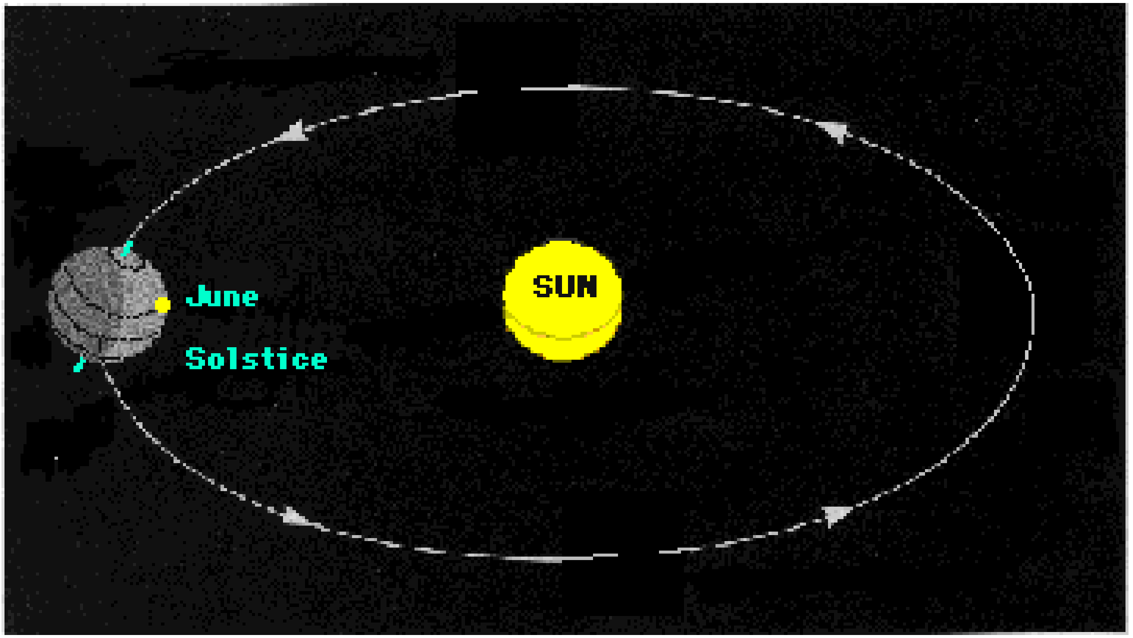
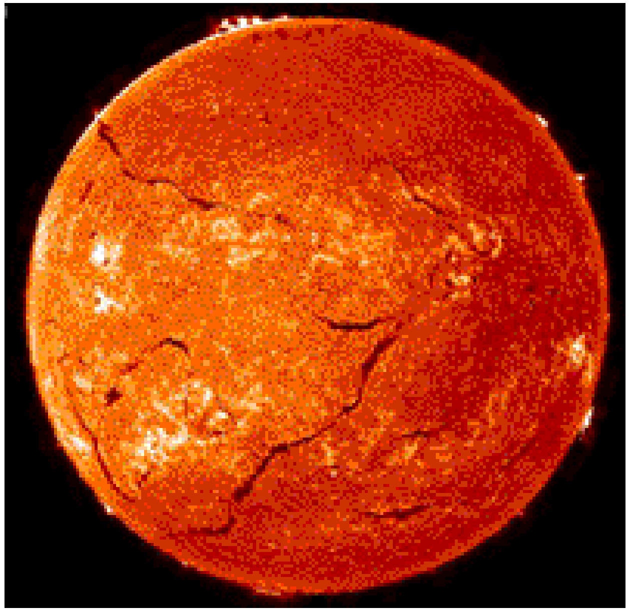
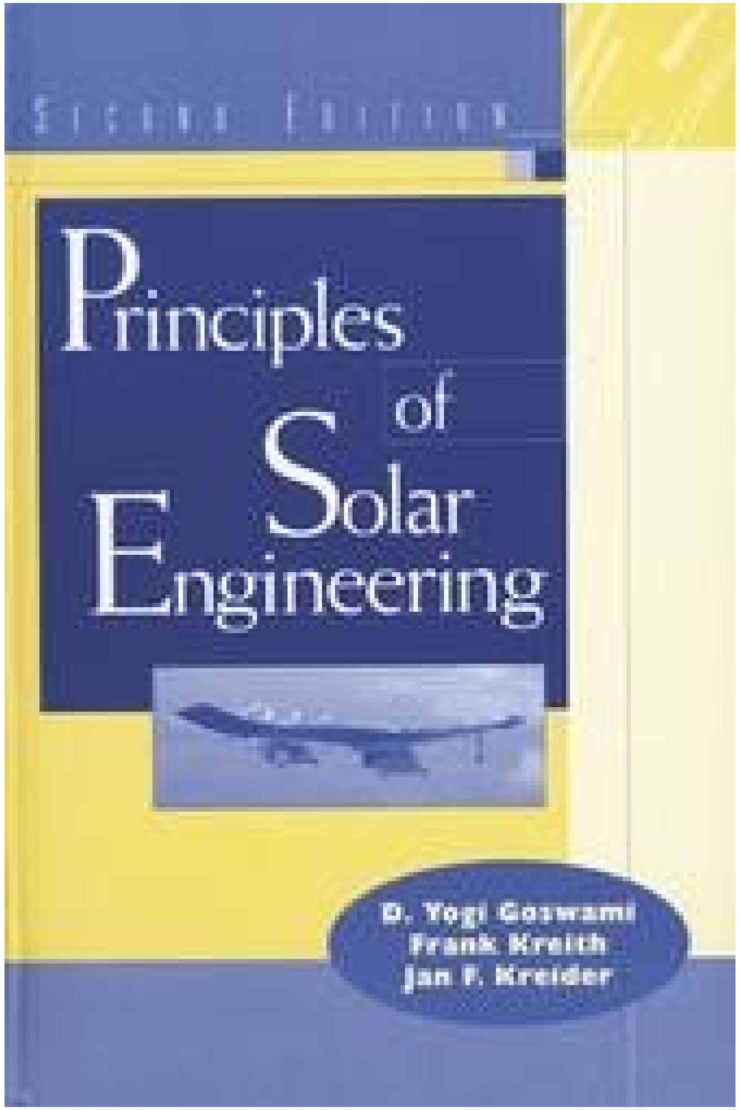
Principle of Solar Engineering

太陽能工程





Principles of Solar Engineering

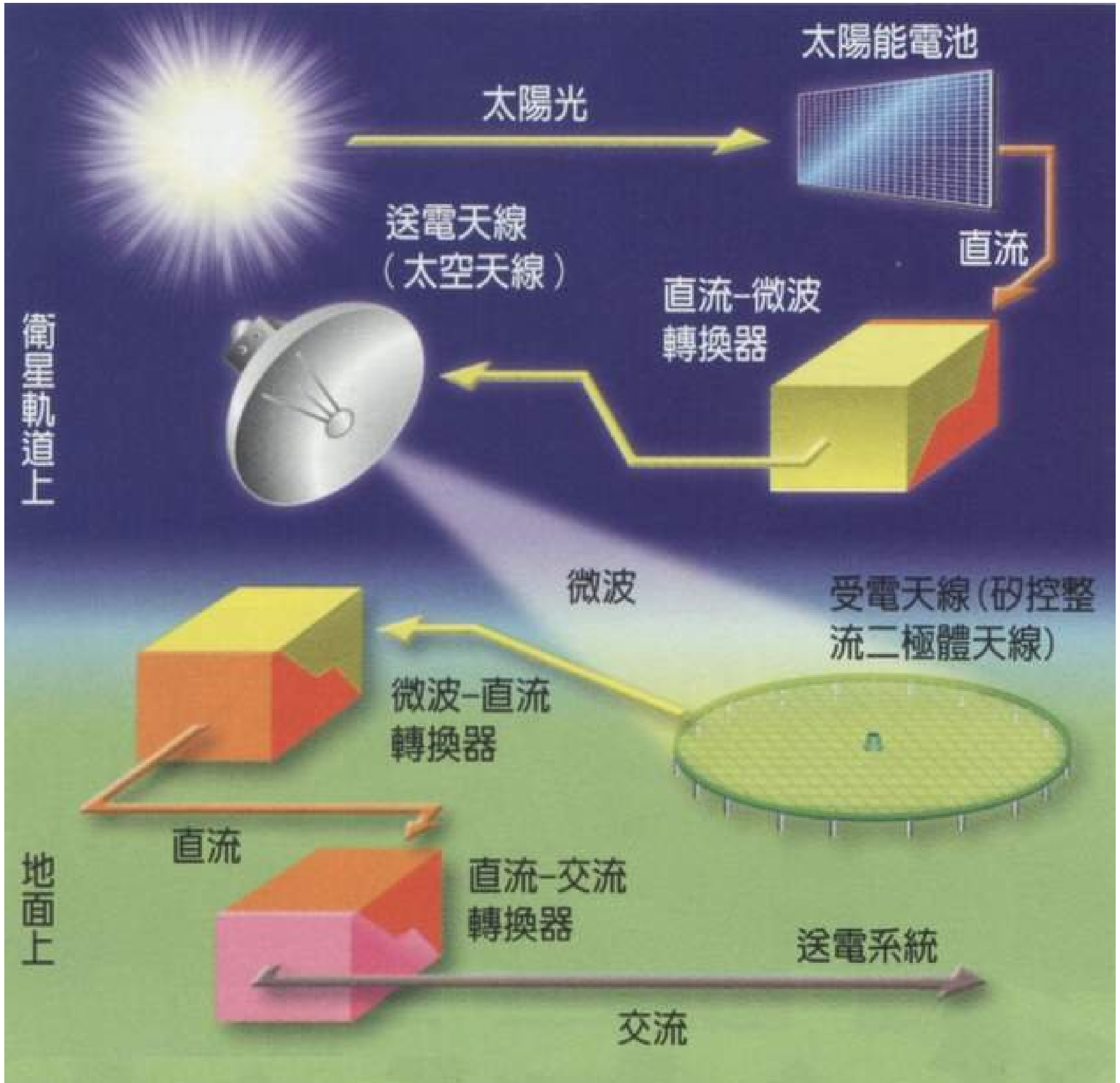


Frank Kreith and Jan F. Kreider

INTRODUCTION TO SOLAR ENERGY AND ITS CONVERSION FOR USE ON EARTH

1

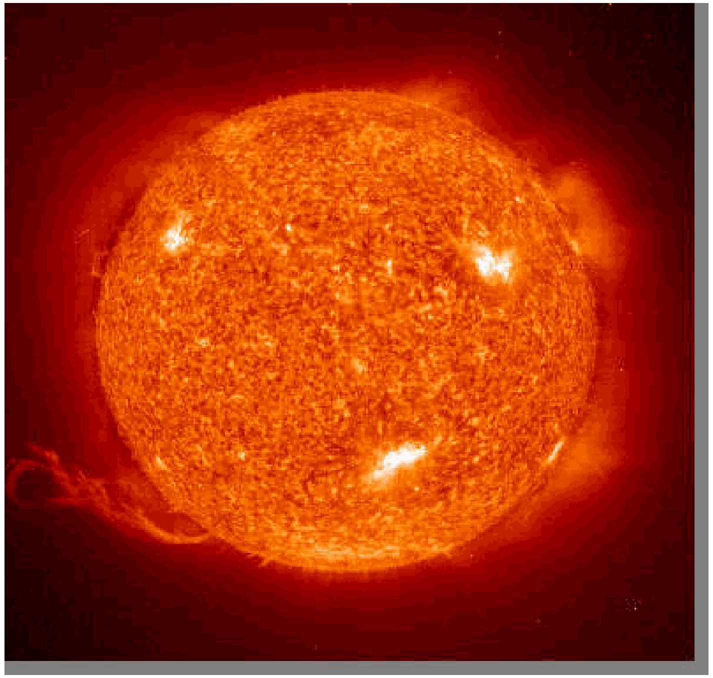
Part (I)



太空太陽能發電系統圖

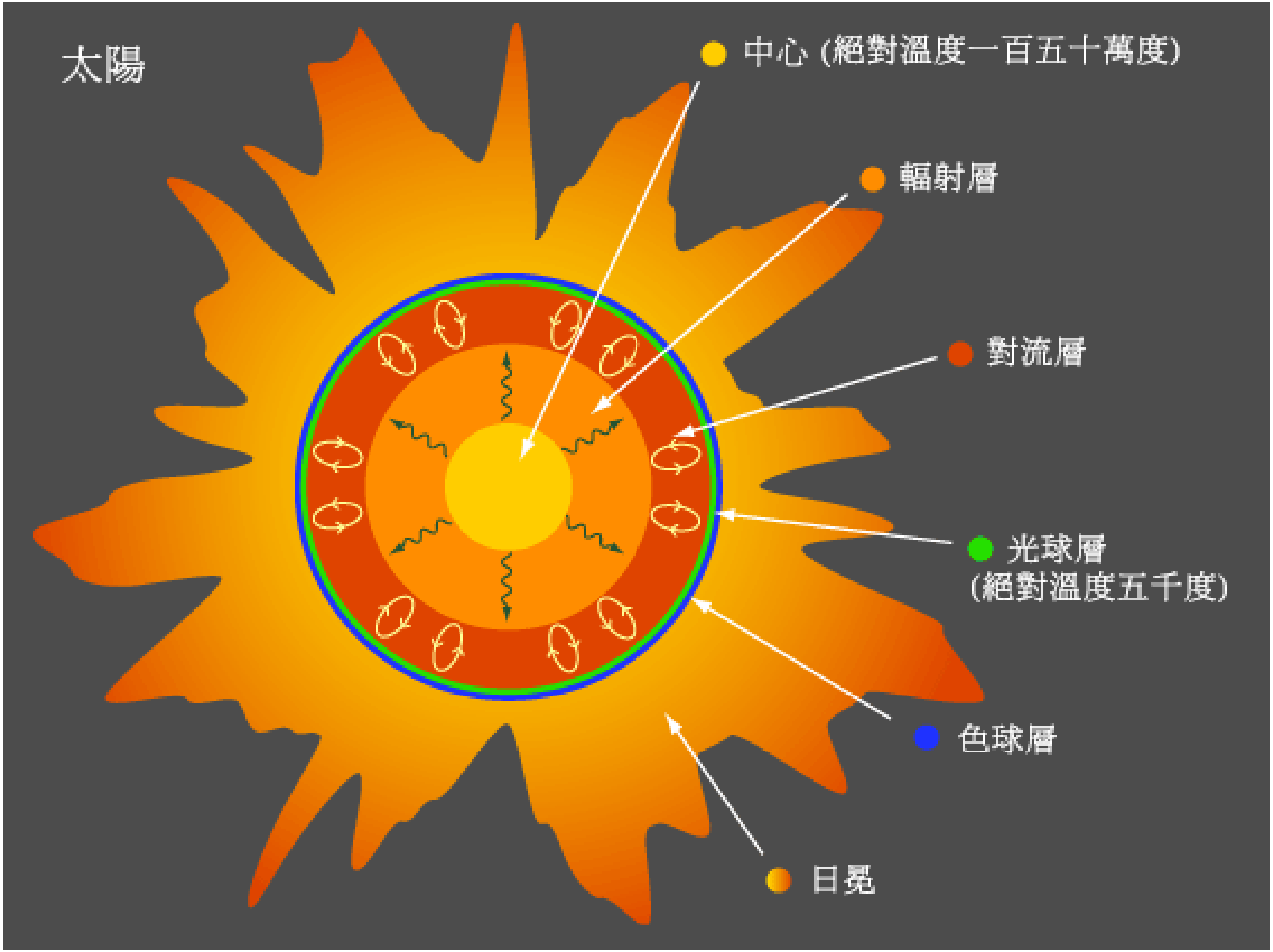
在人造衛星上以太陽能電池產生的直流電能，轉換爲微波後以送電天線送回地面上．地面上的受電裝置再以矽控整流二極體天線接收微波再轉換爲直流後再轉換爲交流電送進輸電系統去．

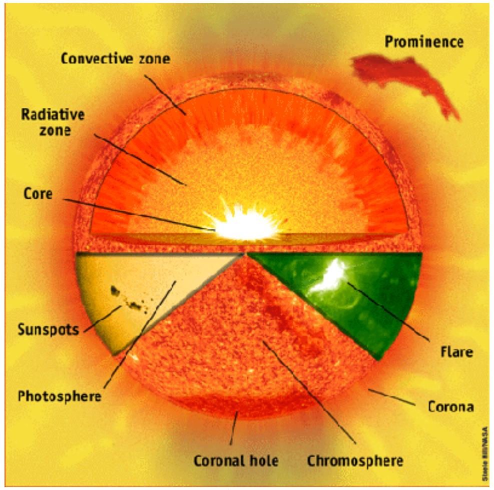
About the Sun 認識太陽

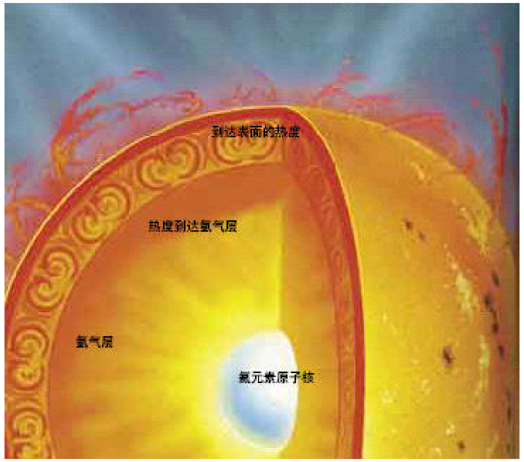


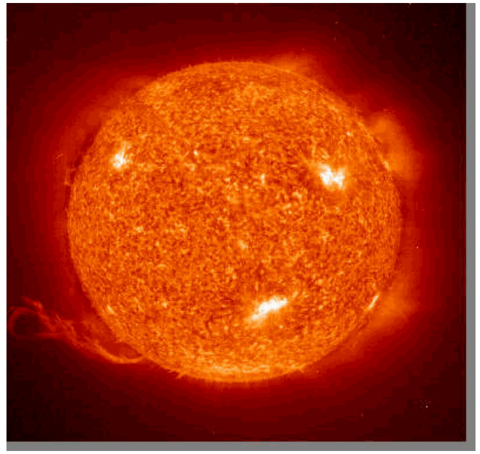
太陽是太陽系的心，其質量佔全太陽系總質量的99.8%，故太陽的引力控制整個太陽系，使其他天體繞著太陽公轉。主要成分為90.965%氫、8.889%氦，以及少量氧、碳、氖、氮、鐵、鎂、矽及硫。

Cross-section of the Sun









● **核心(Core):**厚約175,000km.太陽在這裡以質子－質子鏈把氫轉換成氦,從中釋放能量(核融合) 核心的溫度高達1千4百萬度以上

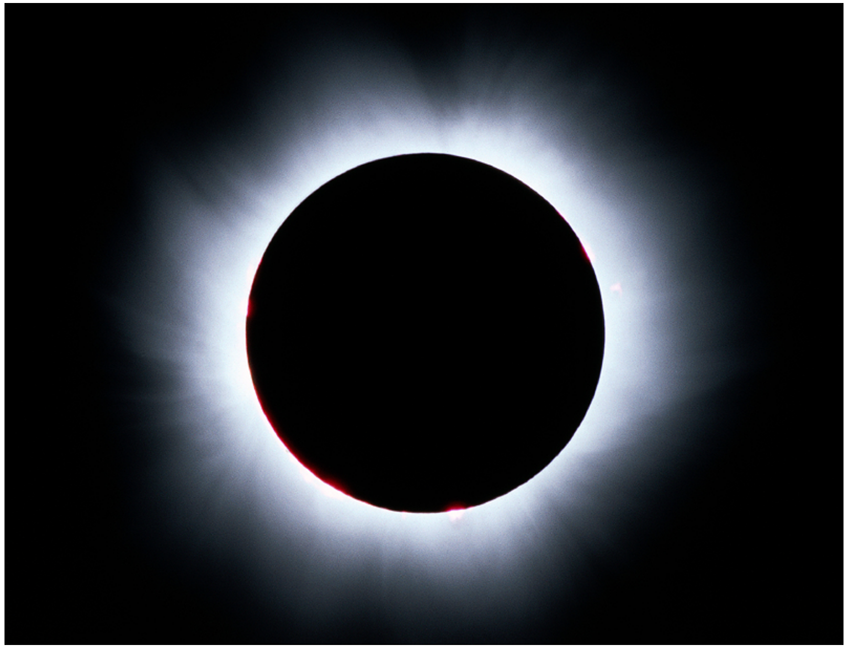
●**輻射層(Radiation Zone):**厚約325,000km.質子－質子鏈產生的能量在此層以輻射的形式離開太陽(光子必須離開太陽,否則太陽會熱到爆炸!), 光子在這裡會吸收發射再吸收再發射,所以光子以極緩慢的速度前進,光子離開輻射層要200,000年,所以科學家這麼看重從核心直接逃出來的中微子.

●**對流層(Convective Envelope):**厚約200,000km.這一層的氣體比輻射層的氣體更不透光,輻射帶走能量已經不能再湊巧,恆星要選取另一種方法帶走能量,這就是對流.

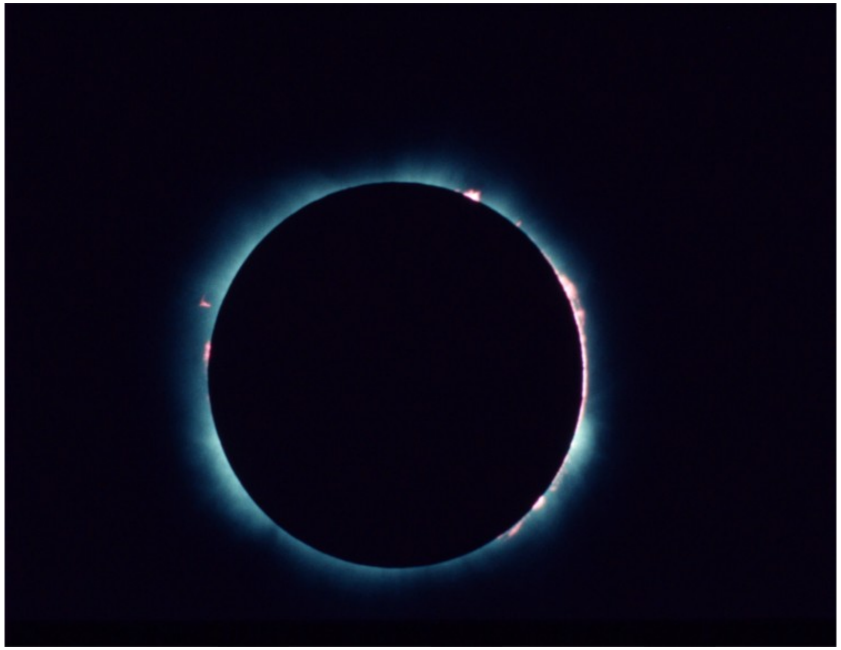
●**光球層(Photosphere):**厚約500km.光球層就是我們平時所見的太陽表面,這裡氣體由對流層走出來,故有起有伏,從多普勒效應得知上升氣流有藍移(靠近我們),下降氣流有紅移(遠離我們),於是形成一個個泡泡,叫做米粒組織(Photospheric Granulation),每個泡泡有2000km闊,並可持續10分鐘.

●**色球層(Chromosphere):**厚約1,500km.色球層有明顯的溫度上升,達到400,000K!而色球層的密度又是極低,故它是透光的,這裡有由光球層噴上來的高速氣流,這些氣流叫做針狀體(spicules),氣流以32km/s的速度被噴到10,000km並持續數分鐘.色球層的高溫是因為太陽磁場的影響

●**日冕(Corona):**日冕其實有滿月那麼光,可是基於光球層比它更光,故把日冕的光輝蓋過了,最佳觀賞日冕的時候是日全蝕時(平時絕對不要用眼直接去望太陽,那是件很危險的事!!!!會盲眼!日冕這麼高溫的原因是由於太陽磁場的影響.



