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right]}{\frac{k}{k-1}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right]} - 1$$

$$= \eta_{pol}\frac{\left(\frac{p_2}{p_1}\right)^{1/\eta_{pol}\cdot(k-1)/k} - 1}{\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1} - 1$$

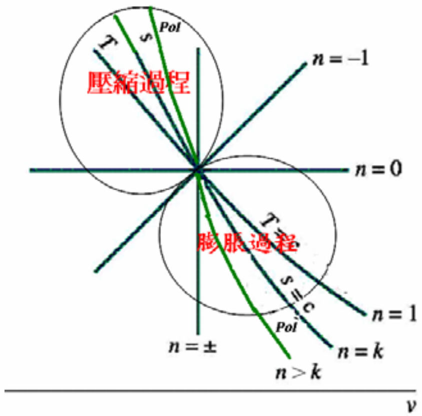


間冷却之多段壓縮

有限

● 若段數爲 z 時的 f_∞ 值爲 f，則 $f = f_\infty\left(1 - \frac{1}{z}\right)$

$$1 + f_\infty = \eta_{pol} / \eta_{ad} \Rightarrow \eta_{pol} = \eta_{ad}(1 + f)$$



● 若壓縮分爲 z 段，中間無冷却，則全段全壓力揚程

$$\Sigma H = H_{ad}(1 + f) = H_{ad}\left(1 + f_\infty\left(1 - \frac{1}{z}\right)\right)$$

■例題

一二段鼓風機進氣量 200 m^3/min ，進氣壓力爲 1.033 kgf/cm^2 abs ，進氣溫度爲 20°C ，排氣總壓爲 3500 mmAq ，若絕熱效率爲 74 % ，不使用中間冷卻，試求所需之揚程爲若干？

解：由已知條件知壓力比 $\dfrac{p_2}{p_1} = \dfrac{1.033 \times 10^4 + 3500}{1.033 \times 10^4} = 1.339$

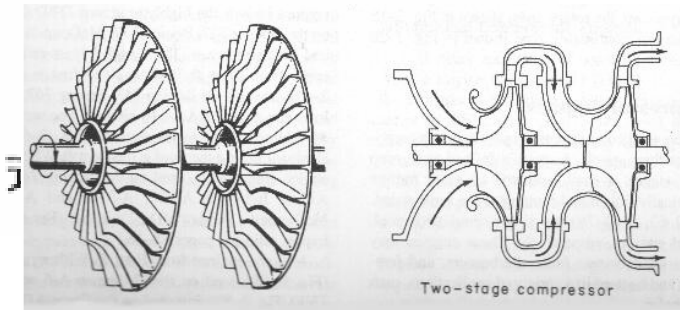
所需之揚程爲 $\Sigma H_z = H_{ad}(1 + f)$ 其中 H_{ad} 爲

$$H_{ad} = \frac{k}{k-1} R T_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right]$$

$$= \frac{1.4}{1.4 - 1} \times 29.27 \times (273 + 20) [1.339^{(1.4-1)/1.4} - 1]$$

$$= 2611 \quad \text{kgf-m/kgf}$$

將 $\eta_{ad} = 0.74$ 及 $(p_2/p_1) = 1.339$ 代入



$$0.74=\frac{1.339^{\frac{1.4-1}{1.4}}-1}{1.339^{\frac{1}{\eta_{pol}}\times\frac{1.4-1}{1.4}}-1}$$

$$\eta_{ad}=\frac{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}-1}{\left(\frac{p_2}{p_1}\right)^{\frac{1}{\eta_{pol}}\cdot\frac{k-1}{k}}-1}$$

$$\Rightarrow\quad 0.74\times1.339^{\frac{1}{\eta_{pol}}\times\frac{1.4-1}{1.4}}-0.74=1.339^{\frac{1.4-1}{1.4}}-1$$

$$\Rightarrow\quad 1.339^{\frac{1}{\eta_{pol}}\times\frac{1.4-1}{1.4}}=\frac{1}{0.74}\left(1.339^{\frac{1.4-1}{1.4}}-1+0.74\right)$$

$$=1.1175$$

兩邊各取對數得

$$\frac{1}{\eta_{pol}}\times\frac{1.4-1}{1.4}=\frac{1}{\ln 1.339}\times\ln 1.175$$