日冕The Corona



色球層的影像

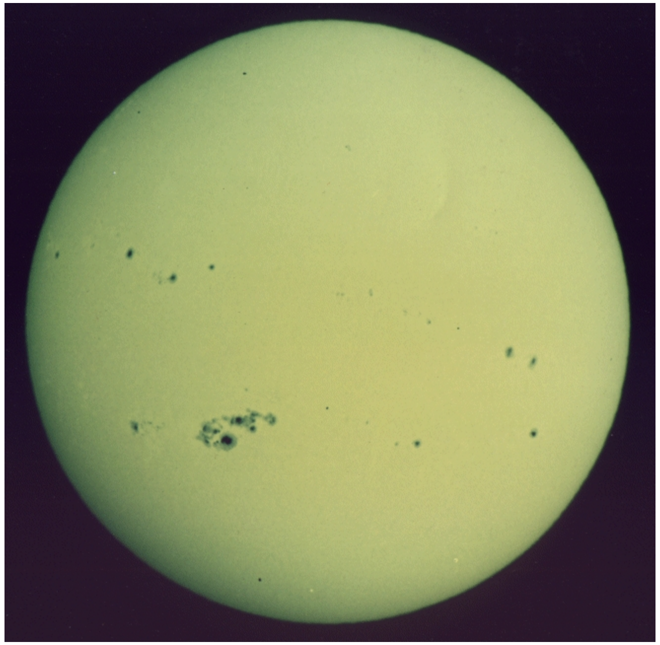
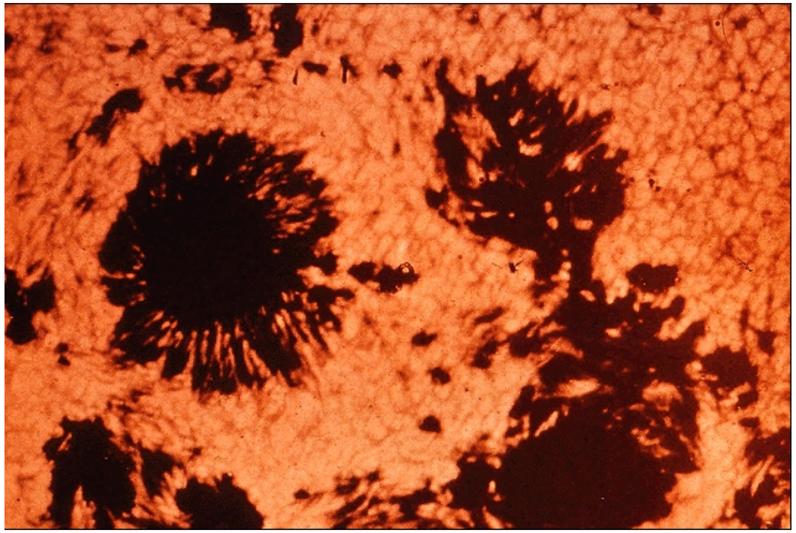
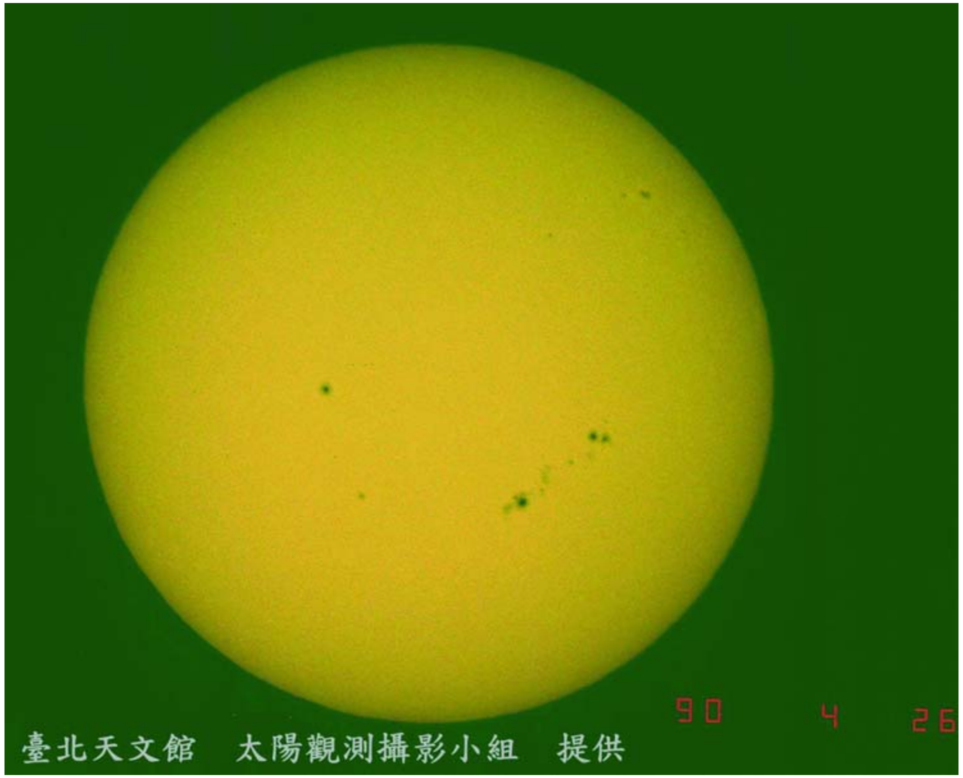


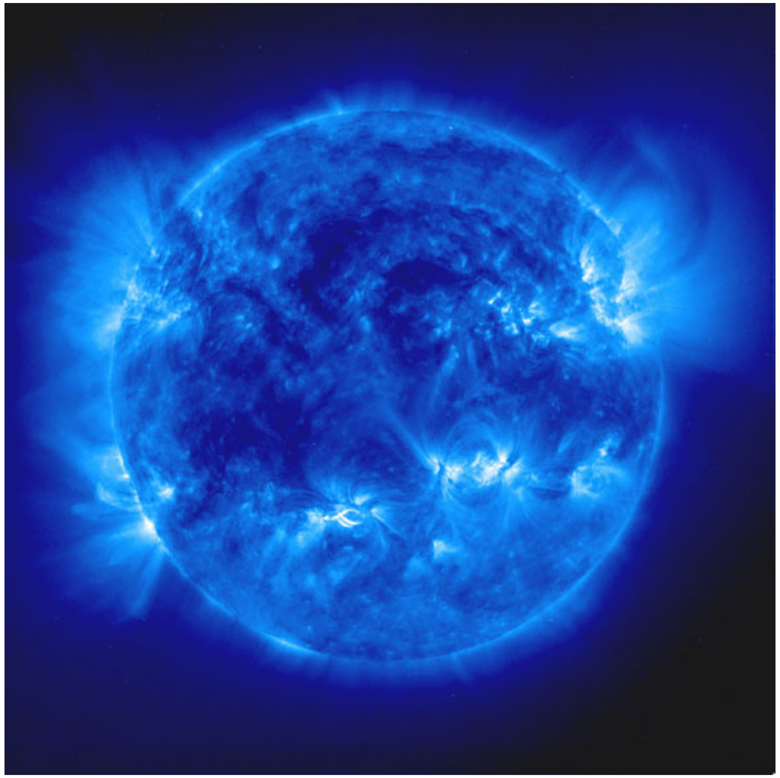
Photo of the Sun's photosphere光球層的影像



黑子群的近照

太陽黑子(Sunspot)是太陽光球表面上最明顯的現象，美國天文學家海爾（George Zllery Hale, 1868-1938）於西元 1908 年發現太陽黑子和磁場有關。太陽黑子是太陽表面局部磁場較強的地方，溫度較低故較周圍黑暗。





陽物理特性The Sun's physical data

下表列出太陽的一些物理特性：

特性	太陽
質量	2 x 10^{30} kg （地球質量的335,000倍）
直徑	1.4 million km （地球直徑的109倍）
密度	1400 kg/m^3 (水的密度為1000 kg/m^3)
年齡	約為46億年
光度 (向外發散出的能量)	4 x 10^{23}瓩
表面溫度	約5500 ˚C (5800 K)
核心溫度	約1千4百萬度
化學組成 （以原子質量計算）	74.5%氫、23.5%氦和2%重元素（如氧、碳、氮等）
化學組成 （以原子數量計算）	94 %氫、6%氦和重元素

The amount of **energy** from the Sun that **reaches the Earth** **annually** is **4×10^{18}** Joules.

$$4 \times 10^{18} \text{ Joules/ Year} \div 365 \text{ Days/ Year} = 1 \times 10^{16} \text{ Joules/ Day}$$

$$1 \times 10^{16} \text{ Joules/ Day} \div 24 \text{ Hours/ Day} = 4 \times 10^{14} \text{ Joules/ Hour}$$

The amount of **energy consumed** annually **by the world's** population is about **3×10^{14} Joules**.

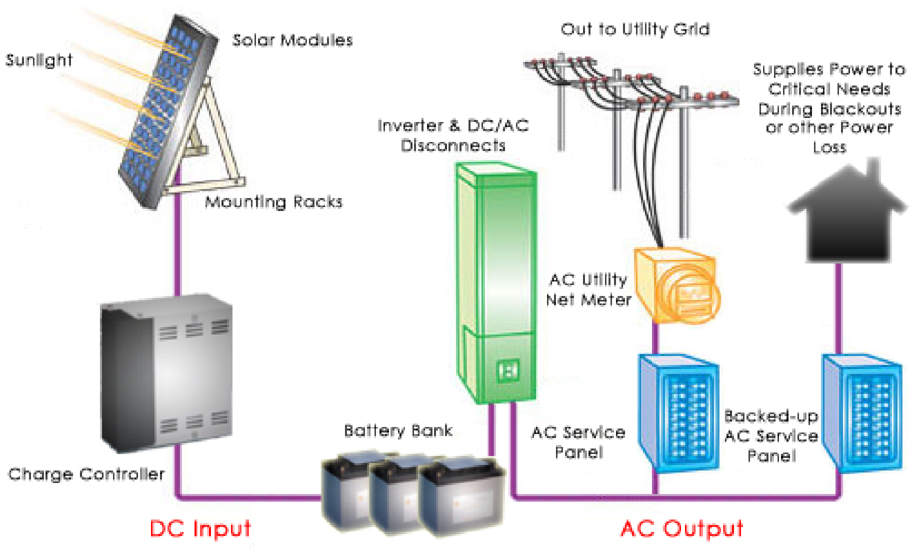
太空中發電！　太陽能轉換成電力

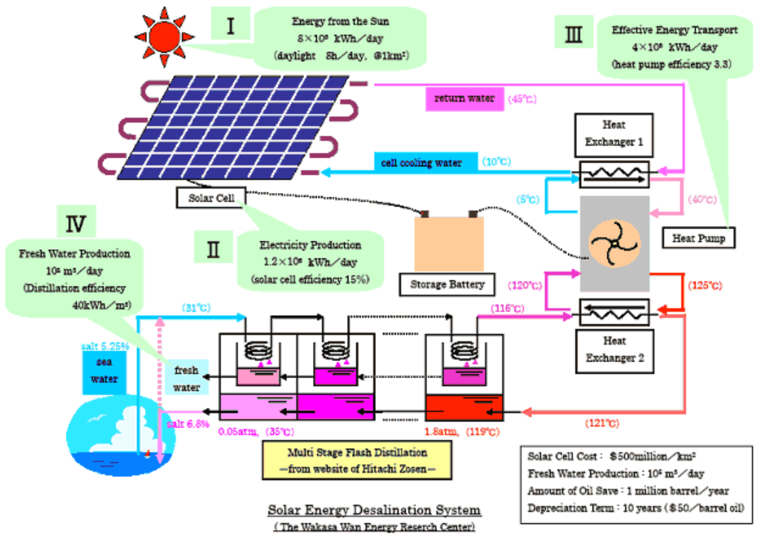
科學家幫人類尋找新的替代性能源，又有新的創意，他們把腦筋動到了外太空，計畫有效率地收集太空中的太陽能，直接在太空中發電；因為經過估算，太陽輻射的能量，大約有10億座發電廠的電量總和，儘管實驗還在初步階段，不過已經讓科學家大為振奮。

太陽能輸出的能量，相當於每秒1億個原子彈爆炸，科學家認為，只要利用一小部分，以電力形式送回地球，就能提供地球所需能源。首先是要設計出高效率的太陽能電池，並且以這種很薄的涅菲爾透鏡，將陽光聚集在太陽能電池上，讓電池效率增強8倍。科學家約翰曼金斯：「我們的任務是找出新能源，須符合經濟性，存量充足，而且能夠再生。」科學家估計，太空中的太陽輻射可以製造出，相當10億座大型電廠的輸出電量，因此科學家將設計出能夠傳送電力的微波束，從大空穿過大氣層到達地面。 另外，還必須依賴日本賀陽教授的傳送器原型，利用微波隔空發射電力，電流透過銅製的傳送器具，產生的電荷會發射電磁能量波，穿過稀薄空氣，經過實驗，小組人員以手機實驗，成功地幫手機充電。研究人員：「這是很大的進展，我們把電以微波，傳送到12呎外。」研究人員在山區進行實驗，以微波束傳送電力，通過大氣層，儘管在技術上，仍有許多困難需要克服，不過也為人類帶來的新希望。

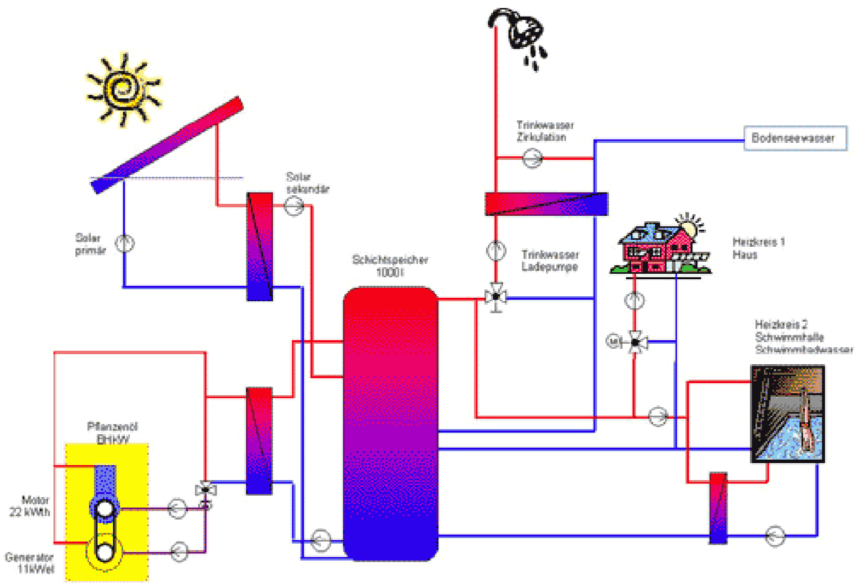
1.1 The Time Scale of Fossil Fuels and The Solar energy Option

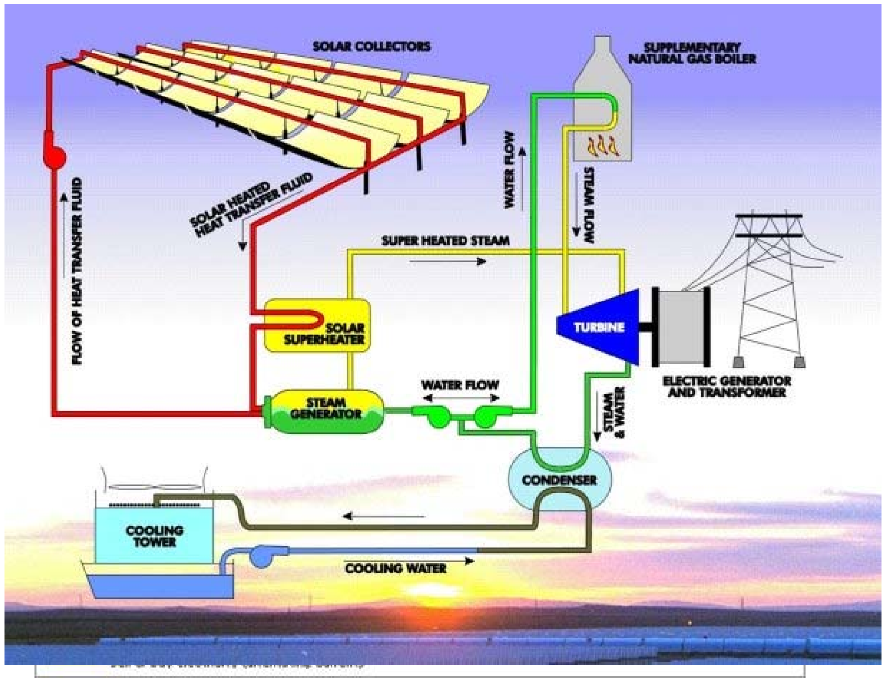
- all the energy consumed by humans has come from the sun. Coal, oil, and natural gas, hydroelectric generators, windmills
- Solar energy is an essentially inexhaustible source potentially capable of meeting a significant portion of the nation's future energy needs with a minimum of adverse environmental consequences.
- The future of solar power development will depend on how we deal with a number of serious constraints, including scientific and technological problems, marketing and financial limitations, and political and legislative actions favoring conventional and nuclear power
- The design and analysis of solar energy systems are based upon well-established engineering knowledge, including thermodynamics, fluid mechanics, heat transfer, basic physics and chemistry, calculus and ordinary differential equations, and economics





sea water using **solar**-**energy**





1.2 The Time Scale of Fossil Fuels and The Solar Energy Option

☛ The rate of production of a nonrenewable energy resource increases with time, reaches a maximum, and then decays.

Coal, oil, and natural gas

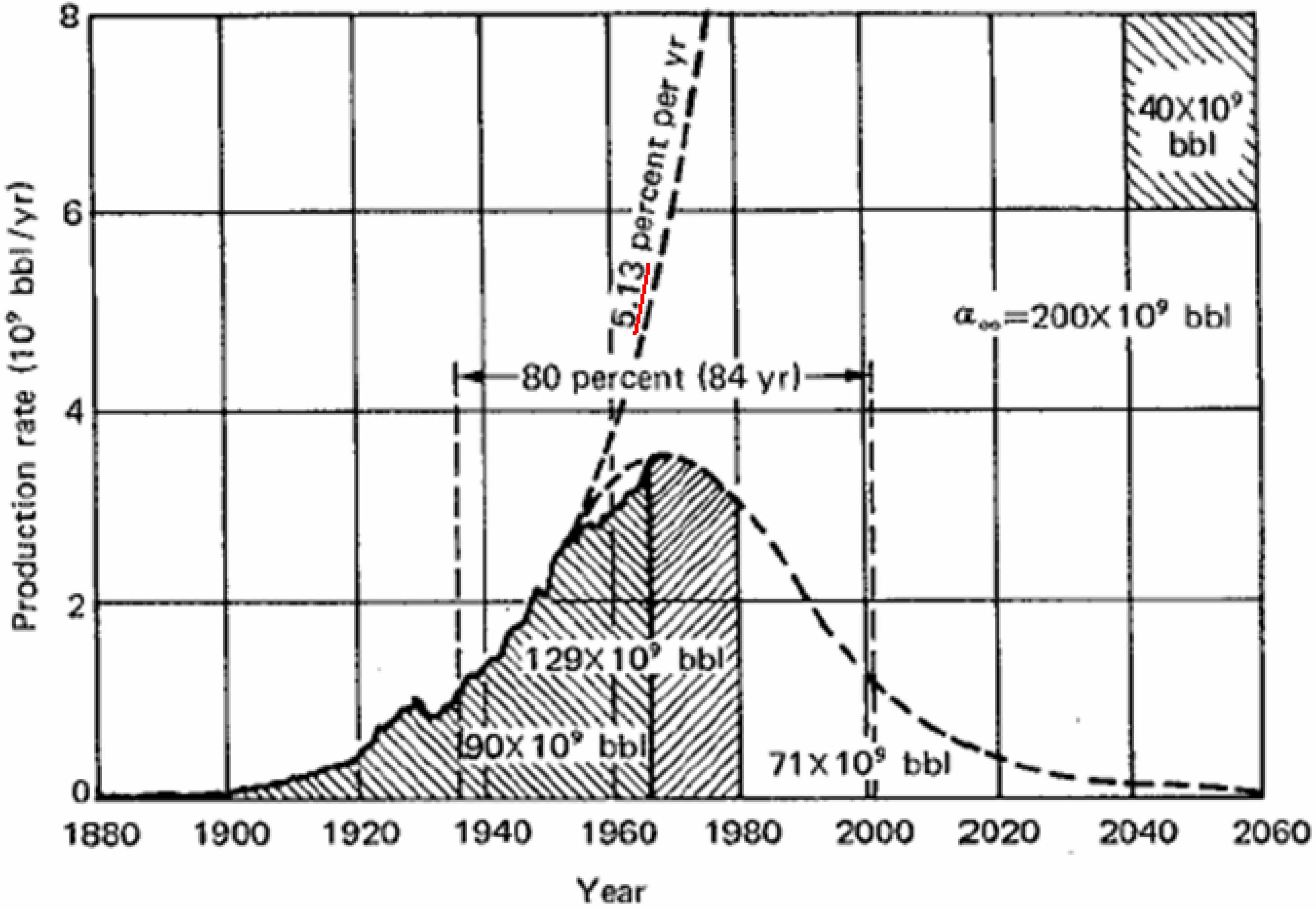


FIGURE 1.1 Complete cycle of U.S. production of petroleum liquids. Redrawn from (18).

☛ Of course, the peaking in the production rate of a given energy source does not imply the end of its availability. It does mean, however, that the end of its availability is in sight and that when peaking occurs some other source must become increasingly available to supplement the diminishing source if all the energy requirements are to be met. In case no other source is available at the time the peak in production rate is reached, some energy-consuming activities would obviously have to be curtailed.

☛ To avoid an energy catastrophe it is therefore imperative that accurate predictions of future energy supply and demand be available to decision makers.

☛ **According to current estimates, about 88% world's fossil fuel supply is coal, It is important to know when it will be exhausted, because this will show much time is available to develop and deploy other energy sources, which are essential to the survival of industrial society.**

Hubbert, a research physicist in the U.S. Department of the Interior, predicted in 1962 that the rate of production of coal would reach a maximum somewhere between the years 2100 and 2150, as shown in Fig. 1.2.

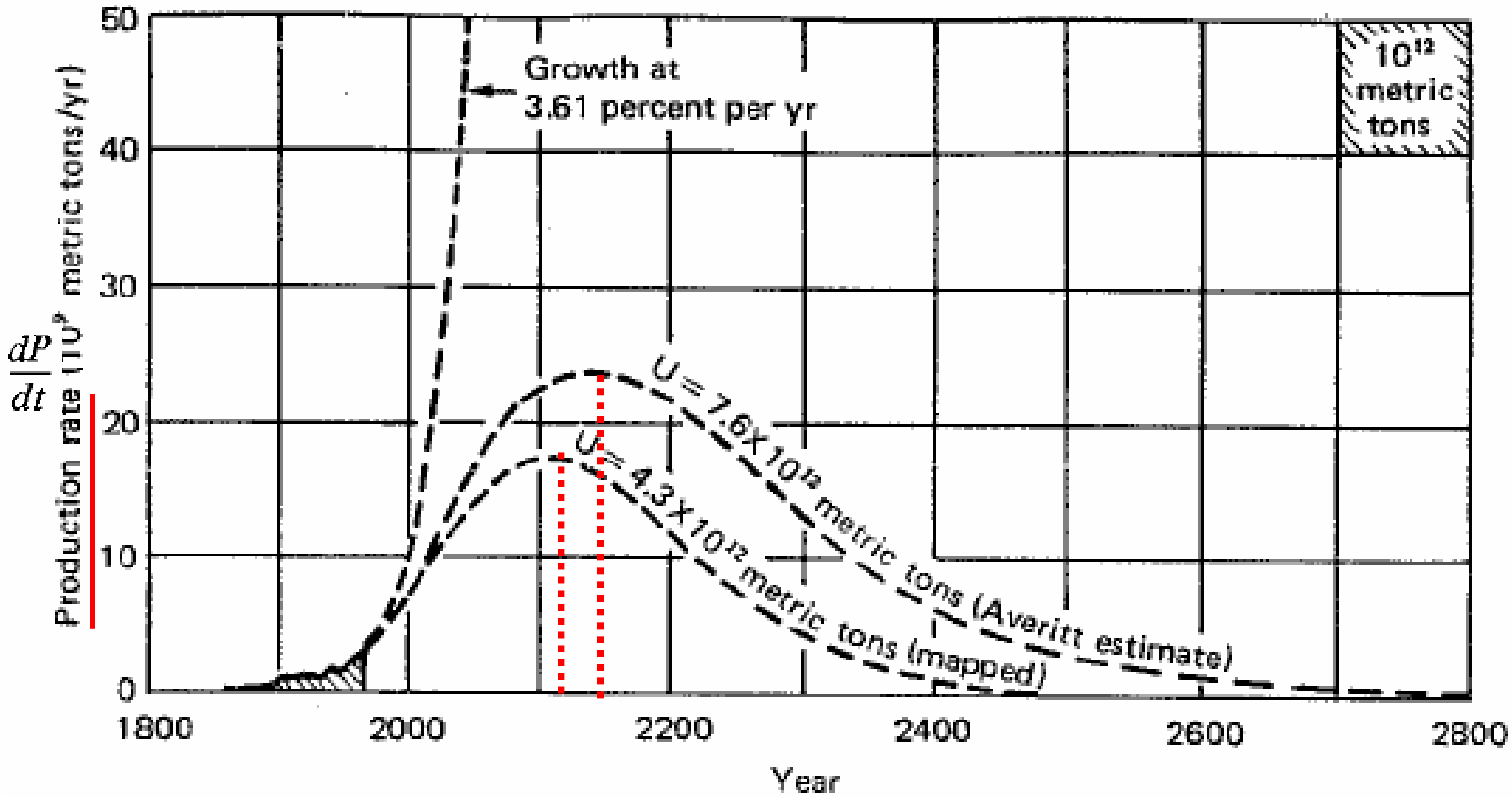


FIGURE 1.2 Complete cycles of world coal production for two values of U. Redrawn from (18).

The complete world cycle prediction of Elliott and Turner (11) for all nonrenewable fossil resources is shown in Fig. 1.3

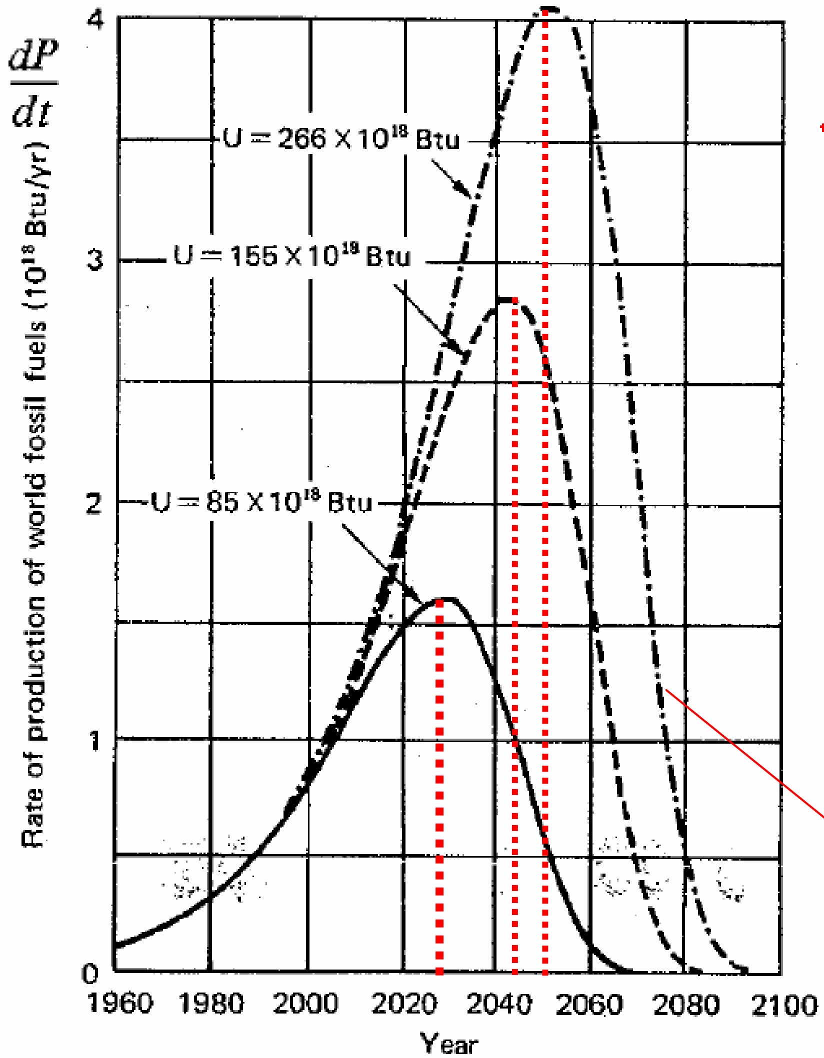


FIGURE 1.3 Effect of the value of the resource base on the projected rate of world fossil fuel yields according to Elliott and Turner (11).