得出 $\eta_{pol}=0.751$

$$f_{\infty}=\frac{\eta_{pol}}{\eta_{ad}}-1=\frac{0.751}{0.74}-1=0.015$$

$$f=f_{\infty}\left(1-\frac{1}{z}\right)=0.015\left(1-\frac{1}{2}\right)$$

$$=0.0075$$

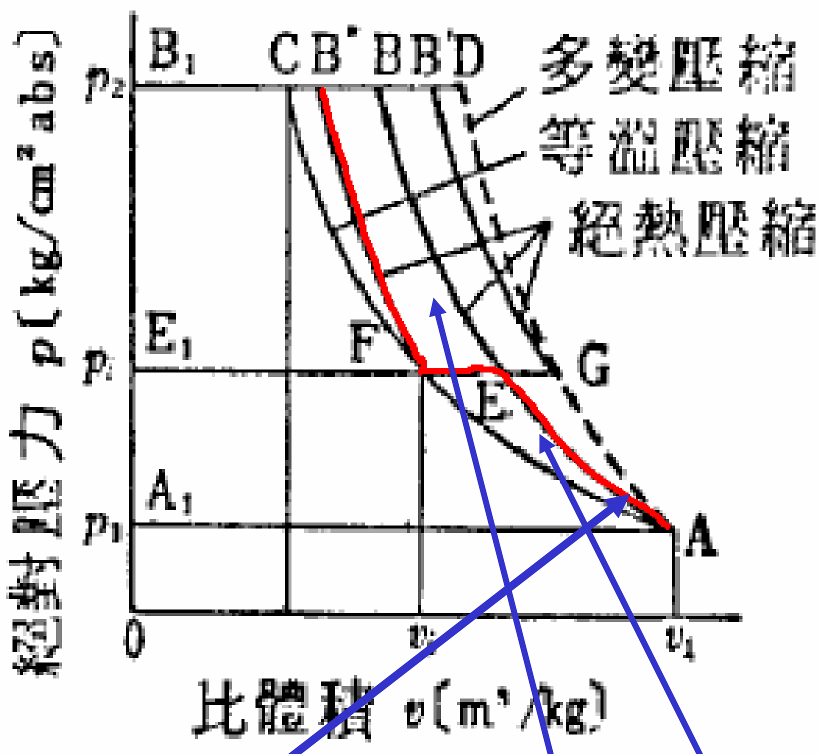
$\Rightarrow$ 得出所需之揚程爲

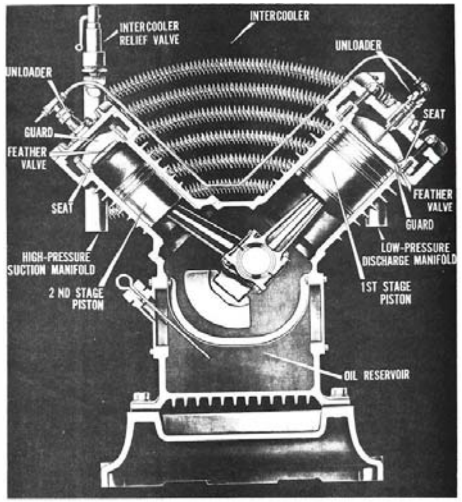
$$\Sigma H_z=H_{ad}(1+f)=2611(1+0.0075)$$

$$=2631\ \text{kgf-m/kgf}$$

● 具有中間冷卻的 2 段壓縮

於是三段壓縮時之功 W 爲





$$W=\frac{k}{k-1}p_1v_1\left[\left(\frac{p_i}{p_1}\right)^{(k-1)/k}-1\right]+\frac{k}{k-1}p_iv_i\left[\left(\frac{p_2}{p_i}\right)^{(k-1)/k}-1\right]$$

AFC 爲等溫壓縮線，$p_1v_1=p_iv_i$

$$=\frac{k}{k-1}p_1v_1\left[\left(\frac{p_i}{p_1}\right)^{(k-1)/k}+\left(\frac{p_2}{p_i}\right)^{(k-1)/k}-2\right]\ (\text{kgf-m/kgf})$$