According to Elliott and Turner

the rate of production of a resource is related to the demand and the amount remaining by an expression of the type

$$\frac{dP}{dt} = f(D)f(s)$$

where P = cumulative production of the resource
t = time
$f(D)$ = function of demand
$f(s)$ = function of supply

$$\frac{dP}{dt} = C_1 \frac{P}{U}\left(1 - \frac{P}{U}\right) \rightarrow -\ln\left(\frac{U}{P} - 1\right) = \frac{C_1}{U}t + C_2$$

$$C_2 = -\frac{C_1}{U}t_m \rightarrow P = \frac{U}{1 + \exp - (C_1 t/U + C_2)}$$

$$\rightarrow P = \frac{U}{1 + \exp - [(C_1/U)(t - t_m)]} \propto U$$

ation (1.6) is similar in form to the relation used by Hubbert

預測一

Table 1.1 gives some estimates of the total energy contents of the world's supply of recoverable fossil fuels according to Hubbert (19). It is apparent that the world production of fossil fuels will peak between the years 2030 and 2050

TABLE 1.1 Energy Contents of the World's Initial Supply of Recoverable Fossil Fuels[a]

Fuel	Quantity	Energy content 10^{21} J(th)	Energy content 10^{15} kW · hr(th)	Percent
Coal and lignite	7.6×10^{12} tons[b]	201	55.9	88.8
Petroleum liquids	2000×10^{9} bbl (272×10^{9} tons)	11.7	3.25	5.2
Natural gas	$10{,}000 \times 10^{12}$ ft^3 (283×10^{12} m^3)	10.6	2.94	4.7
Tar-sand oil	300×10^{9} bbl (41×10^{9} tons)	1.8	0.51	0.8
Shale oil	190×10^{9} bbl (26×10^{9} tons)	1.2	0.32	0.5
Totals		226.3	62.9	100.0

[a]From Hubbert, M. King, Energy Resources for Power Production, in "Environmental Aspects of Nuclear Power Production," International Atomic Energy Agency, Vienna, pp. 13–43, 1971, Table VI.

預測二

TABLE 1.2 Estimates of the Year in Which a Maximum Production Rate of Coal Could Occur[a,b]

Source	Year	Comment
Chase Manhattan Bank, elucidated (5)	2033	Original report states 1500 yr at 1972 rate of consumption; elucidation in text
Brennan (6)	2000	Assumes energy demand stops increasing at 2000
Elliott and Turner (11)	2040	Takes into account use of coal to replace oil
Linden (23)	2010	Assumes mining technology not radically improved from 1970s
Linden (23)	2052	Assumes all coal can be mined
Linden (23)	2073	Highest credible limit for resource base assumed

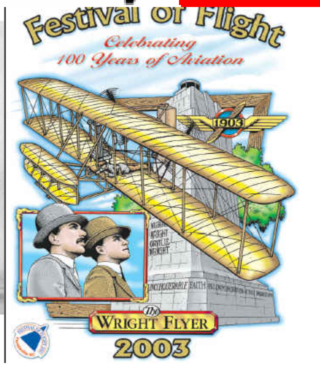
The next question to ask is, "How much time is available to develop other energy sources?" An answer to this question can be given only within the context of the history of science and technology. How long has it taken in the past for a new technology to be introduced into the mainstream of an industrial society? The general estimate is that it takes somewhere between 20 and 40 years for a technology to come to fruition.

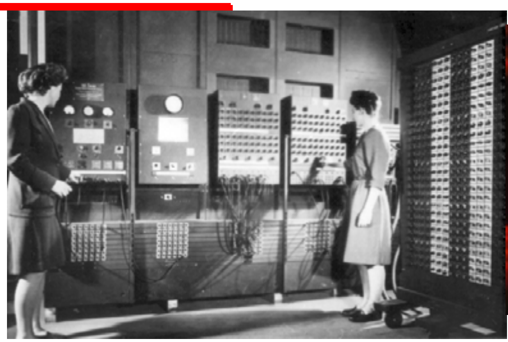
For example

❶ the Wright brothers made their first flight with a heavier-than-air ship in 1903, and aviation became a major industry about 40 years later.

❷ The first high-speed computer was developed at the end of World War II, and a large-scale computer industry developed about 20 years later.

❸ first nuclear chain reaction was set off in 1942, but despite massive governmental funding nuclear power was supplying less than 2 percent of the total energy consumption of the U.S. economy three decades later.





☛ The replacement of current energy-producing system requires careful planning and allocating of current energy resources, and time.

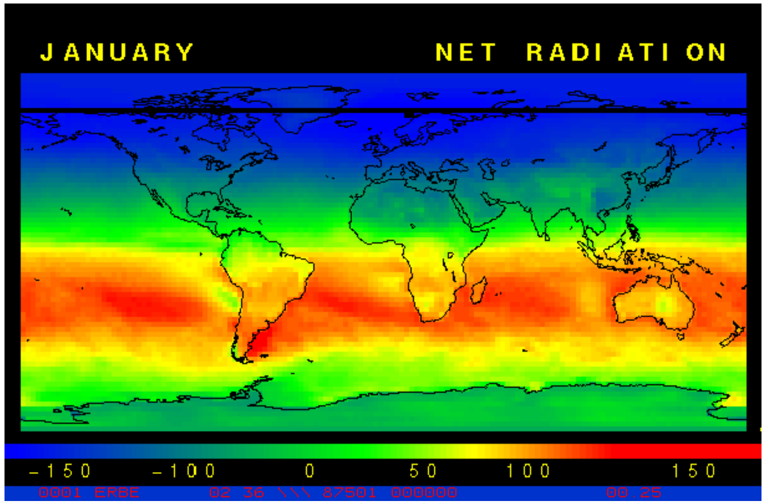
☛ Of all the currently known available sources, only nuclear and solar energy have the potential of supplying large amounts of power within the available time frame.

☛ How much energy each of them can supply and under what conditions one source is more suitable to meet the demand economically and safely than the other.

☛ To build a large-scale solar energy industry must include considerations of the availability of material, work force. And the level of technology required for such an undertaking.

For example

Assuming that all solar conversion plants could be located in the sunny parts of the world where the average daily insolation is $5\ kW \cdot hr/m^2$, about 2.5 million km^2 of land would have to be covered with solar collectors if the average overall solar system conversion efficiency were 10 percent. Assuming further that these solar systems, including all storage and transmission, could be built with the same capital investment as a current coal-fired thermal power plant (about $500/kW) or a nuclear power plant (about $1,000/kW), the required initial investment would be roughly 25 or 50 trillion dollars, respectively. Such an investment over half a century would require about 15-30 percent of the 1974 gross world product per year, according to a study conducted at the International Institute for Applied System Analysis



費用: $q \times A \times \eta \times Cost=$

$5kw\ hr/m^2 \times 2500000 km^2 \times 10\% \times$ (500元/kw or 100元/kw)=25兆美元 or 50兆美元

時間:半個世紀

材料重量:$10kg/m^2$(Flat-plate collector) 50 billion metric tons

TABLE 1.3 Estimates of the Heat, Electric Power, and Fuels to be Supplied by Solar Energy in the United States, as Projected by ERDA[a]

Solar technology	1985	2000	2020
Direct thermal applications (in units of 10^{15} Btu = 1 Q per year)[b]			
Heating and cooling	0.15 Q	2.0 Q	15 Q
Agricultural applications	0.03	0.6	3
Industrial applications	0.02	0.4	2
Total	0.2 Q	3 Q	20 Q
Solar electric capacity (in units of 10^9 W = 1 GWe)[c]			
Wind	10 GWe	20 GWe	60 GWe
Photovoltaic	0.1	30	80
Solar thermal	0.05	20	70
Ocean thermal	0.1	10	40
Total	1.3 GWe	80 GWe	250 GWe
Equivalent fuel energy	0.07 Q	5 Q	15 Q
Fuels from biomass	0.5 Q	3 Q	10 Q
Total solar energy	~1 Q	~10 Q	~45 Q
Projected U.S. energy demand	100 Q	150 Q	180 Q

[a]From (12).

1. 2 Overview of Solar Energy Conversion Method

In accordance with the suggestions of the Solar Energy Panel of NSF/NASA, the following overview of the principal methods

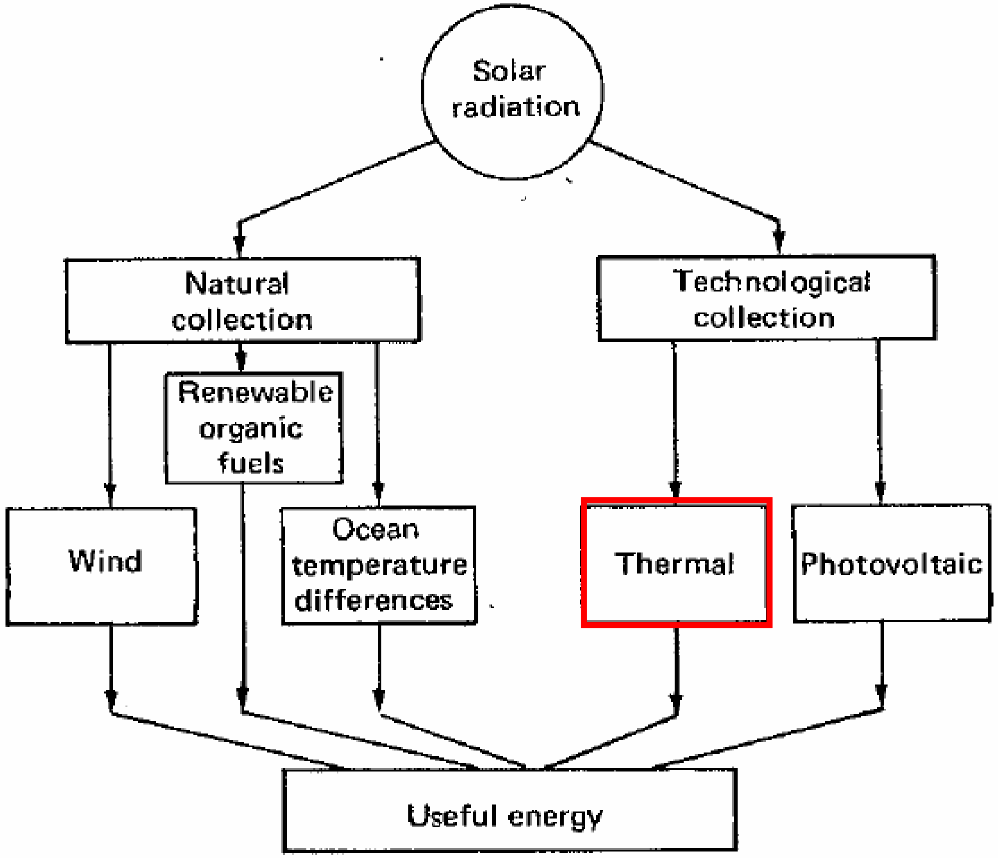


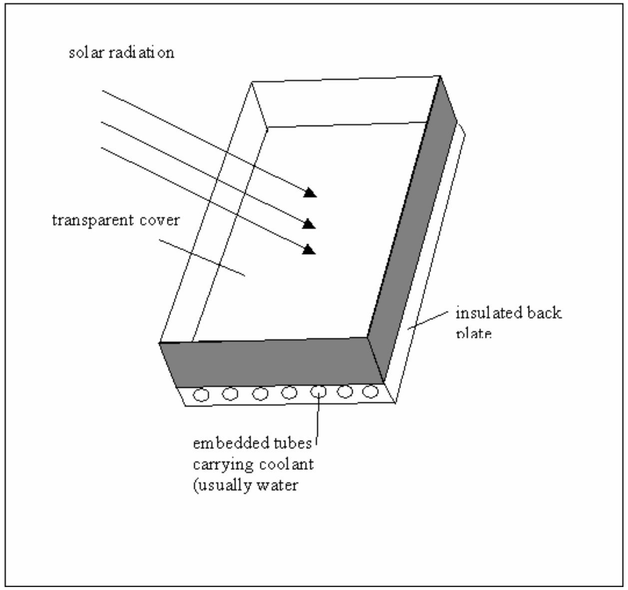
FIGURE 1.4 A summary of schemes for solar energy conversion. Redrawn from (27).

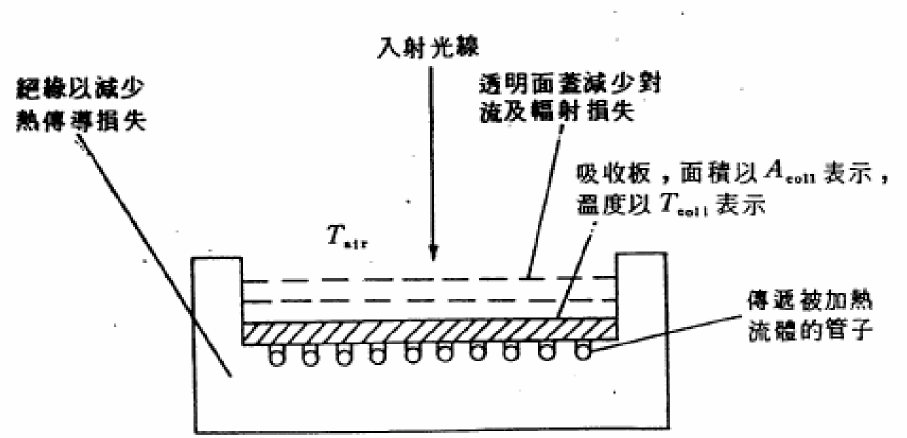
At the present time only the thermal method is sufficiently advanced to compete economically with other sources, including nuclear.

1.2.1 Thermal conversion

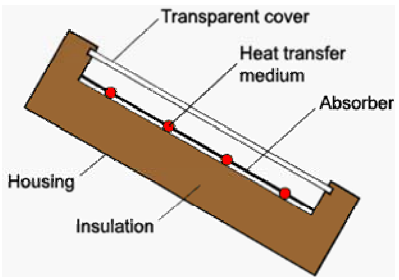
(1) For generting low temperature hot (365°K)

When a dark surface is placed in sunshine, it absorbs solar energy and heats up. Solar energy collectors working on this principle consist of a surface facing the sun, which transfers part of the energy it absorbs to a working fluid in contact with it.



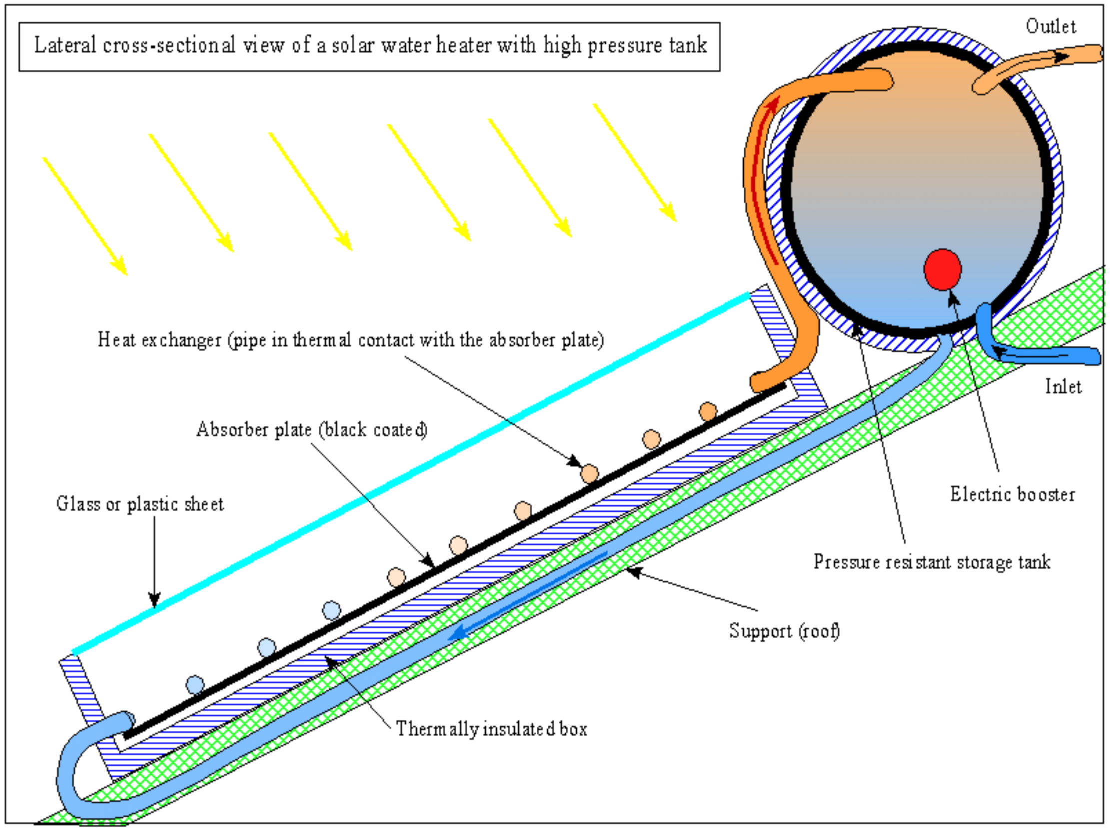


a solar-energy collector.



- These types of thermal collectors suffer from heat losses due to radiation and convection, which increase rapidly as the temperature of the working fluid increases.
- To reduce heat losses to the atmosphere one or two sheets of glass are usually placed over the absorber surface to improve its efficiency.
- Improvements such as the use of selective surfaces, evacuation of the collector to reduce heat losses, and special kinds of glass are used to increase the efficiency of these devices.

Example1 flat -plate collector (365°K)

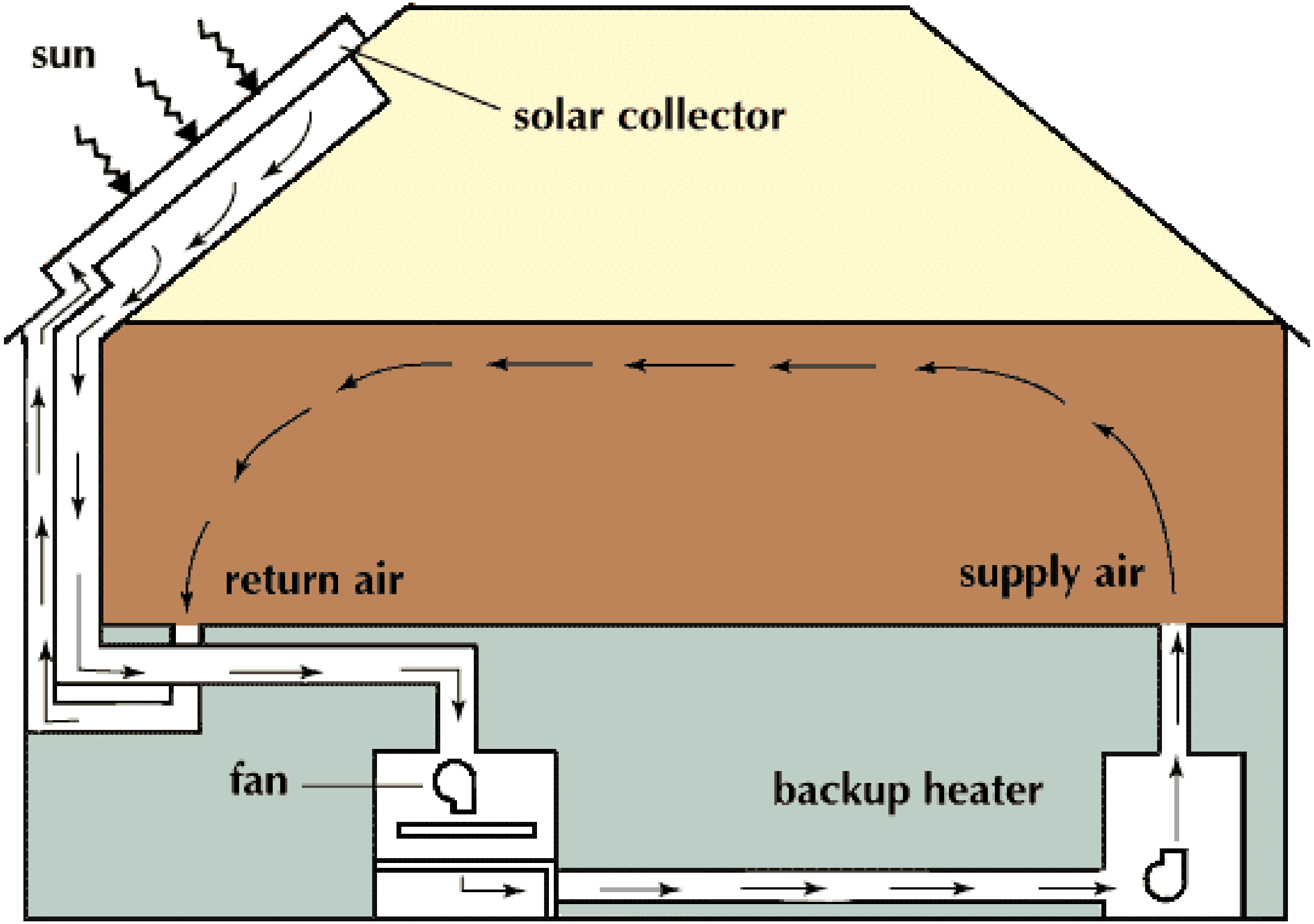


Example1 Flat -plate collector (365°K)

- The thermal utilization of solar energy for the purpose of generating low-temperature heat They are available today for operation over a range of temperatures up to approximately 365 K (200°F).

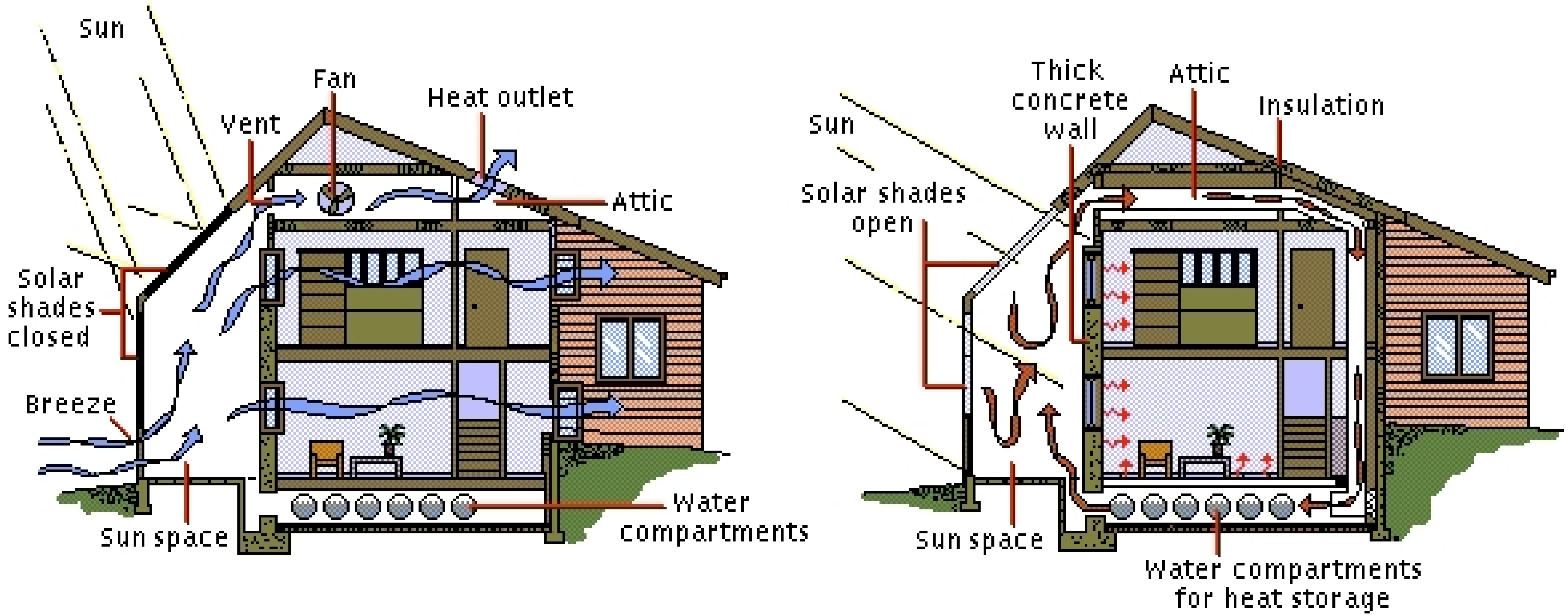
- These collectors are suitable mainly for providing hot service water and space heating and possibly are also able to operate absorption-type air-conditioning systems.

hot service water



solar heating system

space heating and cooling

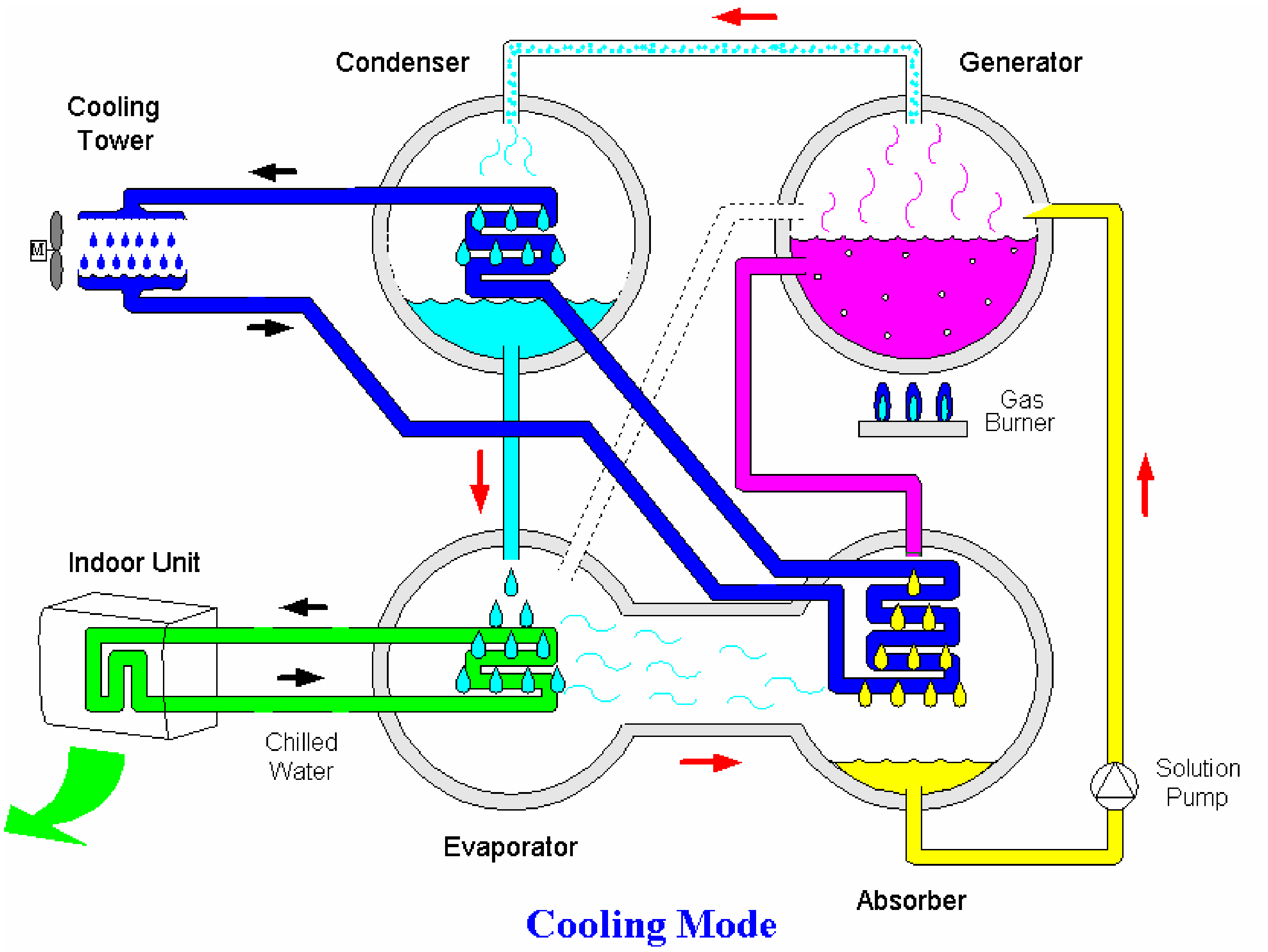


Passive solar cooling
(Summer)

Passive solar heating
(Winter)

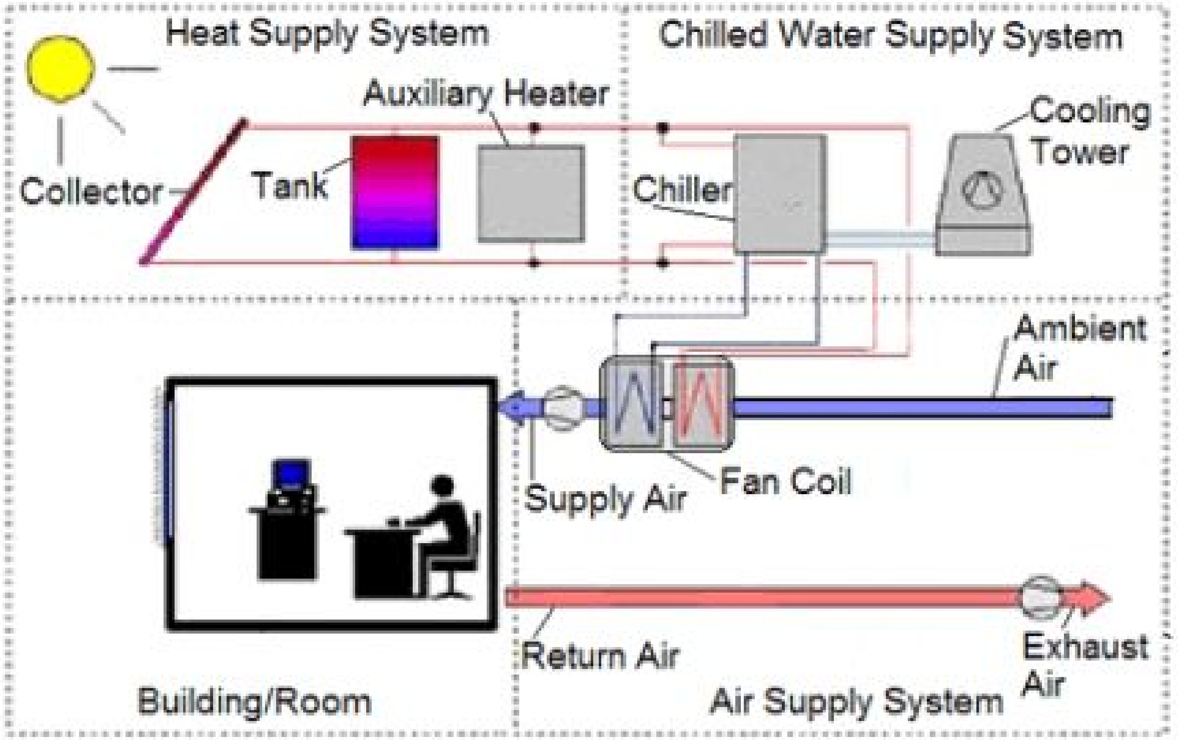
Microsoft Illustration

A Yazaki absorption chiller-heater, using water as the refrigerant, is today's best choice in air conditioning for protecting the environment and reducing the cost of energy. Double-effect cycles and advanced technology ensure high performance and long term reliability



absorption-type air-conditioning system

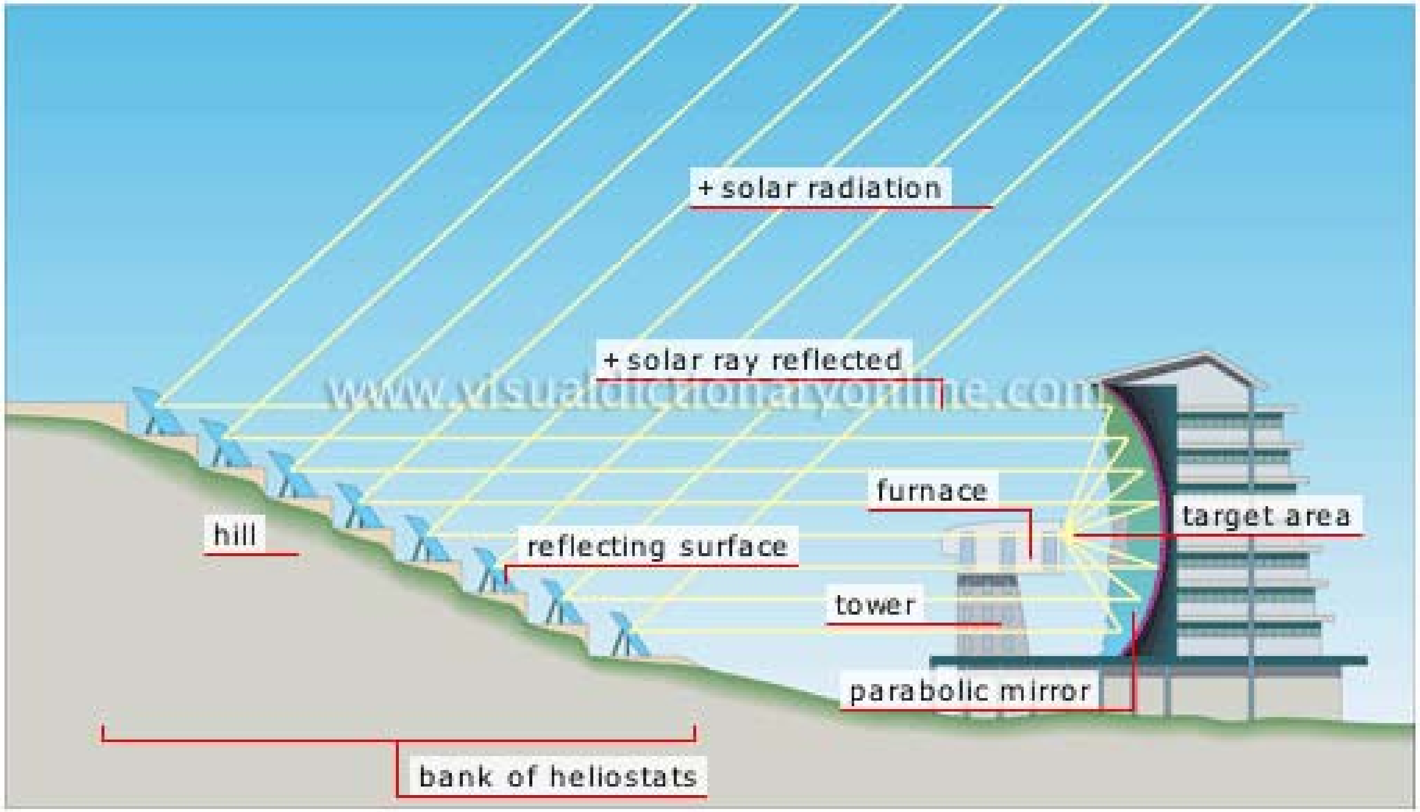
Diagram of A Solar Air Cooling System



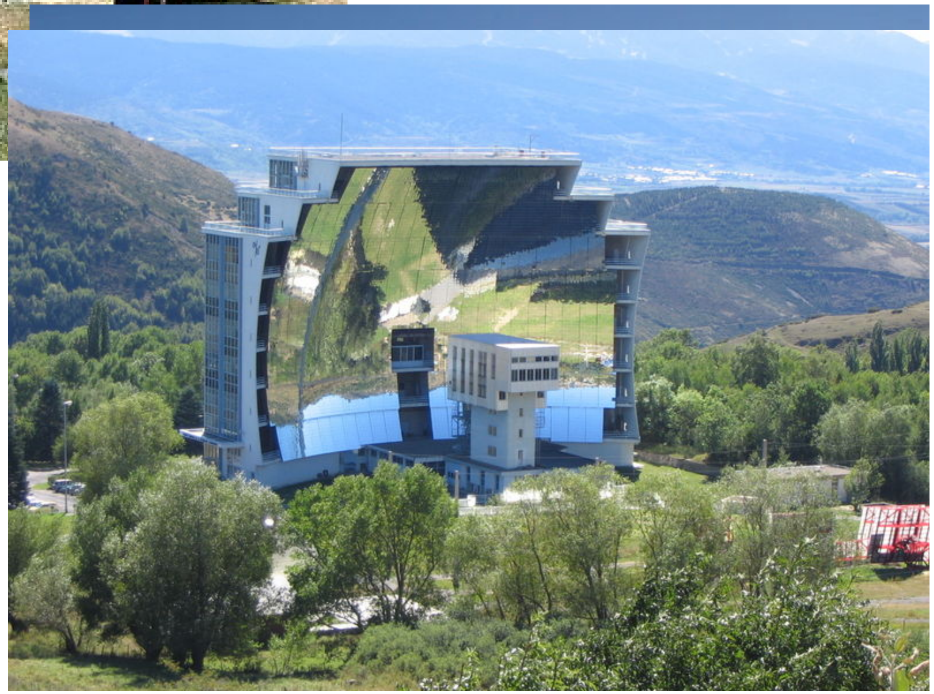
(2) For generting high working temperature

Solar furnace (4000°K)

The generation of higher working temperatures as needed, for example, to operate a conventional steam engine requires the use of focusing devices in connection with a basic absorber-receiver.



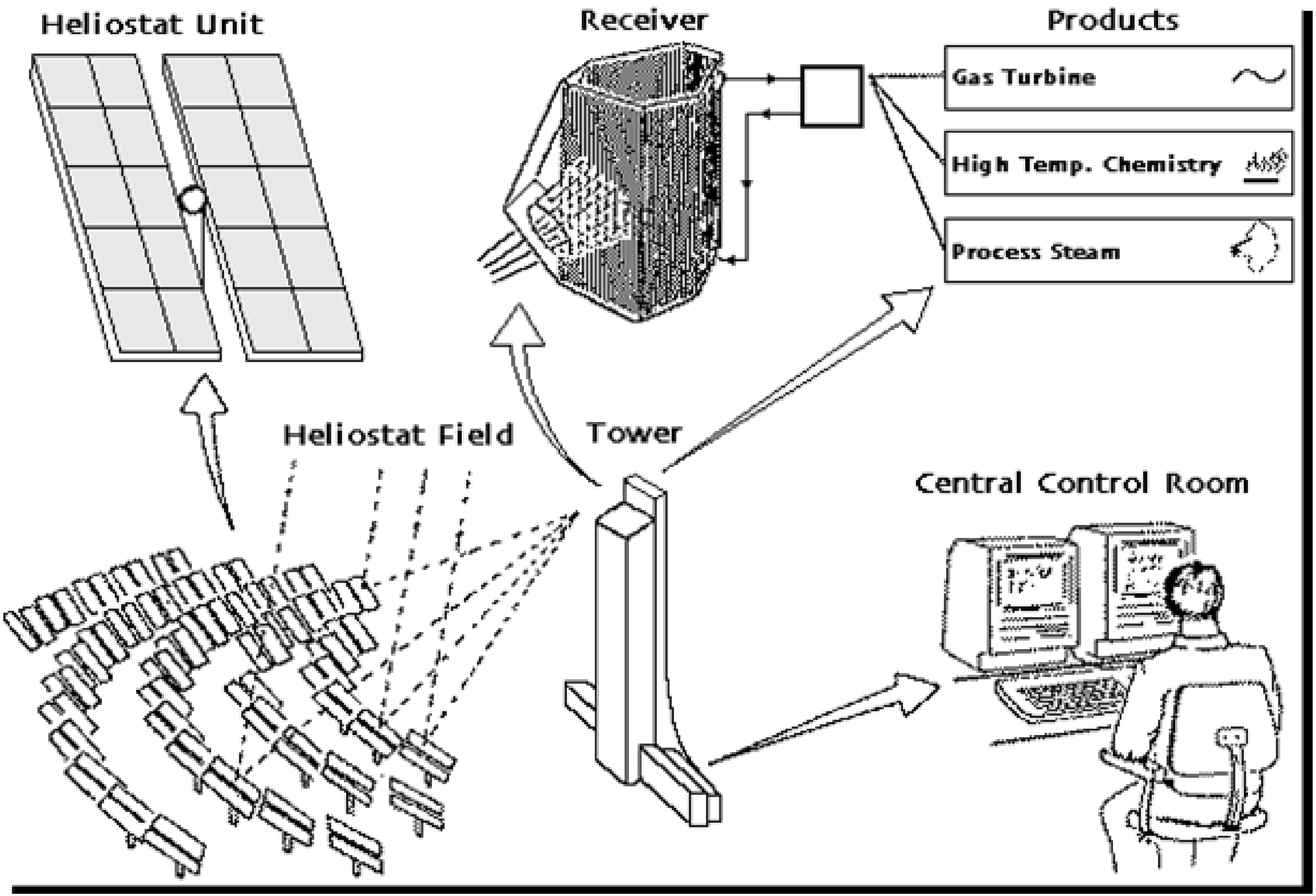
solar furnace



Odeillo solar furnace in France
(4000°K)

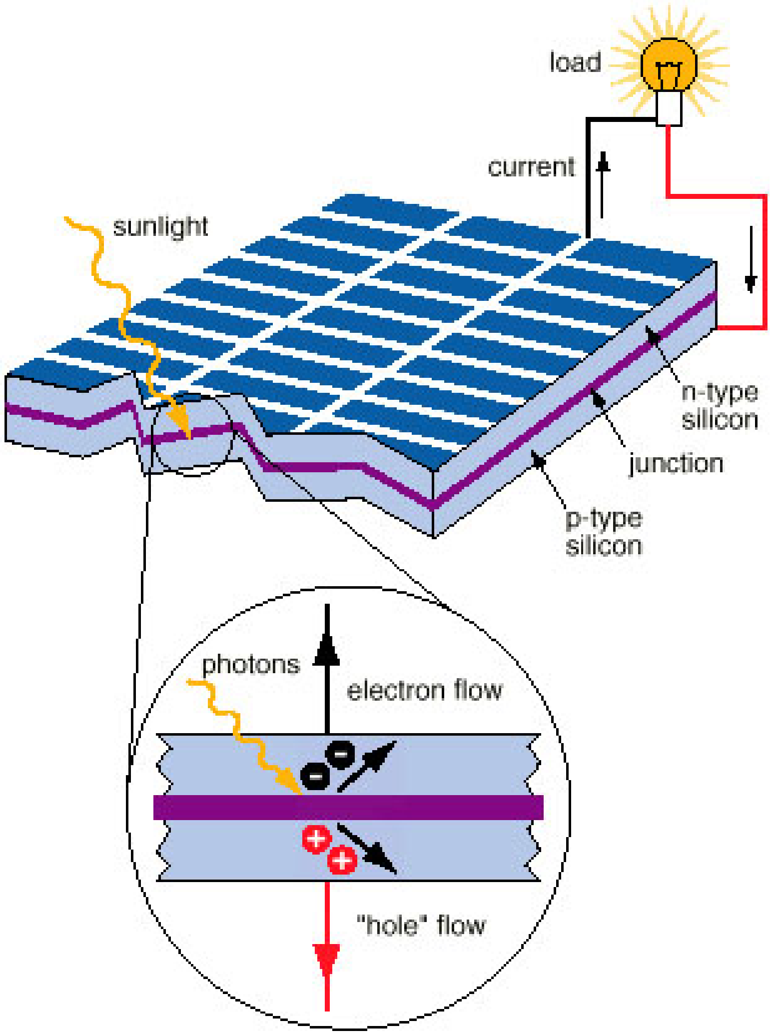
solar Tower

A solar furnace (picture from NREL
http://rhlx01.rz.fh.../pictures/tower.jpg
[phoenix, May 19 2002, last modified Oct 05 2004]


Heliostat Unit
Receiver
Products
Gas Turbine
High Temp. Chemistry
Process Steam
Heliostat Field
Tower
Central Control Room

1.2.2 Photovoltaics Conversion (光電轉換)

The conversion of a solar radiation into electrical energy by means of solar cell



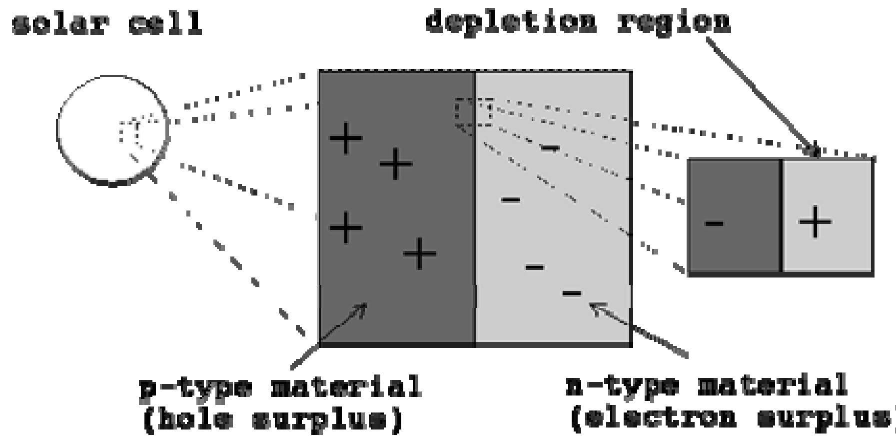


Figure 1: a solar cell.

太陽能路燈

1.2.2 Photovoltaics Conversion (光電轉換)

☛ Photovoltaic solar cells, as opposed to conventional collectors, which convert solar radiation into heat, utilize energetic photons of the incident solar radiation directly to produce electricity. Thus, this technique is often referred to as direct solar conversion. Conversion efficiencies of thermal systems are limited by collector temperatures, whereas the conversion efficiency of photo cells is limited by other factors.

☛ The conversion of solar radiation into electrical energy by means of solar cells has been developed as a part of satellite and space-travel technology. The theoretical efficiency of solar cells is about 24 percent, and in practice efficiencies as high as 15 percent have been achieved with silicon photovoltaic devices. The technology of photovoltaic conversion is well developed, but large-scale application is hampered by

☛ the high price of photo cells, which has been estimated by an ad hoc panel on solar cell efficiencies of the National Academy of Sciences at approximately $100 per electric watt in 1972. This is about 100 times more than electric energy from conventional fossil- and nuclear-fuel electric power plants, and a cost reduction of the order of 100 to 1 will be necessary before photovoltaic conversion can be economically viable.

Photovoltaic panels integrated into the roof of a building

Design concept for a satellite solar power station.