將上式對 p_i 微分，並使之等於零，可決定使 W 爲最小時的理想中間壓力 $\boxed{p_i}$

$$\frac{dW}{dp_i}=\frac{k}{k-1}p_1v_1\left[\frac{k-1}{k}\cdot(p_i)^{-1/k}\cdot p_1^{-(k-1)/k}\right.$$

$$\left.+\frac{1-k}{k}\cdot(p_i)^{(1-2k)/k}\cdot p_2^{(k-1)/k}\right]$$

$$=p_1v_1\left[p_i^{-1/k}\cdot p_1^{-(k-1)/k}-p_i^{(1-2k)/k}\cdot p_2^{(k-1)/k}\right]$$

$$=0$$

即 $$p_i^{-1/k}\cdot p_1^{-(k-1)/k}=p_i^{(1-2k)/k}\cdot p_2^{(k-1)/k}$$

解上式可得

$$\boxed{p_i=(p_1p_2)^{1/2}\quad 或 \quad \frac{p_i}{p_1}=\frac{p_2}{p_i}}$$

在理想中間壓力下，壓縮時所需之最小功W_{min}為

$$p_i = (p_1 p_2)^{1/2} \quad 或 \quad \frac{p_i}{p_1} = \frac{p_2}{p_i}$$

代入

$$W = \frac{k}{k-1} p_1 v_1 \left[\left(\frac{p_i}{p_1}\right)^{(k-1)/k} - 1 \right] + \frac{k}{k-1} p_i v_i \left[\left(\frac{p_2}{p_i}\right)^{(k-1)/k} - 1 \right]$$

$$= \frac{k}{k-1} p_1 v_1 \left[\left(\frac{p_i}{p_1}\right)^{(k-1)/k} + \left(\frac{p_2}{p_i}\right)^{(k-1)/k} - 2 \right] \text{(kgf-m/kgf)}$$

$$W_{min} = \frac{k p_1 v_1}{k-1} \left[2 \left(\frac{p_i}{p_1}\right)^{(k-1)/k} - 2 \right]$$

$$= \frac{2k}{k-1} \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/2k} - 1 \right] p_1 v_1$$

● 同理，具有中間冷却的 z 段壓縮，使功爲最小的 ($z-1$) 個理想中間壓力，係要使 z 段壓縮之壓力比均相同，即

$$\frac{p_{i1}}{p_1}=\frac{p_{i2}}{p_{i1}}=\frac{p_{i3}}{p_{i2}}=\cdots\cdots=\frac{p_2}{p_{i,z-1}}=\sqrt[z]{\frac{p_2}{p_1}}$$

而所需之最小功 W_{min} 爲

$$W_{min}=\frac{zk}{k-1}p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/zk}-1\right]\ (\text{kgf-m/kgf})$$