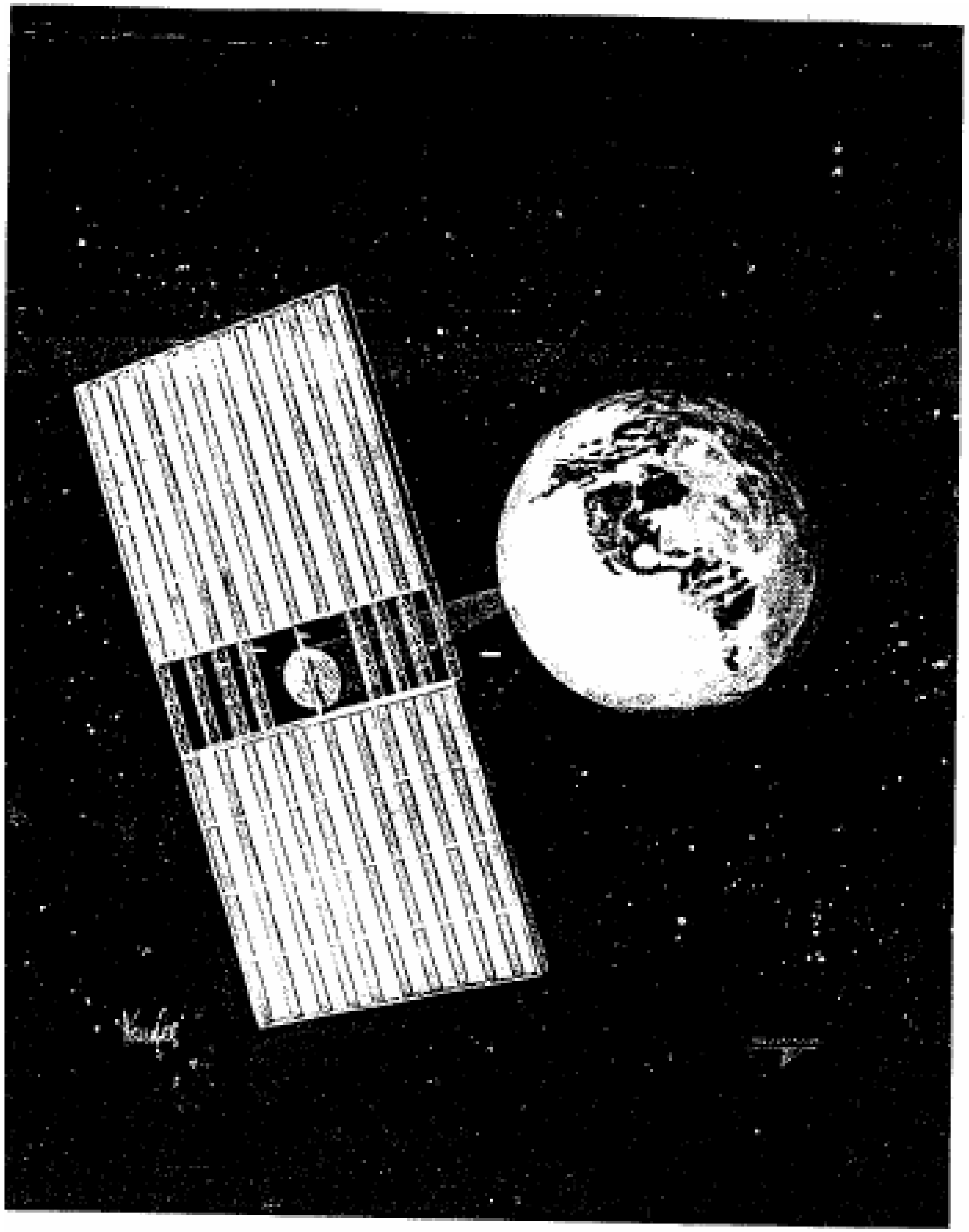


FIGURE 1.5 Design concept for a satellite solar power station. Courtesy of Arthur D. Little, Inc.

space solar power satellite

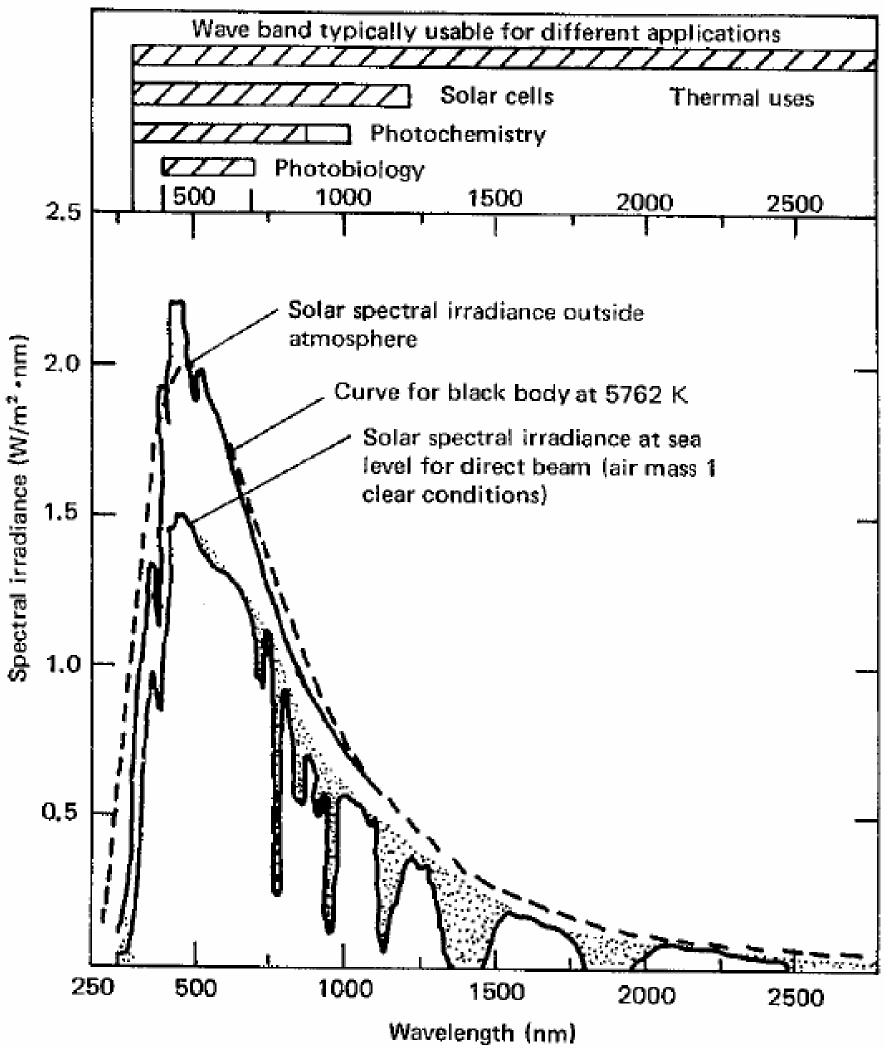
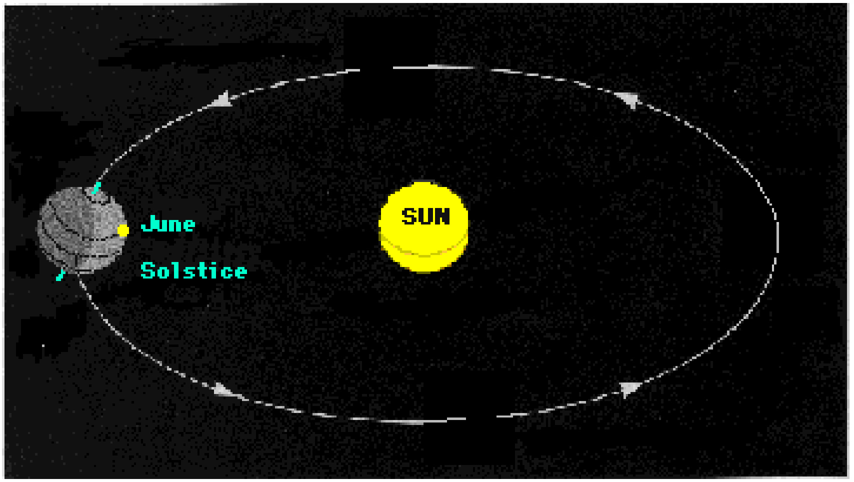
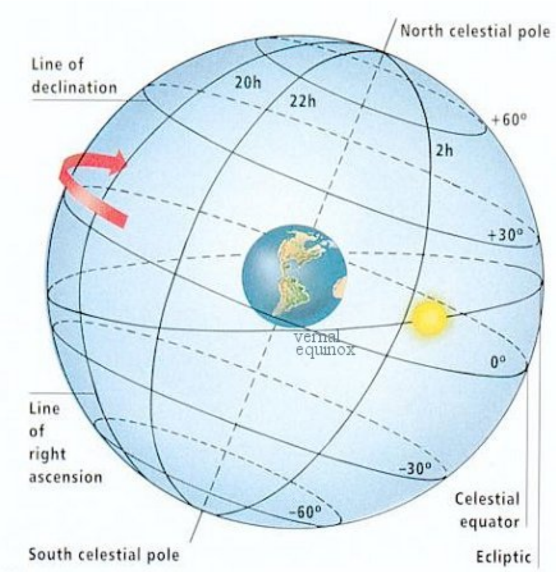


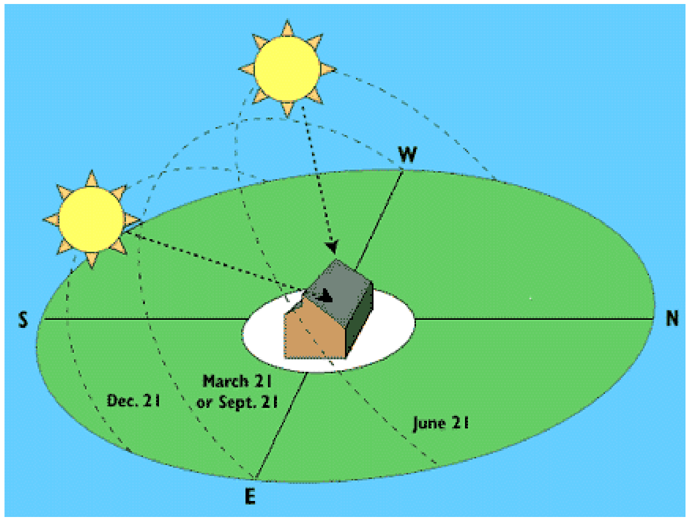
FIGURE 1.6 Spectral irradiance curves for direct sunlight extraterrestrially and at sea level with the sun directly overhead. Shaded areas indicate absorption due to atmospheric constituents, mainly H_2O, CO_2, and O_3. Wavelengths potentially utilized in different solar energy applications are indicated at the top.

FUNDAMENTALS OF SOLAR RADIATION

2



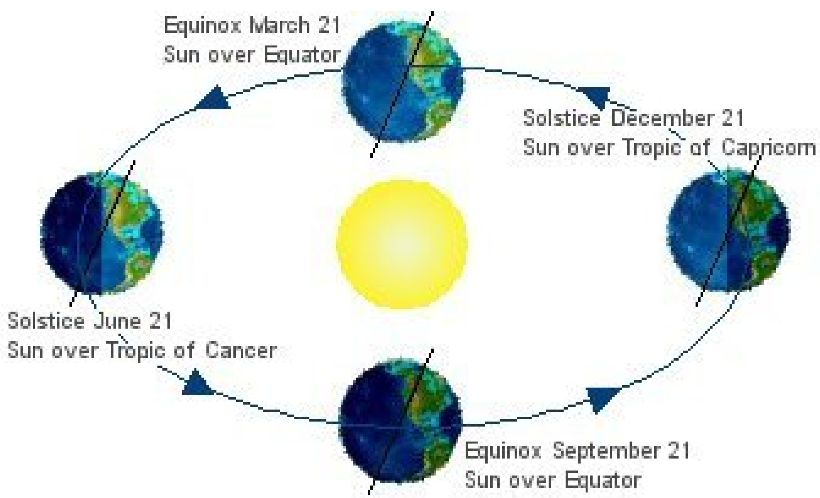


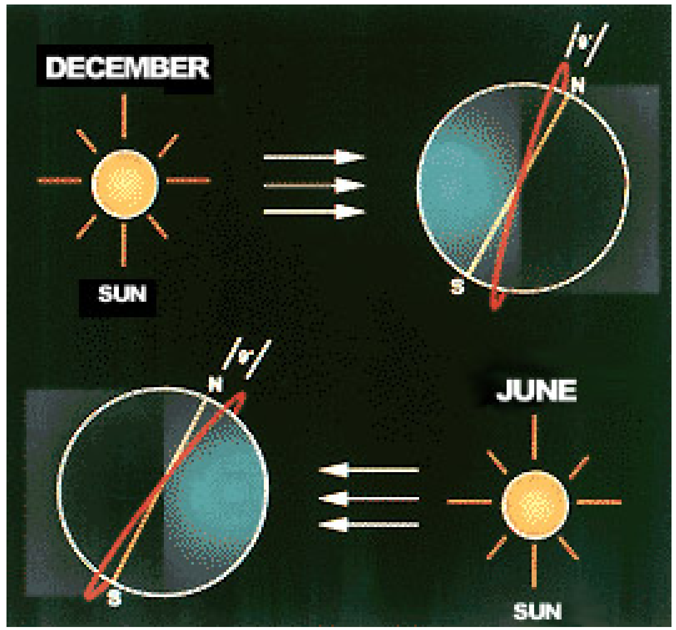


The subject of this chapter---

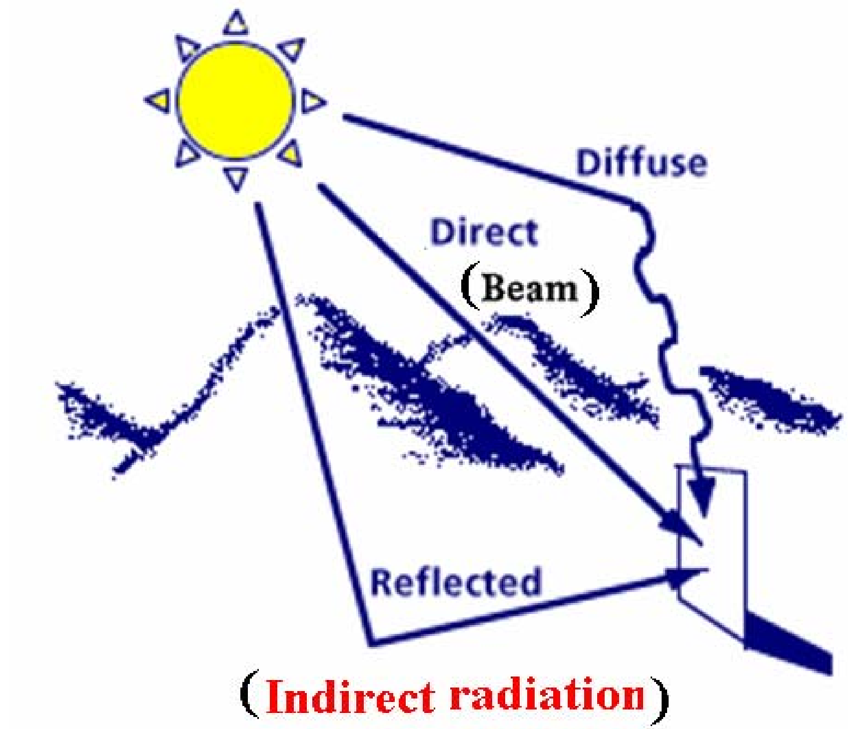
To make a **quantitative analysis** of the **sun's motion** and **its energy flux** on earth:

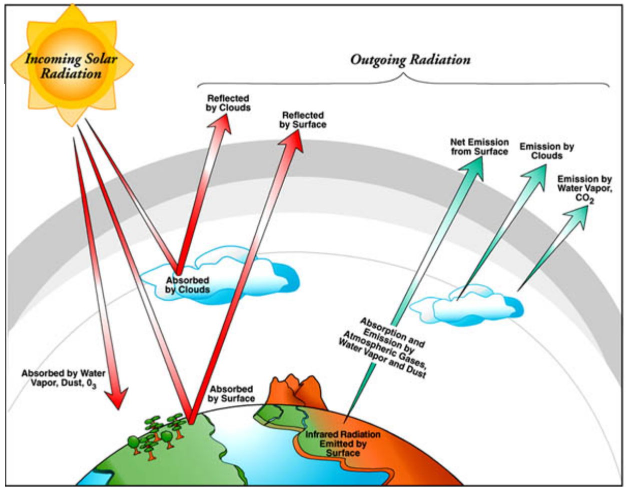
1. Based on the **season and daily**, the motion is analyzed.
2. The measurement and prediction of solar radiation on **monthly, daily, and hourly** time scales





Terminology





Beam radiation Solar radiation intercepted by a surface with negligible direction change and scattering in the atmosphere. Beam radiation is also referred to as *direct radiation.*

Diffuse radiation Solar radiation scattered by aerosols, dust, and by the Rayleigh mechanism; it does not have a unique direction. d

Total radiation The total of diffuse and beam radiation; total radiation is sometimes referred to as *global radiation.*

※ **Direct radition+ Diffuse radiation+Indirect radiation =Total radiation**

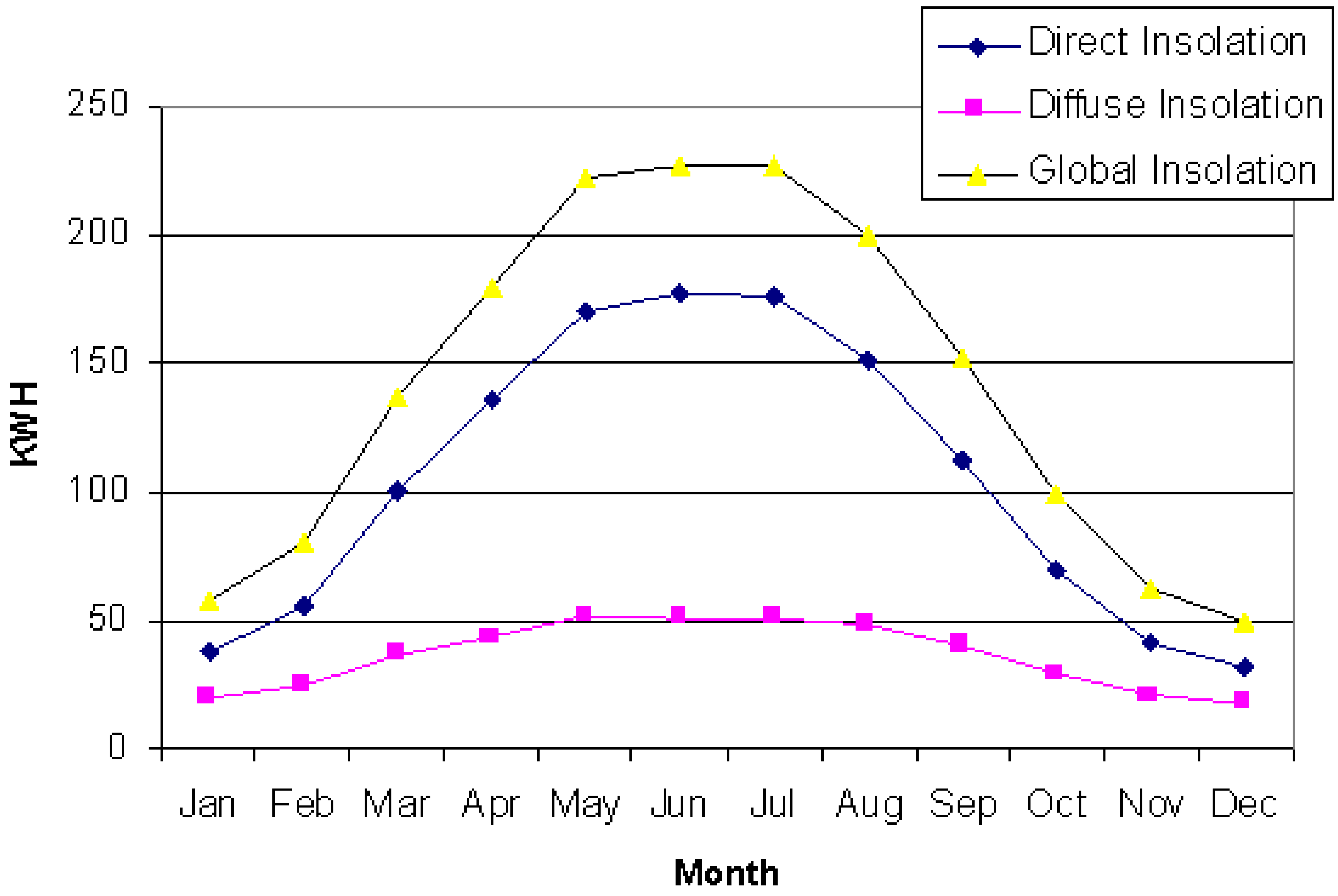
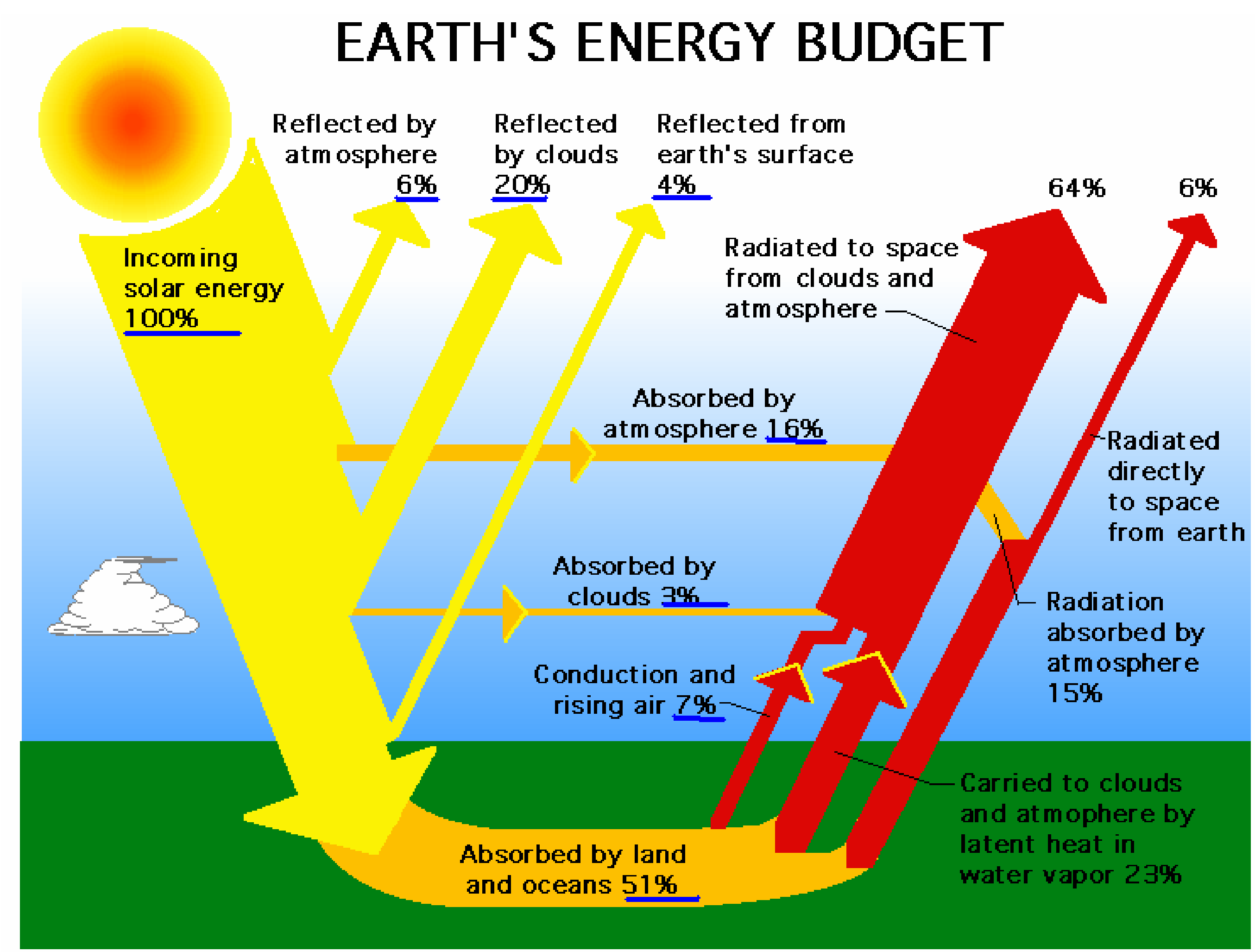
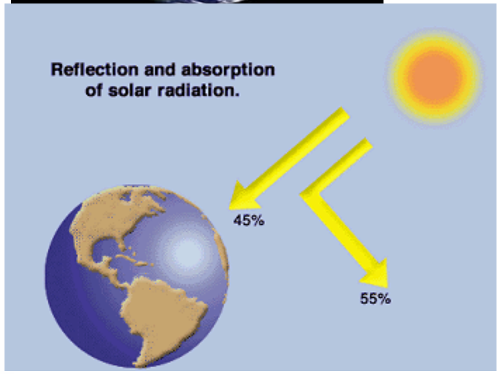
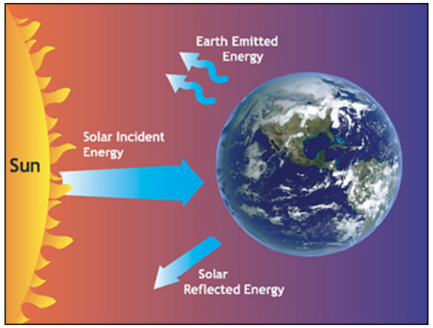


Fig. Annual pattern of direct, diffuse and global insolation for average clear day for the weather station at the research meadow

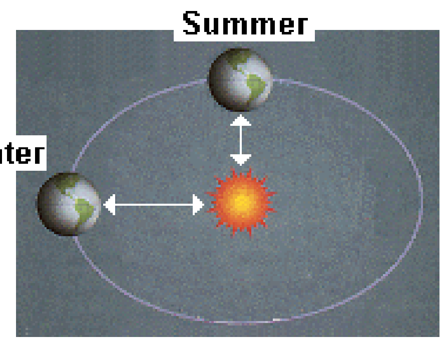






Of this amount 30 percent is reflected to space,
47 percent is converted to low-temperature heat and reradiated to space,
and 23 percent powers the evaporation/precipitation cycle of the biosphere;
less than 0.5 percent is represented in the kinetic energy of the wind and waves
and in photosynthetic storage in plants.

2.1.1 The Solar Constant



solar constant I_0 : The average amount of solar radiation

$$I_0 = 1.353\ \text{kW/m}^2 = 429\ \text{Btu/hr} \cdot \text{ft}^2 \quad (\pm 1.6\ \text{percent}).$$
$$= 1.94\ \text{cal/cm}^2 \cdot \text{min}$$

since the sun-earth orbit is elliptical, the sun-earth distance varies by ±1.7 percent during a year, and the extraterrestrial radiation also varies slightly by the inverse-square law

TABLE 2.1 Annual Variation of Solar Radiation from Orbital Eccentricity

Date	Radius vector[a]	Ratio of flux to solar constant	Solar radiation (kW/m^2)
Jan. 1	0.9832	1.034	1.399
Feb. 1	0.9853	1.030	1.394
Mar. 1	0.9908	1.019	1.379
Apr. 1	0.9993	1.001	1.354
May 1	1.0076	0.985	1.333
June 1	1.0141	0.972	1.312
July 1	1.0167	0.967	1.308
Aug. 1	1.0149	0.971	1.312
Sep. 1	1.0092	0.982	1.329
Oct. 1	1.0011	0.998	1.350
Nov. 1	0.9925	1.015	1.373
Dec. 1	0.9860	1.029	1.392

[a]Ratio of sun-earth distance to mean sun-earth distance.

$$\frac{(I_o)_{\text{actual}}}{I_o}$$

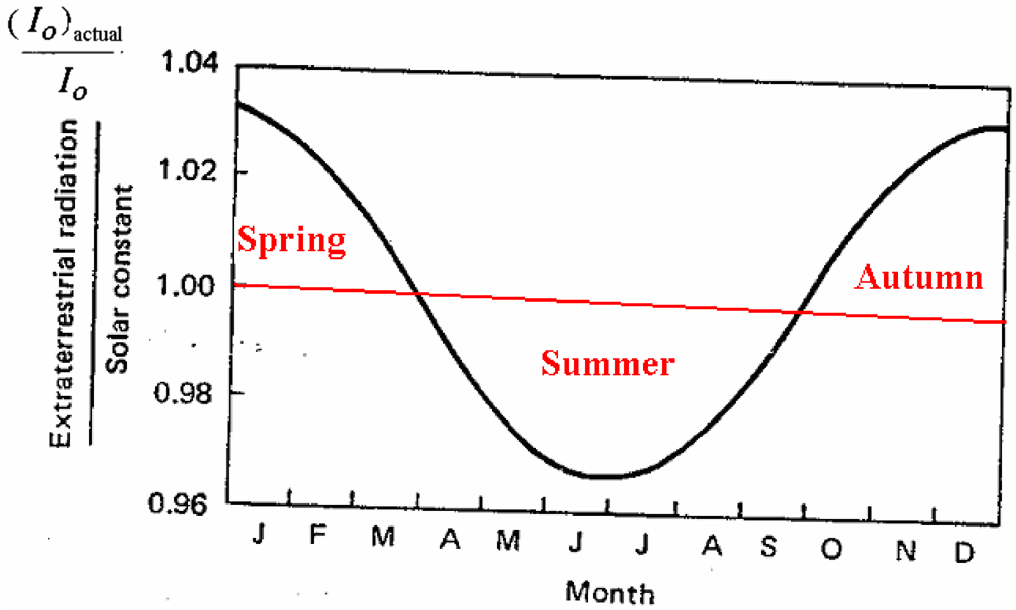
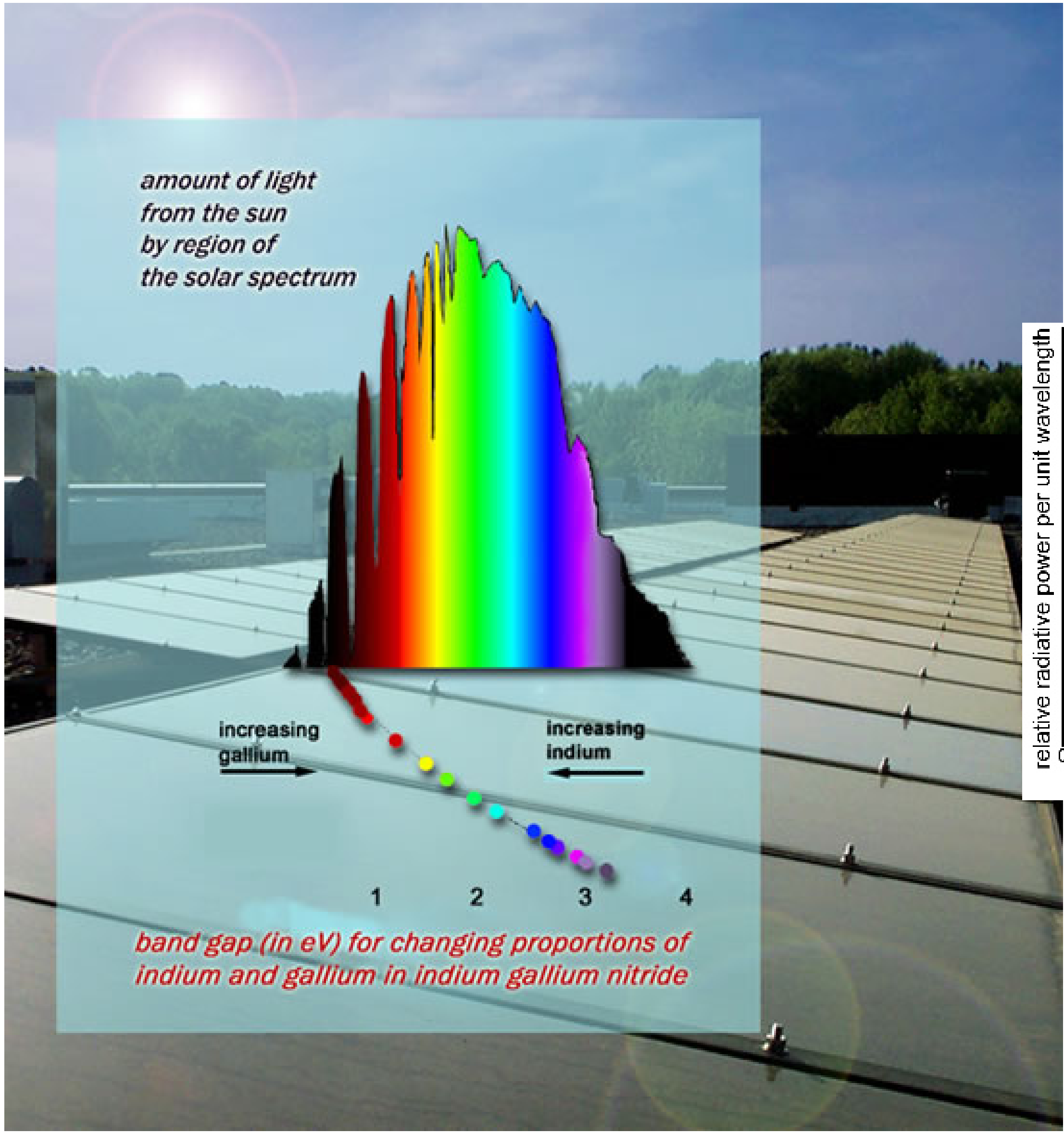
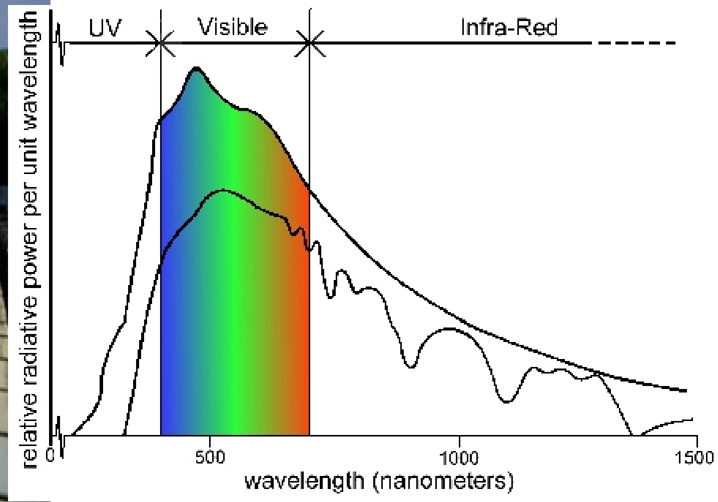


FIGURE 2.2 Effect of the time of year on the ratio of extraterrestrial radiation to the nominal solar constant.


amount of light
from the sun
by region of
the solar spectrum
increasing gallium
increasing indium
1
2
3
4
band gap (in eV) for changing proportions of
indium and gallium in indium gallium nitride


UV
Visible
Infra-Red
relative radiative power per unit wavelength
0
500
1000
1500
wavelength (nanometers)

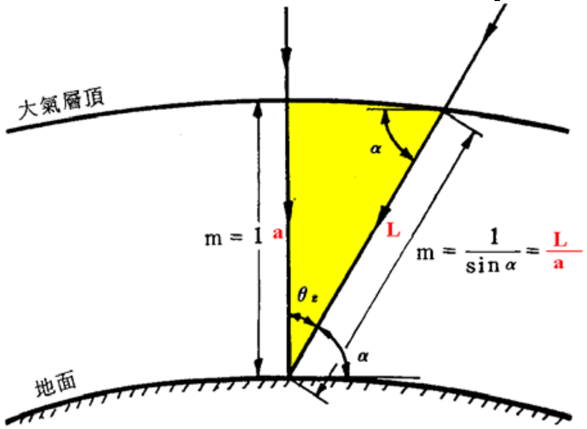
2.2.2 TERRESTRIAL SOLAR RADIATION ESTIMATION—BEAM COMPONENT

Air Mass Ratio m —

the dimensionless path length of sunlight through the atmosphere

$$m = \frac{L}{a} = \sec\theta_z = \frac{1}{\sin\alpha} \quad (90^\circ > \alpha > 0^\circ)$$

α : solar-altitude angle （太陽高度角）



太陽在大氣之入射路徑

$m = 1$ the sun is overhead.

$m = 2$ solar-altitude angle is 30°

$m = 0$ extraterrestrial radiation

2.2.2 TERRESTRIAL SOLAR RADIATION ESTIMATION—BEAM COMPONENT

(1) Solar Radiaion under clear skies

Bouger's law equation—calculating atmospheric absorption of solar radiation for clear skies

$$I_b = I_o e^{-km} \qquad (2.1)$$

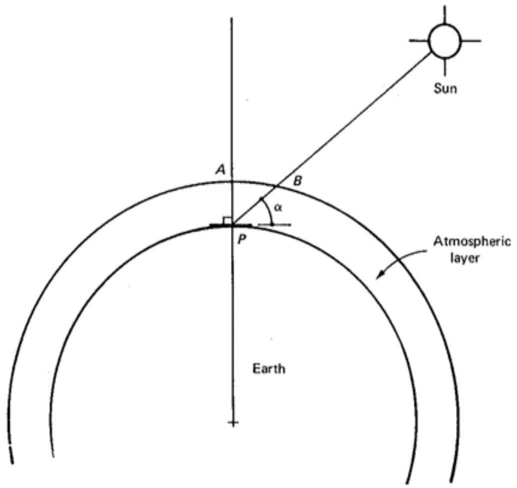


FIGURE 2.4 Air mass definition; air mass $m = BP/AP = \csc\alpha$, where α is the altitude angle. The atmosphere is idealized as a constant thickness layer.

I_b the terrestrial intensity of beam radiation

I_o the extraterrestrial intensity of beam radiation

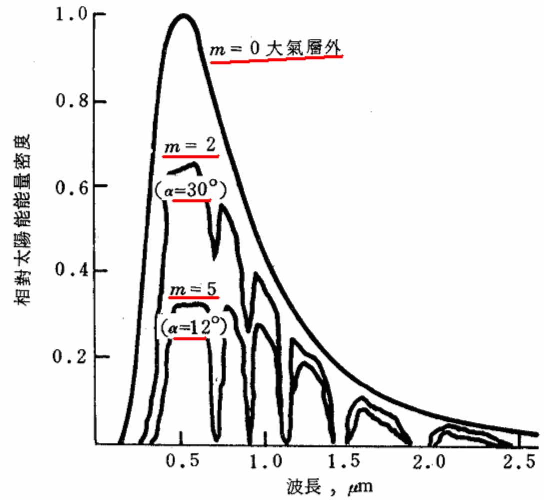
k an absorption constant for the atmosphere

For clear skies atmospheric transmittance $\qquad I_b = I_o e^{-km} \qquad (2.1)$

$$\boxed{\bar{\tau}_{\text{atm}} (\equiv I_b / I_o)} = 0.5\left(e^{-0.65 m(z,\alpha)} + e^{-0.095 m(z,\alpha)}\right) \quad \pm 3\% \text{ accuracy} \qquad (2.2)$$

$$m(z,\alpha) = m(0,\alpha) \frac{p(z)}{p(0)}$$

$$= \left[1229 + (614 \sin \alpha)^2\right]^{1/2} - 614 \sin \alpha \; \frac{p(z)}{p(0)}$$



通過大氣層太陽輻射之變化

I_b is the terrestrial intensity of beam radiation

I_o is the extraterrestrial intensity of beam radiatio

$m(0,\alpha)$ is the air mass m at sea level (altitude 0)

$m(z,\alpha)$ is the air mass m at an altitude z above sea level

α is the solar-altitude angle

p is the atmospheric pressure

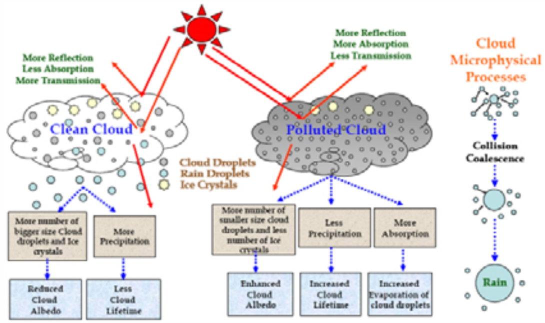
The surface beam radiation I_b is given by

$$\boxed{I_b = I_o \bar{\tau}_{\text{atm}}} \qquad (2.5)$$

Equations (2.2) and (2.4) represent an accuracy improvement over Eq. (2.1), since they include curvature effects.

Modified Transmittance Equation

for particulates air pollution and water vapor in the air



modifications $\qquad \bar{\tau}_{atm} = a_0 + a_1 e^{-k \csc \alpha}$

a_0, a_1, and k are functions only of altitude and visibility shown in Table 2.3

TABLE 2.3 Coefficients a_0, a_1, and k Calculated for the 1962 Standard Atmosphere for Use in Determining Solar Transmittance $\bar{\tau}_{atm}$[a]

	Altitude above sea level (km)					extrapolated values.
	0	0.5	1	1.5	2	$(2.5)^b$
			23 km haze model			
a_0	0.1283	0.1742	0.2195	0.2582	0.2915	(0.320)
a_1	0.7559	0.7214	0.6848	0.6532	0.6265	(0.602)
k	0.3878	0.3436	0.3139	0.2910	0.2745	(0.268)
			5 km haze model			
a_0	0.0270	(0.063)	0.0964	(0.126)	(0.153)	(0.177)
a_1	0.8101	(0.804)	0.7978	(0.793)	(0.788)	(0.784)
k	0.7552	(0.573)	0.4313	(0.330)	(0.269)	(0.249)

[a] Adapted from Hottel (5) by permission.
[b] Values in parentheses indicate interpolated or extrapolated values.

TABLE 2.3 Coefficients a_0, a_1, and k Calculated for the 1962 Standard Atmosphere for Use in Determining Solar Transmittance $\bar{\tau}_{atm}$[a]

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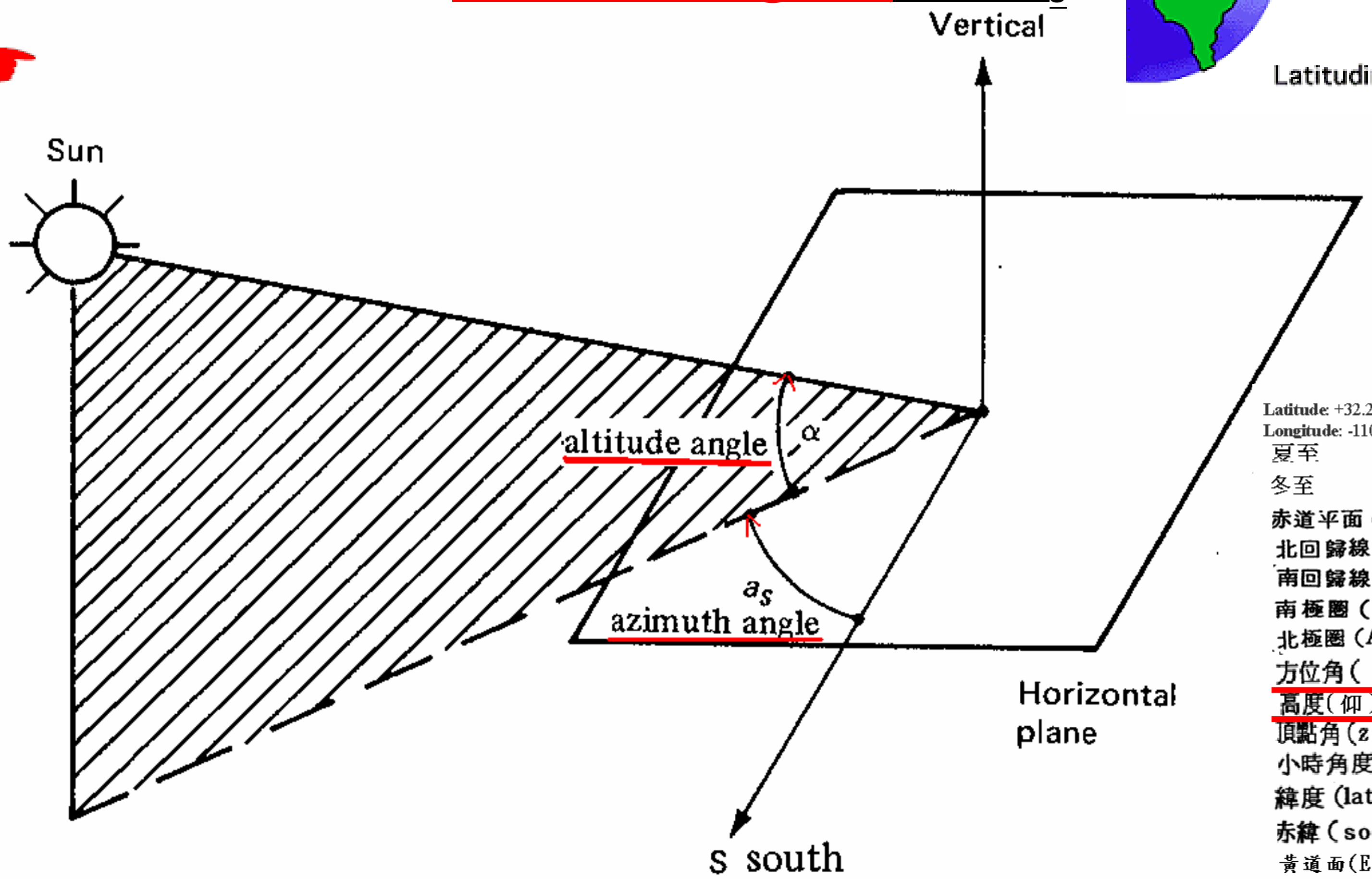
k	0.7552	(0.573)	0.4313	(0.330)	(0.269)	(0.249)

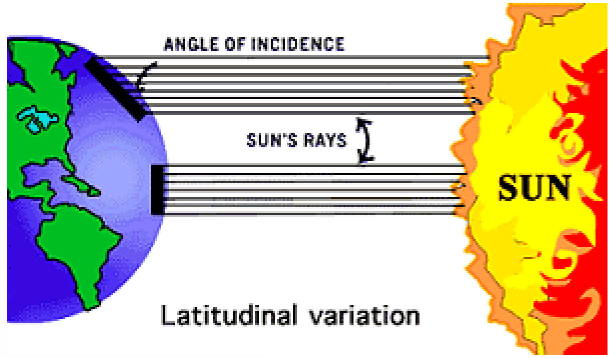
[a]Adapted from Hottel (5) by permission.
[b]Values in parentheses indicate interpolated or extrapolated values.

2.3 SOLAR RADIATION GEOMETRY

Terminology ---- Altitude Angle (β , or α) and Azimith Angles (γ , or a_s)

☛



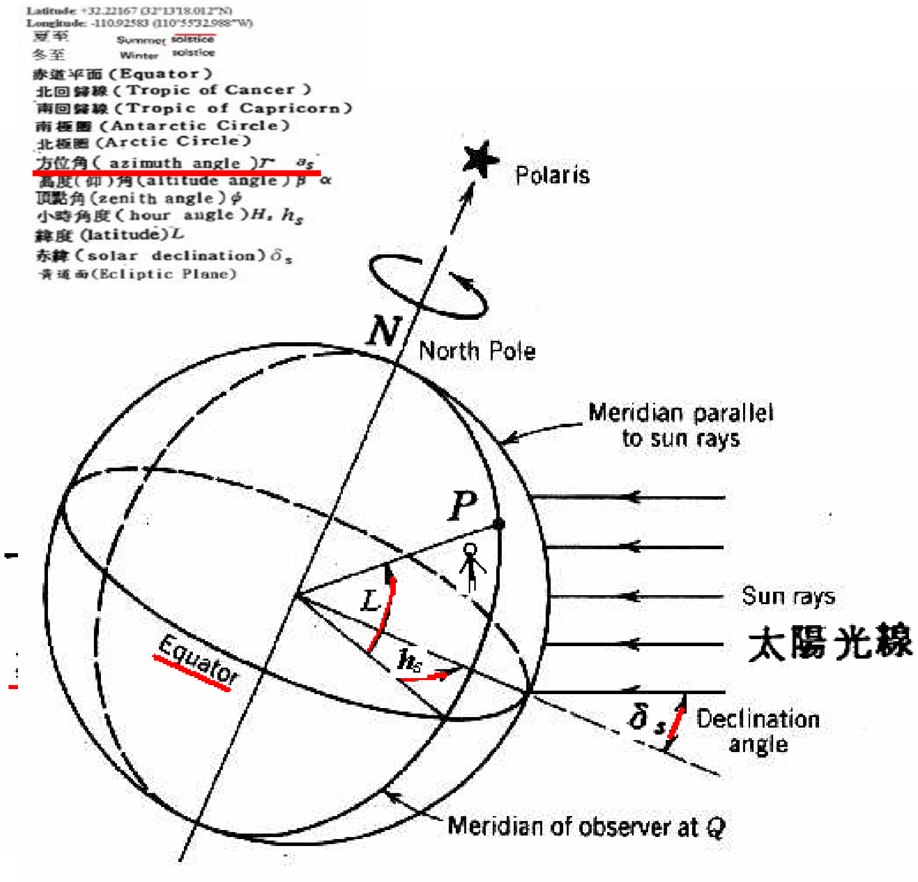


Latitude: +32.22167 (32°13'18.012"N)
Longitude: -110.92583 (110°55'32.988"W)

夏至 Summer solstice
冬至 Winter solstice
赤道平面 (Equator)
北回歸線 (Tropic of Cancer)
南回歸線 (Tropic of Capricorn)
南極圈 (Antarctic Circle)
北極圈 (Arctic Circle)
方位角 (azimuth angle) γ a_s
高度(仰)角 (altitude angle) β α
頂點角 (zenith angle) ϕ
小時角度 (hour angle) H, h_s
緯度 (latitude) L
赤緯 (solar declination) δ_s
黃道面 (Ecliptic Plane)

FIGURE 2.6 Diagram showing solar-altitude angle α and solar-azimuth angle a_s. See also Fig. 2.9a.

Terminology ---- Solar Hour Angle (h_s), Solar Declination (δ_s), Altitude (L)



L 點表緯度 (latitude)

H_s 表小時角度 (hour angle)

δs 赤緯 (solar declination)

1 Solar Hour Angle=360°/24=15°

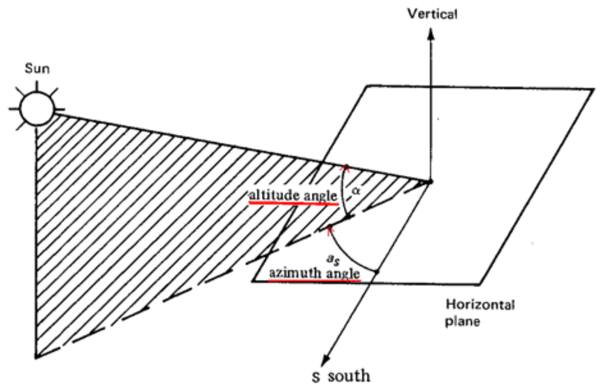
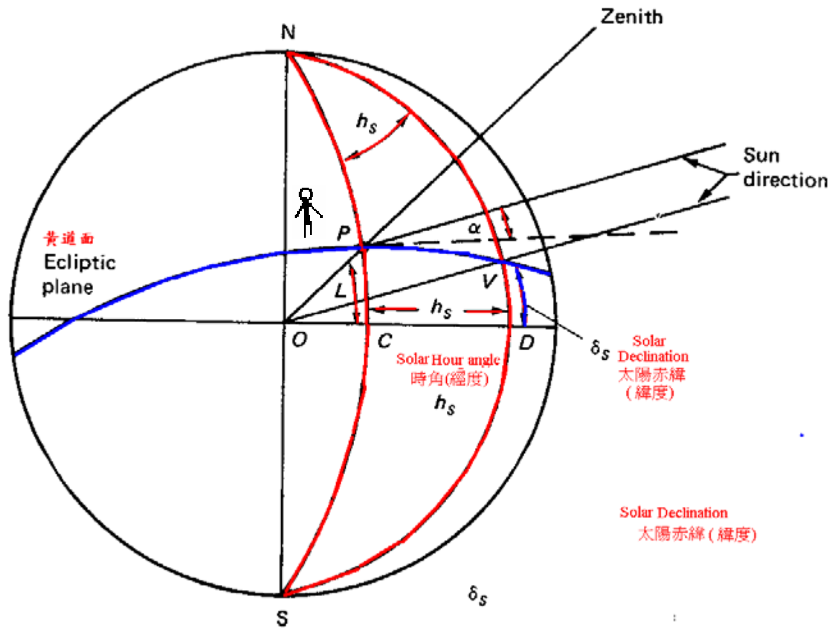


FIGURE 2.6 Diagram showing solar-altitude angle α and solar-azimuth angle a_s. See also Fig. 2.9*a*.



太陽時間(Solar time) h_s

在計算太陽能系統時都是以太陽時間(solar time)來表示，並應用太陽中午時 $h_s=0$，而每一錶上所示小時(watch hour)等於 15 度的經度(longitude)且早上時 h_s 以正號表示，下午時 h_s 以負號表示。

例 1.1 在太陽中午以前 1 小時，即 11:00a.m.