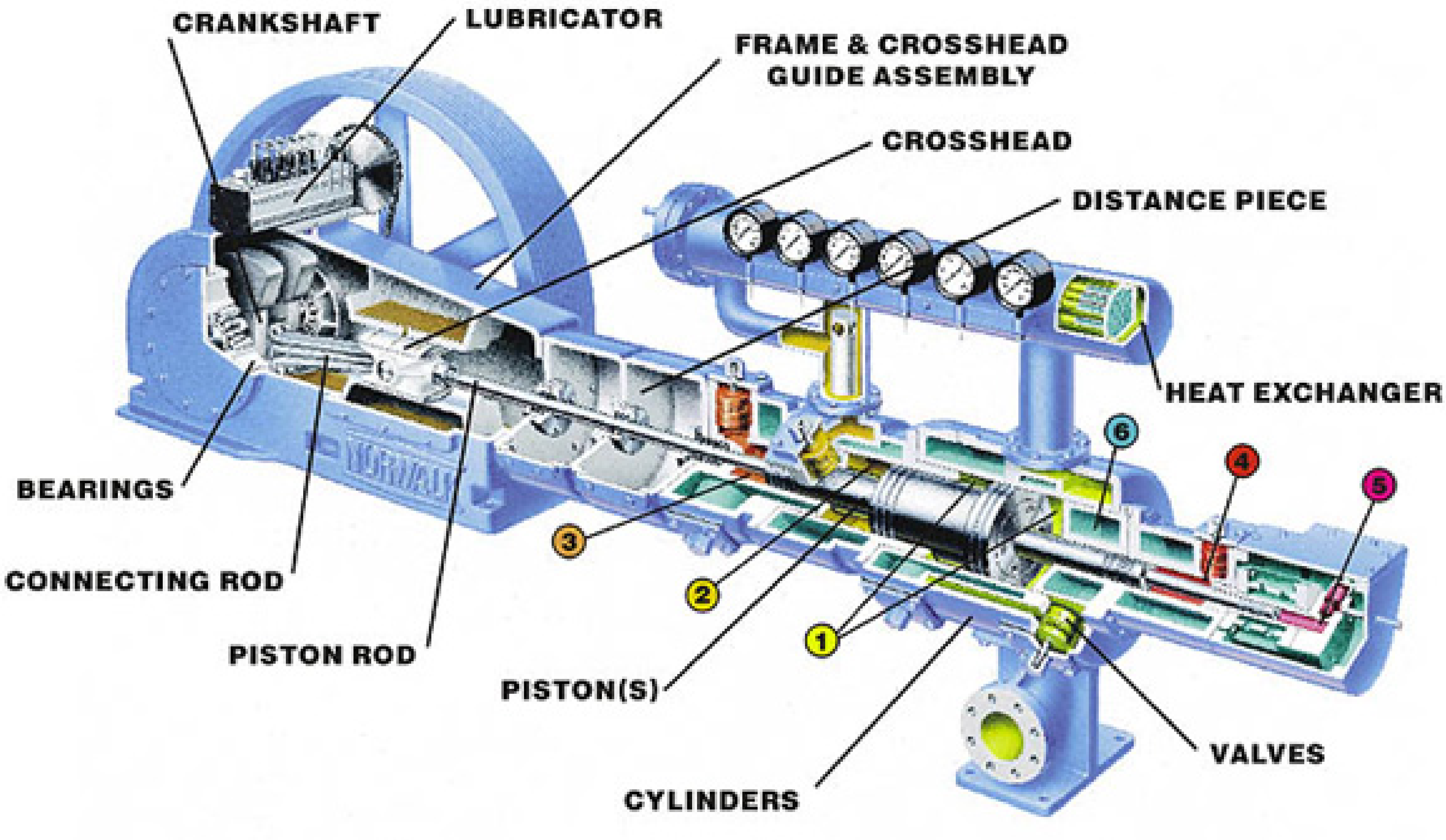
若風量爲 Q_1 (m³/min) 時，理論絕熱壓縮動力爲

$$L_{adz}=\frac{Q_1p_1}{6120}\frac{zk}{k-1}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/zk}-1\right]\quad(\text{kw})$$

For ease of identification, each stage of compression is labeled and color coded in accordance with the table below:

5-stages reciprocating compressor
With cooling water

1	First Stage	2	Second Stage	3	Third Stage
4	Fourth Stage	5	Fifth Stage	6	Cooling Water



例題10.6:

試求滿足進氣量 $200 m^3/min$ ，排氣壓力（全壓）爲3500 mmAq 的二段輪機鼓風機（turbo blower）所需之揚程。設進氣壓力爲 $1.033\ kgf/cm^2$ abs ，進氣溫度爲 20°C ，絕熱效率爲 74 % ，不使用中間冷卻。

解： 壓力比 $p_2/p_1 = (1.033 \times 10^4 + 3500) / 1.033 \times 10^4 = 1.339$

鼓風機全絕熱揚程

$$H_{ad} = \frac{L_{ad}}{\gamma Q} = \frac{k}{k-1} \frac{p_1}{\gamma} \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right] = \frac{k}{k-1} R T_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right]$$

$$= \frac{1.4}{1.4-1} \times 29.27 \times (273+20) \left[(1.339)^{(1.4-1)/1.4} - 1 \right]$$

$$= 2620\ kgf\text{-}m/kgf$$

$$\eta_{ad} = \frac{\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1}{\left(\frac{p_2}{p_1} \right)^{1/\eta_{pol} \cdot (k-1)/k} - 1}$$

將 $p_2/p_1 = 1.339$ ， $\eta_{ad} = 0.74$ ，代入

$$(1.339) \left[^{(1.4-1)/1.4} \cdot \frac{1}{\eta_{pol}} \right] = \{ \left[(1.339)^{(1.4-1)/1.4} - 1 \right] / 0.74 \} + 1$$

$$\frac{1.4-1}{1.4}\cdot\frac{1}{\eta_{pol}}=0.381 \quad , \quad \text{所以 } \eta_{pol}=0.75$$

$$f_{\infty}=\frac{\eta_{pol}}{\eta_{ad}}-1 \Rightarrow f_{\infty}=(0.75/0.74)-1=0.012$$

$$f=f_{\infty}\left(1-\frac{1}{z}\right)$$

$$f=0.012\times\left(1-\frac{1}{2}\right)=0.006$$

於是所求之揚程爲 $\Sigma H=2620\times(1+0.006)=2636\,\mathrm{m}$