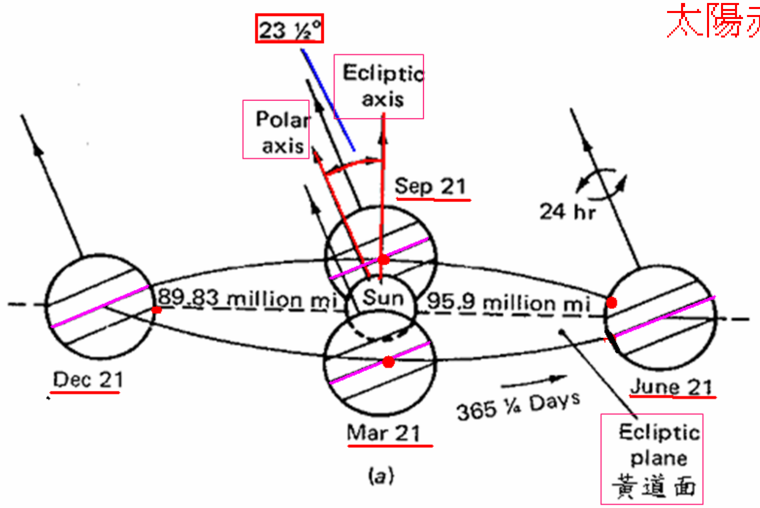
$$h_s=+15°$$

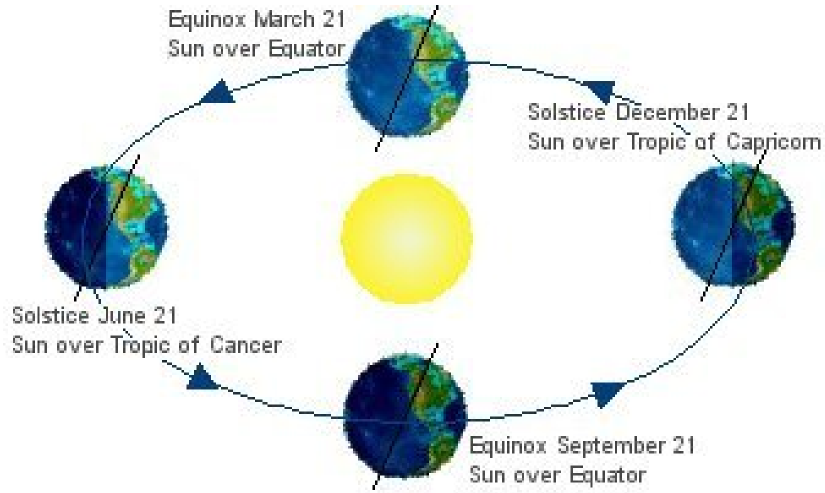
在太陽中午之後 4 小時 37 分

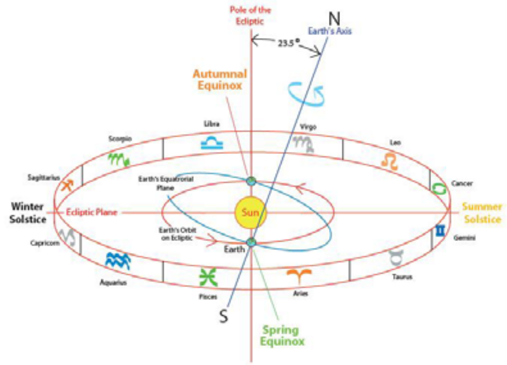
$$h_s=-4(15)-\frac{37}{60}(15)=-69.25°$$

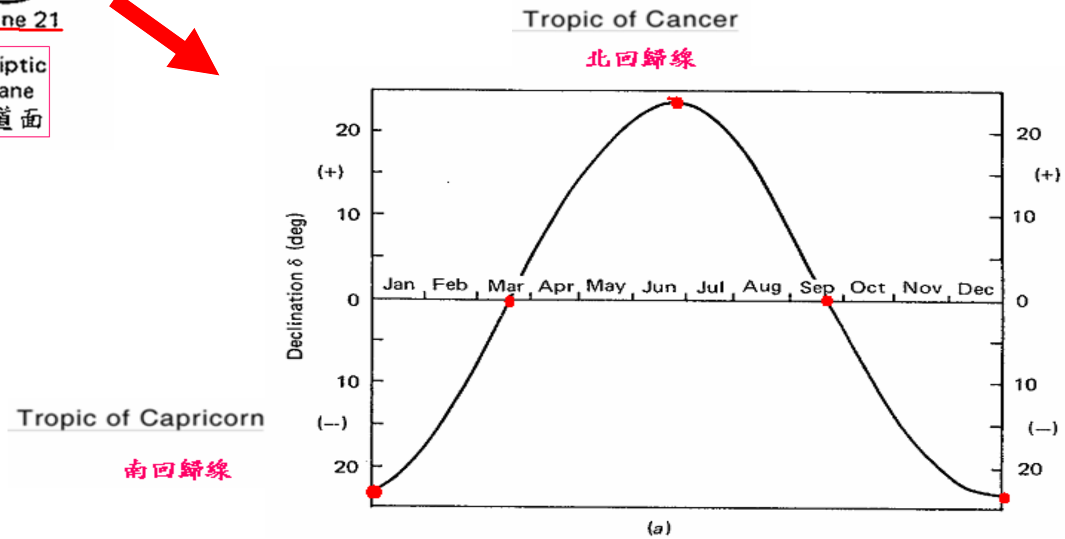
☛ Graph to determine the solar declination.

太陽赤緯(緯度)









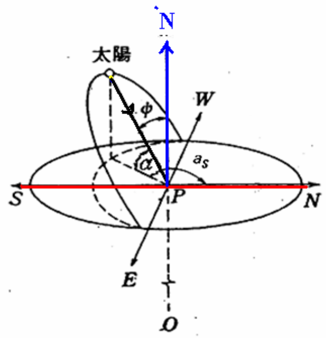
- 赤緯(solar declination)爲太陽光的入射線向北或向南對應於赤道平面的夾角

，其近似值可由Copper 公式求出

$$\delta_s = 23.45 \sin \left[\frac{360(284+n)}{365} \right]^\circ$$

The relation between ϕ, α, and a_s

太陽高度角 α 太陽方位角

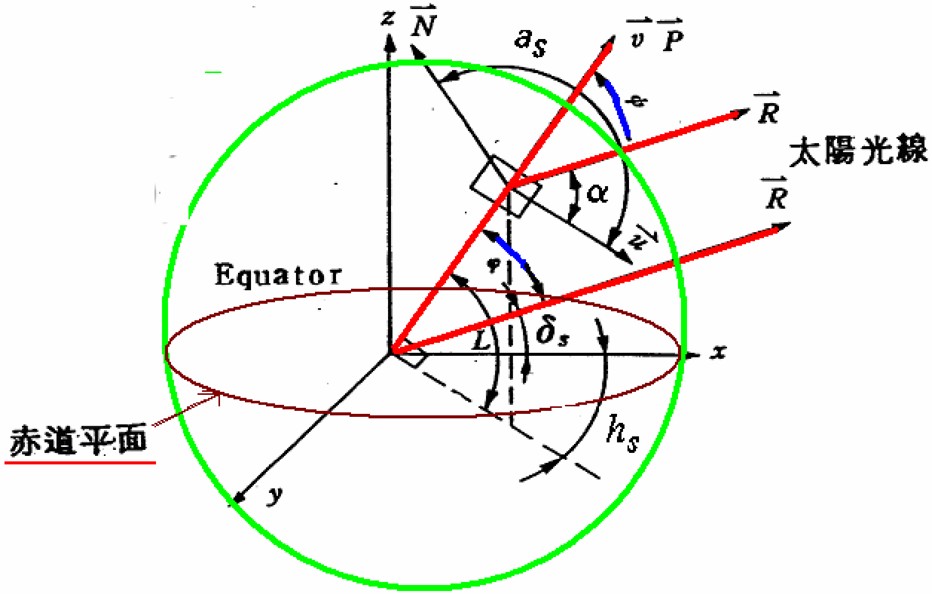


$$\cos\phi = \cos L \cos h_s \cos\delta_s + \sin L \sin\delta_s$$

$$\phi + \alpha = \frac{\pi}{2}$$

$$\sin\alpha = \cos L \cos h_s \cos\delta_s + \sin L \sin\delta_s = \cos\phi$$

$$\cos a_s = \sec\alpha\,[\cos L \sin\delta_s - \cos\delta_s \sin L \cos h_s] = \sin a_s = \frac{\cos\delta_s \sin h_s}{\cos\alpha}$$



$$\delta_s = 23.45 \cdot \sin\left[\frac{360 \cdot (284 + n)}{365}\right]$$

(Copper formulation)

$$\alpha = \arcsin(\sin L \sin\delta_s + \cos L \cos\delta_s \cos H_s)$$

$$\Phi = \arcsin\left(\frac{\cos\delta_s \sin H_s}{\cos\alpha}\right)$$

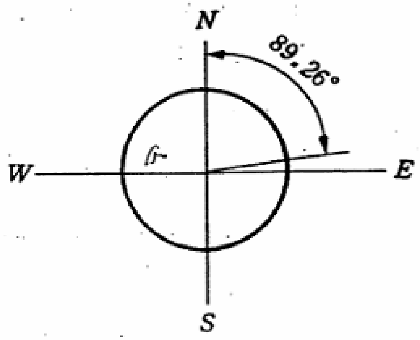
方位角 (azimuth angle) γ a_s

高度(仰)角 (altitude angle) β α

赤緯 (solar declination) δ_s

例1.3　於8月1日太陽時間7：30 a.m.在北緯40°試求太陽的高度角 α 及方位角 a_s

- $\sin\alpha = \cos L \cos h_s \cos\delta_s + \sin L \sin\delta_s \quad \psi = \dfrac{\pi}{2} - \alpha$
- $\cos a_s = \sec\alpha\,[\cos L \sin\delta_s - \cos\delta_s \sin L \cos h_s]$



- $\delta_s = 23.45 \cdot \sin\left[\dfrac{360 \cdot (284+n)}{365}\right]$

方位角（azimuth angle）a_s
高度(仰)角(altitude angle) α
赤緯（solar declination）δ_s

∵　北緯40°　$L = 40°N$

7：30 a.m.　$h_s = 4(15) + \dfrac{30}{60}(15) = 67.5°$

$\delta_s = 18°12' = 18.2°$，$n = 213$

或可由

$$\delta_s = 23.45 \sin \frac{360(284+213)^{\circ}}{365} = 17.91°$$

$$\therefore \sin\alpha = \cos L \cos h_s \cos\delta_s + \sin L \sin\delta_s$$

$$= \cos 40° \cos 67.5° \cos 18.2° + \sin 40° \sin 18.2° = 0.4792$$

$\Rightarrow$ $\alpha = 28.63°$，$\psi = \dfrac{\pi}{2} - \alpha = 90° - 28.63° = 61.37°$

$$\cos a_s = \sec\alpha\,[\cos L \sin\delta_s - \cos\delta_s \sin L \cos h_s]$$

$$= \sec 28.63° \,[\cos 40° \sin 18.2° - \cos 18.2° \sin 40° \cos 67.5°$$

$\Rightarrow$ $a_s = 89.26°$

EXAMPLE 2.3

Using Fig. 2.10, determine the solar altitude and azimuth for March 8 at 10 a.m. Compare the results to those calculated from the basic Eqs. (2.8) and (2.9).

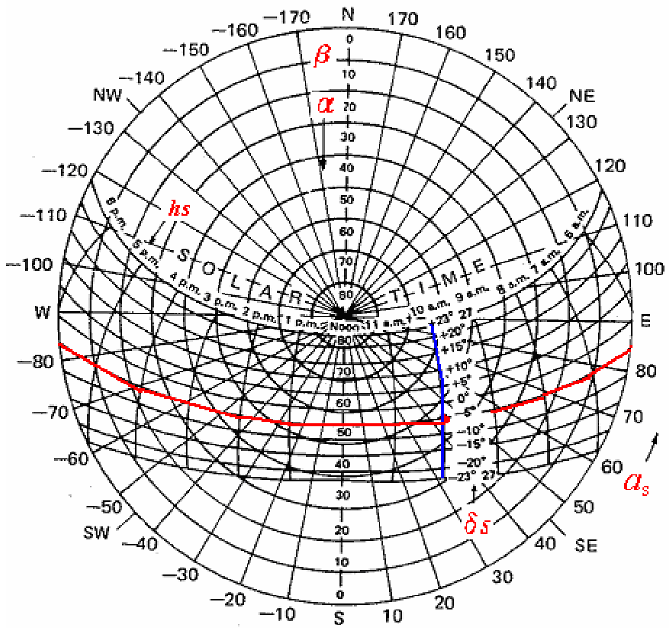
Declination	Approx. dates
$+23°27'$	June 22
$+20°$	May 21, July 24
$+15°$	May 1, Aug. 12
$+10°$	Apr. 16, Aug. 28
$+5°$	Apr. 3, Sep. 10
$0°$	Mar. 21, Sep. 23
$-5°$	Mar. 8, Oct. 6
$-10°$	Feb. 23, Oct. 20
$-15°$	Feb. 9, Nov. 3
$-20°$	Jan. 21, Nov. 22
$-23°27'$	Dec. 22

SOLUTION

On March 8 the solar declination is $-5°$; therefore the $-5°$ sun path is used. The intersection of the 10 a.m. line and the $-5°$ declination line in the diagram represents the sun's location; it is marked with a heavy dot in Fig. 2.10. The sun position lies midway between the $40°$ and $50°$ altitude circles, say at $45°$, and midway between the $40°$ and $50°$ azimuth radial lines, say at $45°$. So

$$\alpha \cong 45°$$

$$a_s \cong 45°$$



Equations (2.8) and (2.9) give precise values for α and a_s:

$$\sin \alpha = \sin (30°) \sin (-5°) + \cos (30°) \cos (-5°) \cos (30°)$$

$$\alpha = 44.7°$$

$$\sin a_s = \frac{\cos (-5°) \sin (30°)}{\cos (44.7°)}$$

$$a_s = 44.5°$$

Therefore, the calculated values are within $0.5°$ (1 percent) of those read from the sun-path diagram.

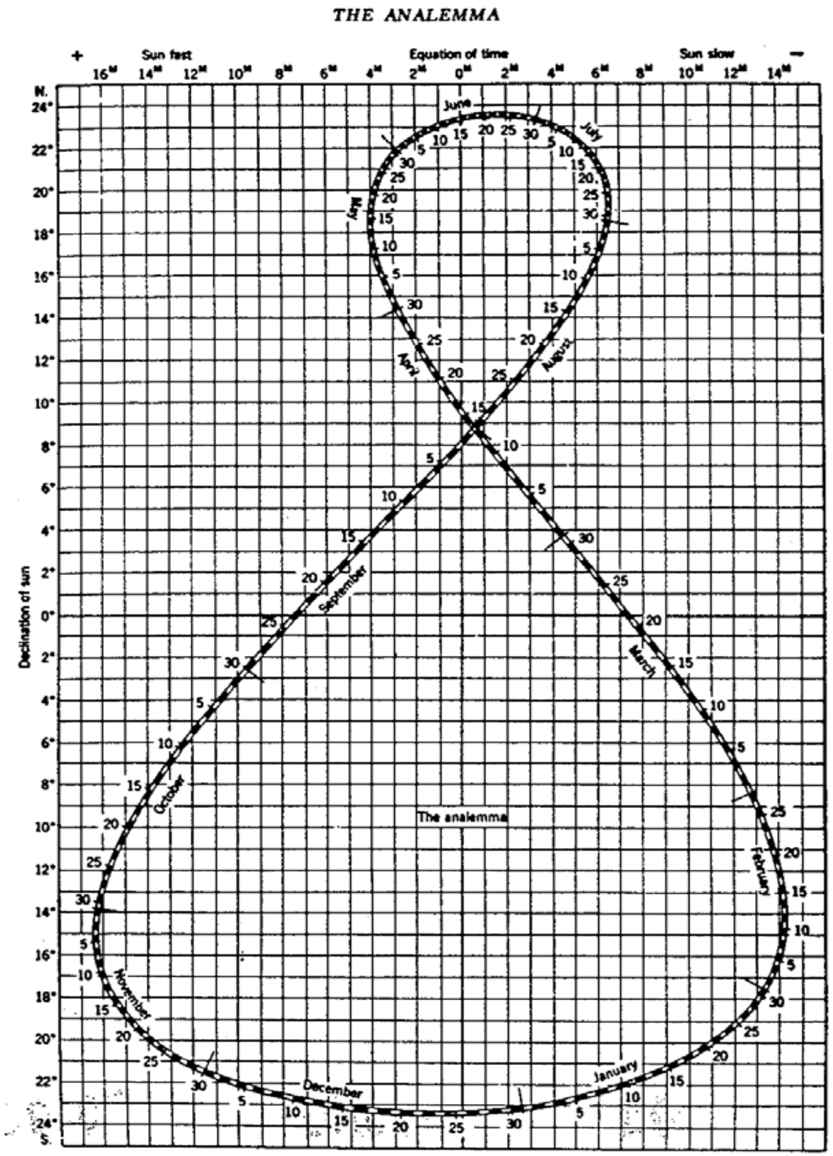
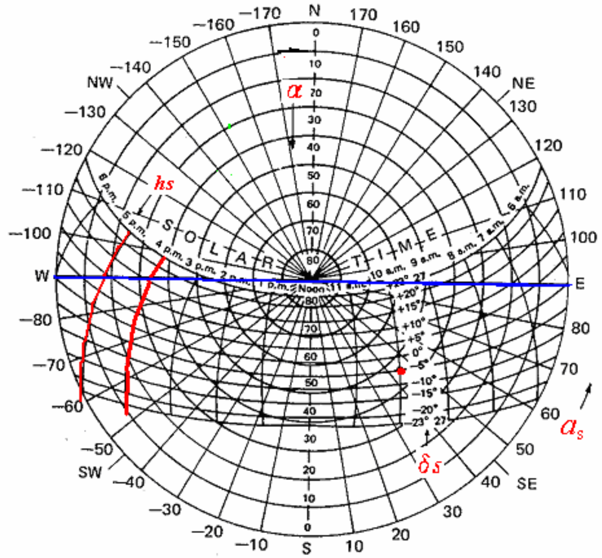
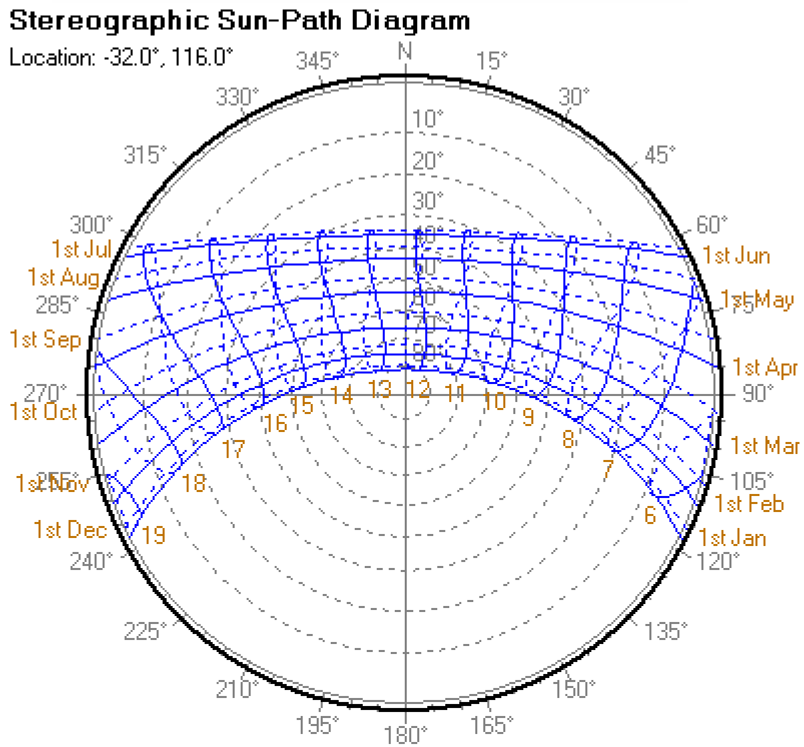
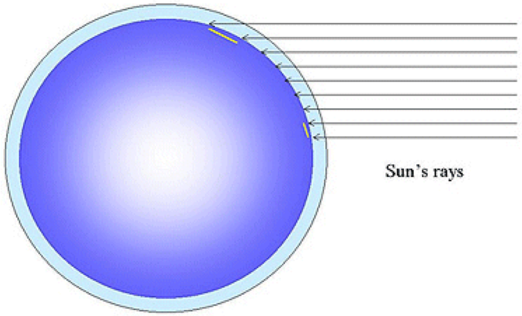


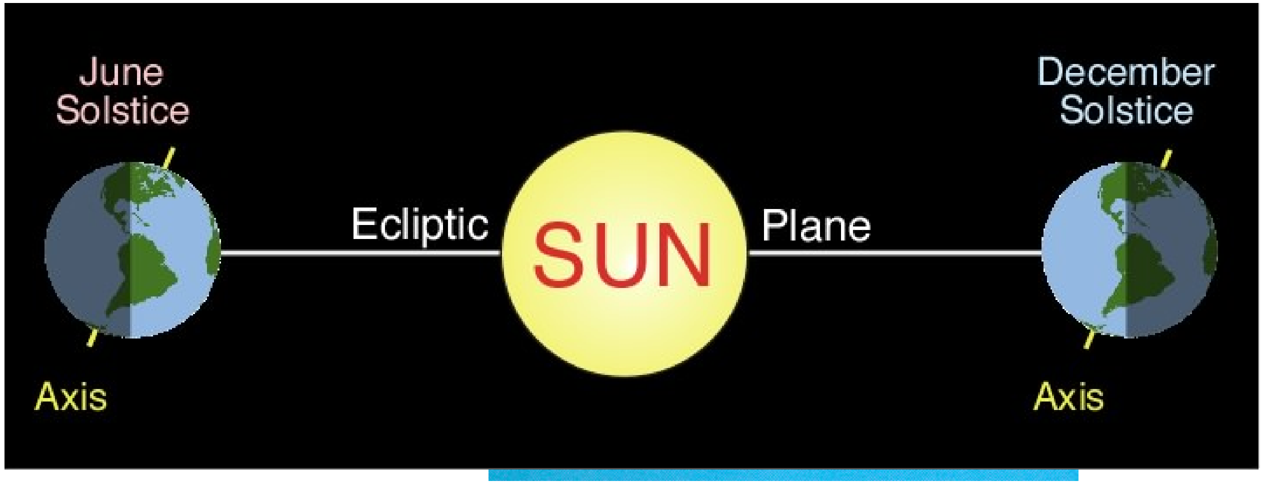
FIGURE 2.16 The analemma—a graphical presentation of the equation of time. Equation of time values are plotted to the right and left of the vertical centerline.



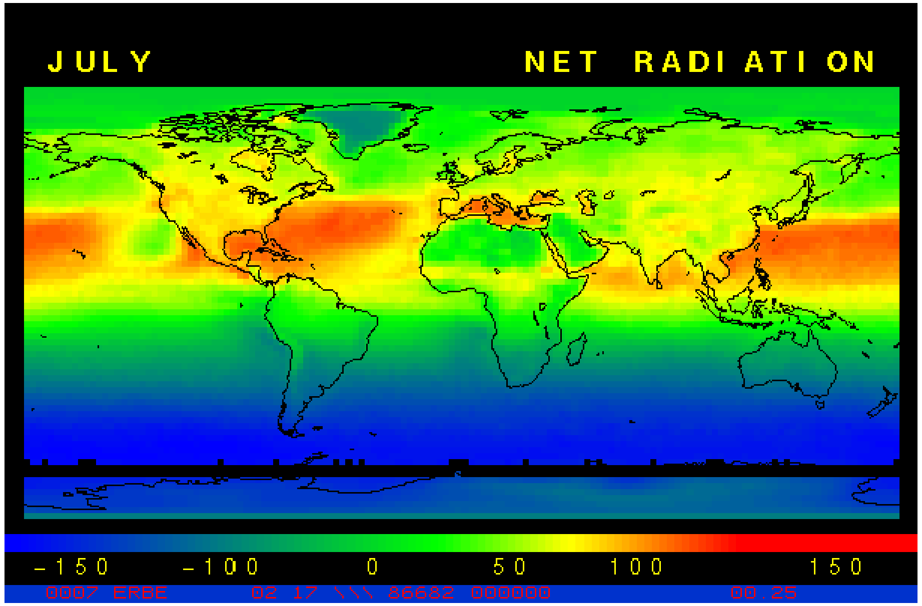




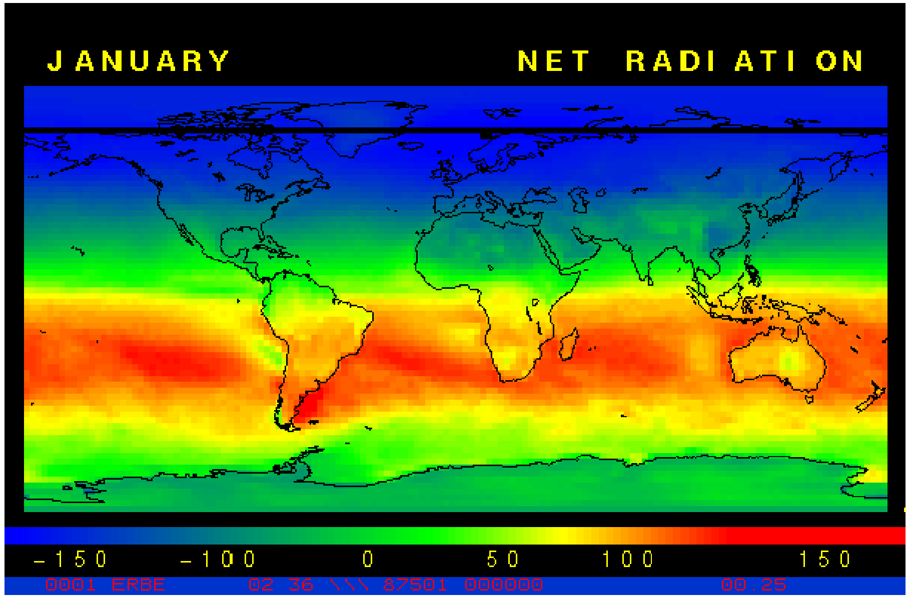
Sun's rays



June Solstice
December Solstice
Ecliptic
SUN
Plane
Axis
Axis



JULY
NET RADIATION
-150 -100 0 50 100 150
0007 ERBE 02 17 \\\ 86682 000000 00.25



JANUARY
NET RADIATION
-150 -100 0 50 100 150
0001 ERBE 02 36 \\\ 87501 000000 00.25

☛ CALCULATION OF RADIATION INTERCEPTED BY SURFACES--BEAM COMPONENT

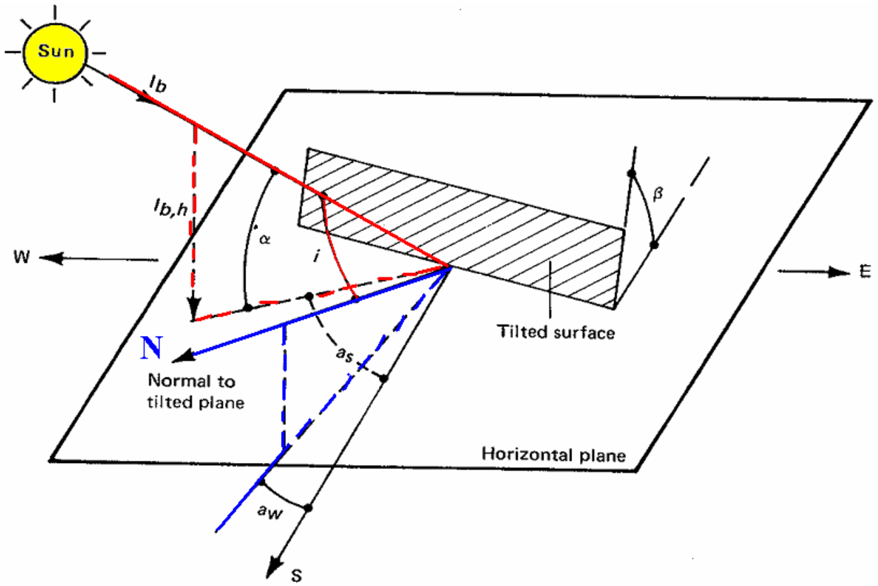
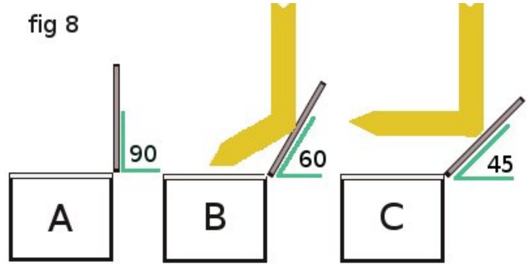


fig 8



- The *incidence angle i* is defined as the angle between the normal to a surface and a line collinear with the sun's rays.

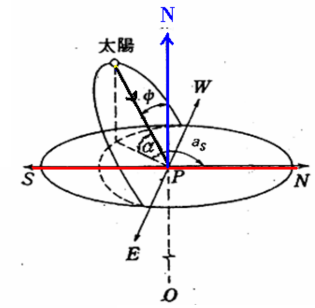
  太陽光之入射角，爲垂直於平面的方向（亦即面之方向）與太陽光入射線的夾角（ incidence angle ）。

- the component of beam radiation $I_{b,c}$ intercepted by the surface of a collector is given by $\boxed{I_{b,c} = I_b \cos i}$

a_w：太陽光之壁方位角（wall solar azimuth）爲在水平面上取垂直於直面的線與太陽光線在水平面上投影的夾角。

(i) *South-facing Horizontal and Vertical Surfaces (Fixed)*

$$\cos\phi = \sin\delta_s \sin L + \cos\delta_s \cos L \cos h_s$$



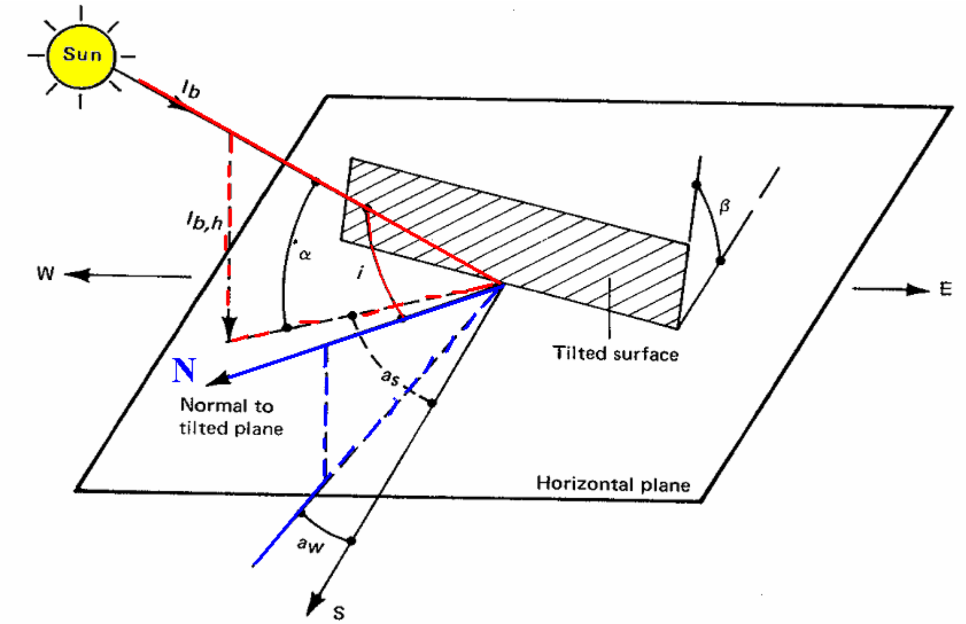
(1) For vertical south-facing surfaces , $a_w = 0,\ \beta = 90°$,

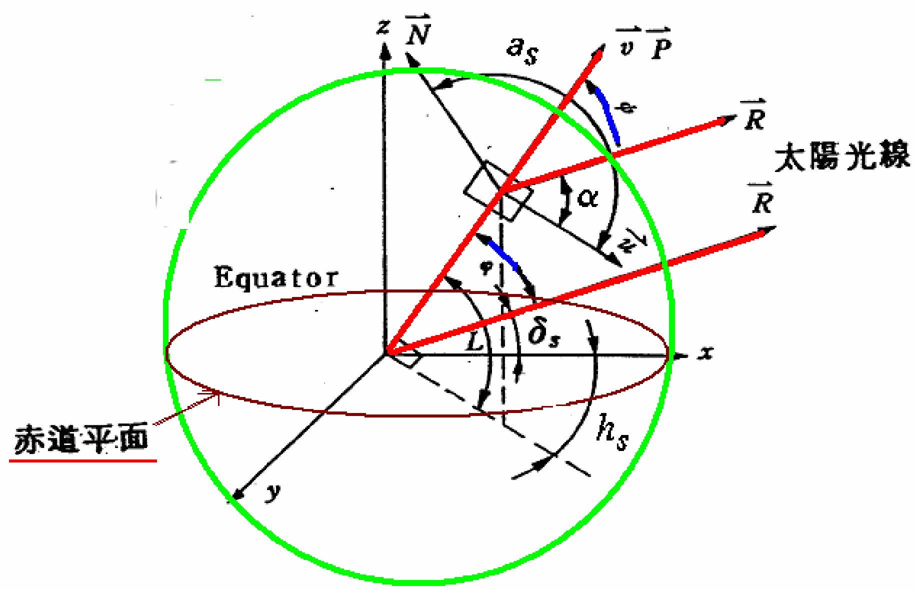
the incidence angle

$$\boxed{\cos i = -\sin\delta_s \cos L + \cos\delta_s \sin L \cos h_s} \qquad \phi + \beta = \frac{\pi}{2}$$

(2) For a horizontal south-facing surface, $\beta = 0$,the incidence angle

$$i = \phi \quad \boxed{\cos i = \sin\delta_s \sin L + \cos\delta_s \cos L \cos h_s\ (= \sin\alpha)} = \cos\phi$$





(1) For vertical south-facing surfaces , $a_w = 0$, $\beta = 90°$,

the incidence angle

$$\boxed{\cos i = -\sin \delta_s \cos L + \cos \delta_s \sin L \cos h_s} \quad \psi + \beta = \frac{\pi}{2}$$

求證：$\cos i = \cos h_s \sin L \cos \delta_s - \cos L \sin \delta_s$

證明：$\vec{P} = \sin L \cos h_s \vec{i} + \sin L \sin h_s \vec{j} - \cos L \vec{k}$

$$\vec{R} = \cos \delta_s \vec{i} + \sin \delta'_s \vec{k}$$

$$\vec{P} \cdot \vec{R} = (\cos L \cos h_s \vec{i} + \cos L \sin h_s \vec{j} + \sin L \vec{k}) \cdot (\cos \delta_s \vec{i} + \sin \delta_s \vec{k})$$

$$\therefore \cos i = \cos h_s \sin L \cos \delta_s - \cos L \sin \delta_s$$

Collector without moving surface (Fixed surface)

(i) *South-facing Horizontal and Vertical Surfaces (Fixed)*

(1) For vertical south-facing surfaces , $a_w = 0$, $\beta = 90°$,

the incidence angle $\cos\phi = \cos L \cos h_s \cos\delta_s + \sin L \sin\delta_s$

$$\boxed{\cos i = -\sin\delta_s \cos L + \cos\delta_s \sin L \cos h_s} \qquad \phi + \beta = \frac{\pi}{2}$$

(2) For a horizontal south-facing surface, $\beta = 0$, the incidence angle

$$\boxed{\cos i = \sin\delta_s \sin L + \cos\delta_s \cos L \cos h_s \ (= \sin\alpha)}$$

(ii) *South-facing Tilted Surfaces* the tilt angle β

The incidence angle for a south-facing tilted surface is

$$\boxed{\cos i = \sin(L - \beta)\sin\delta_s + \cos(L - \beta)\cos\delta_s \cos h_s}$$

(iii) *Non-south-facing Tilted Surfaces*

the incidence angle i: $\boxed{\cos i = \cos(a_s - a_w)\cos\alpha \sin\beta + \sin\alpha\cos\beta}$

(iv) Generalized Equation for Fixed Planar Surfaces

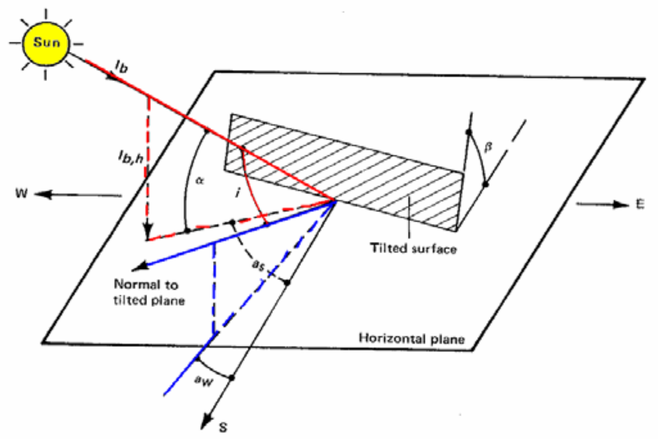
The incidence angle

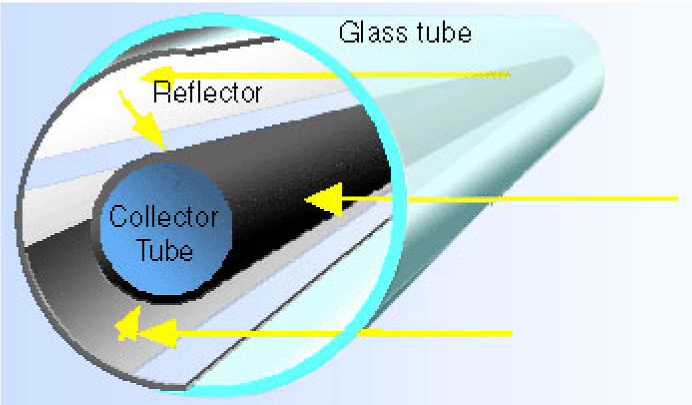
$$\boxed{\begin{aligned}\cos i = {} & \sin \delta_s(\sin L \cos \beta - \cos L \sin \beta \cos a_w) \\ & + \cos \delta_s \cos h_s(\cos L \cos \beta + \sin L \sin \beta \cos a_w) \\ & + \cos \delta_s \sin \beta \sin a_w \sin h_s\end{aligned}}$$

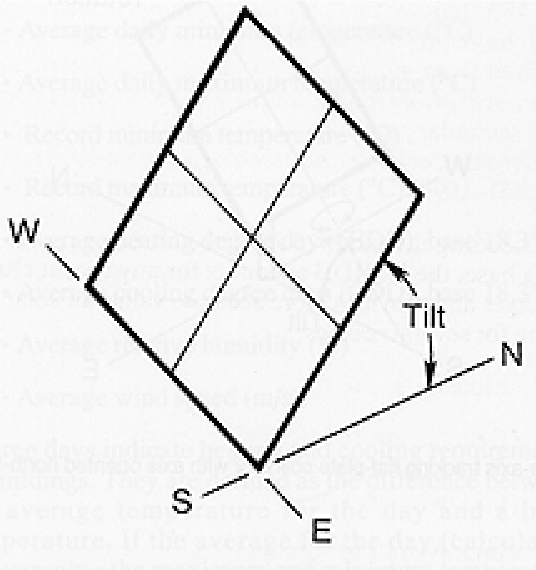
(v) Fixed, Tilted Cylindrical Surfaces in a north-south plane.

The incidence angle tubular solar collectors

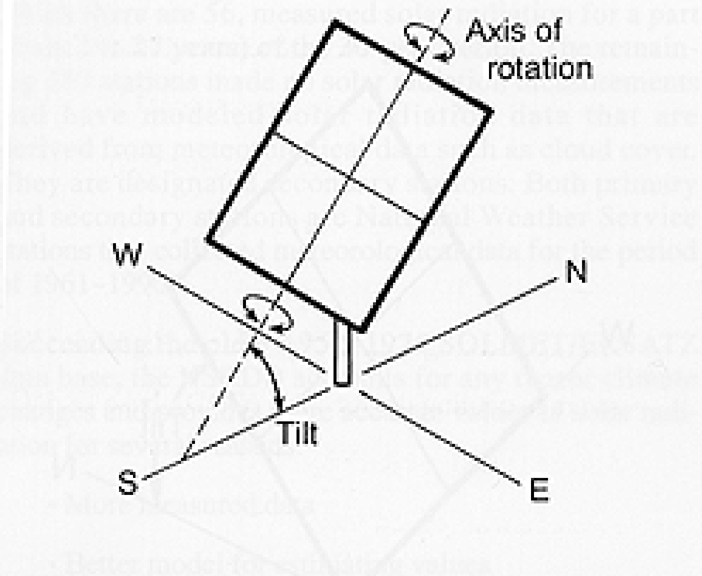
$$\boxed{\cos i = \{1 - [\sin(\beta - L) \cos \delta_s \cos h_s + \cos(\beta - L) \sin \delta_s]^2\}^{1/2}}$$



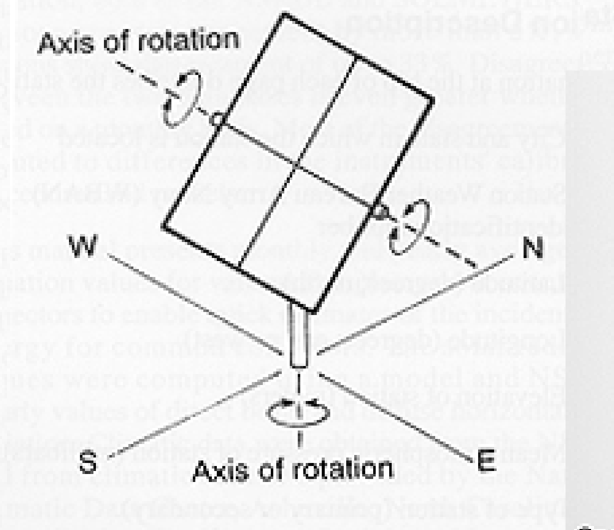




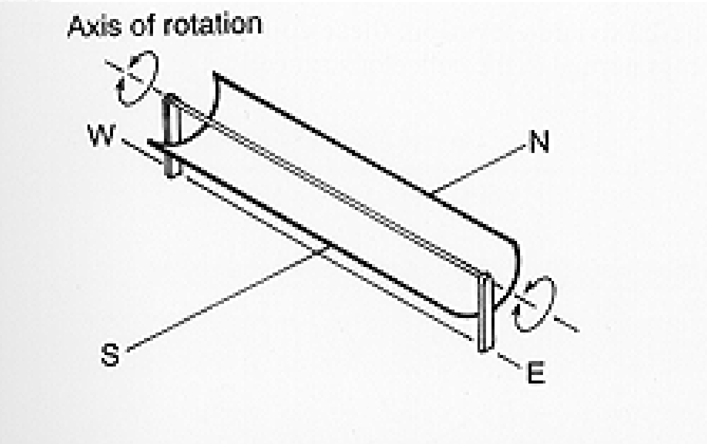
Flat-plate collector facing south at fixed tilt



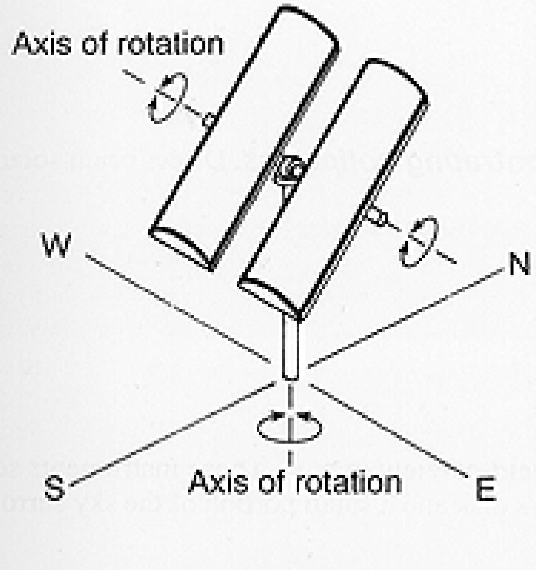
One-axis tracking flat-plate collector with axis oriented north-south



Two-axis tracking flat-plate collector



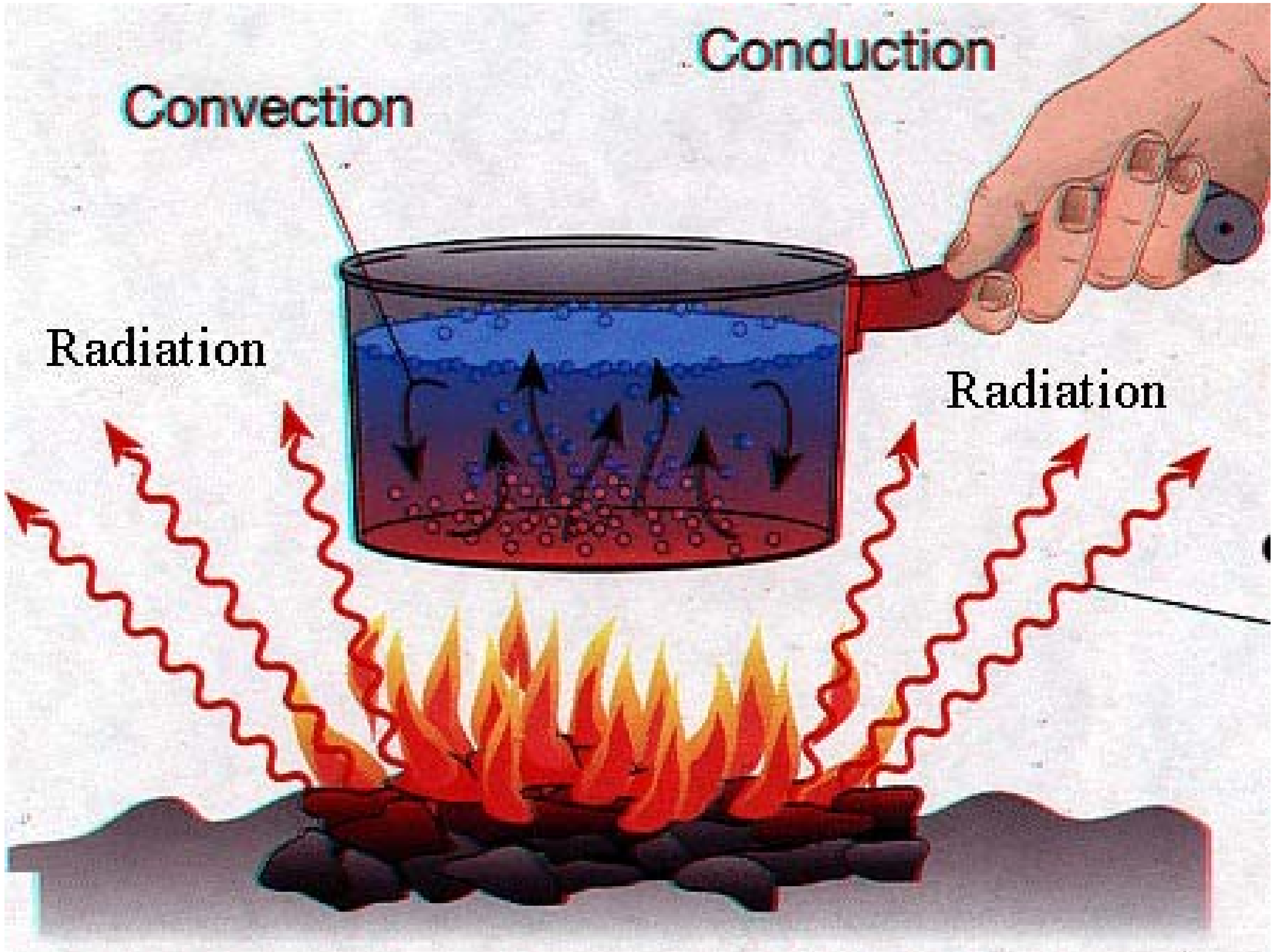
One-axis tracking parabolic trough with axis oriented east-west

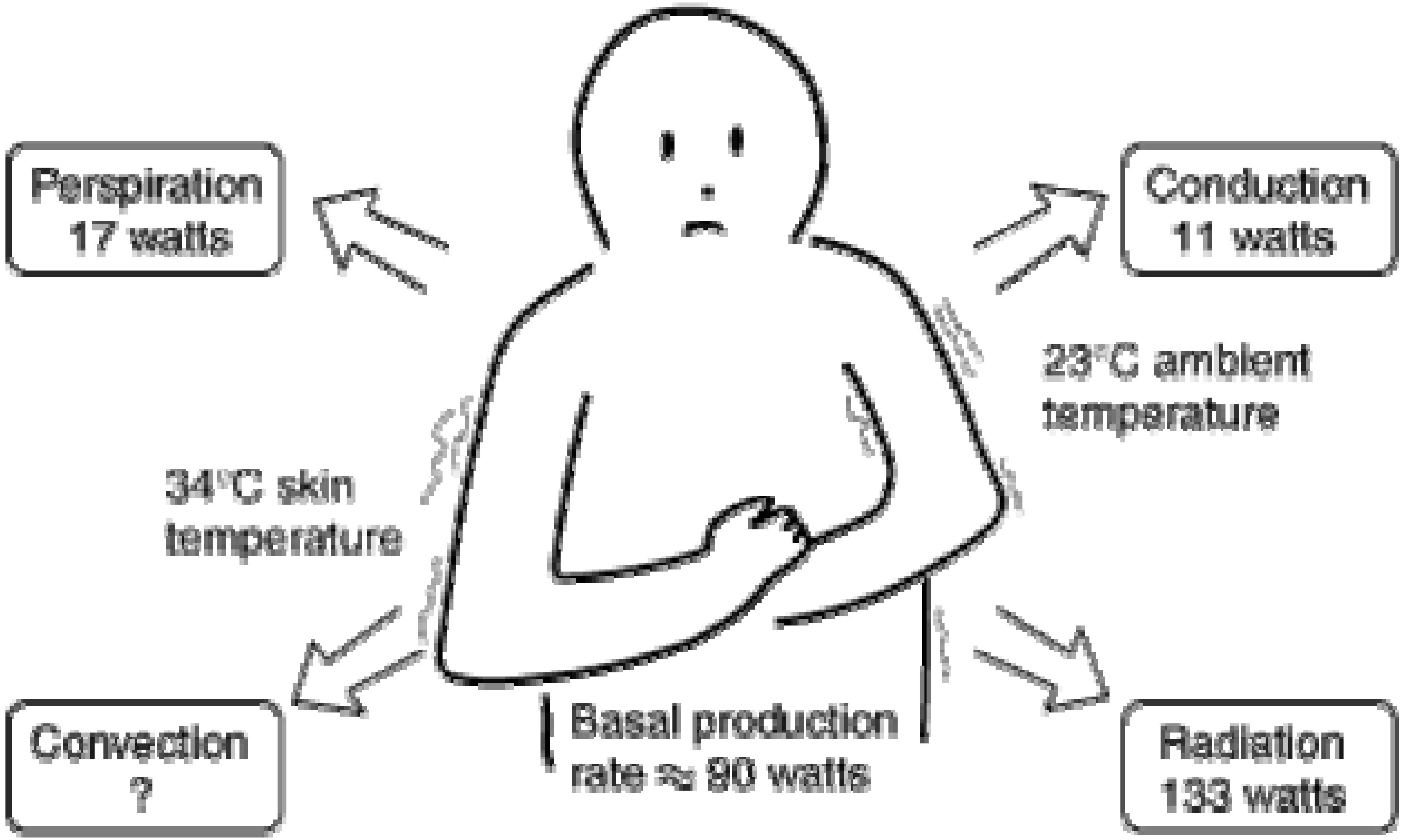


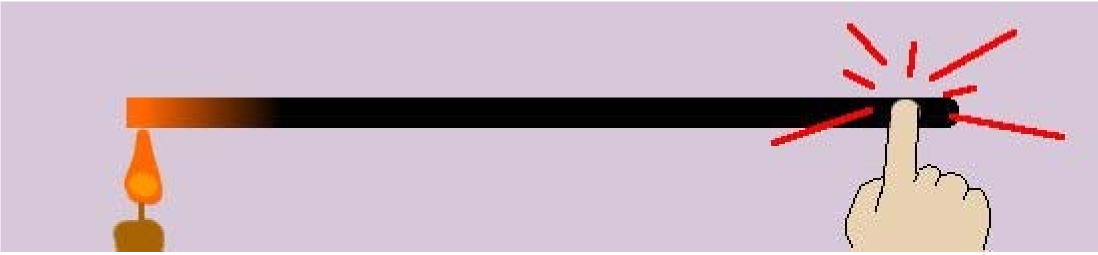
Two-axis tracking concentrator

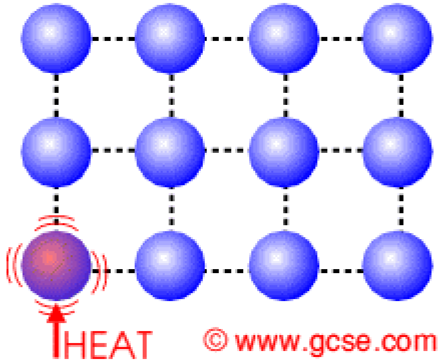
Chap. 3

Radiation Heat Transfer


Conduction
Convection
Radiation
Radiation

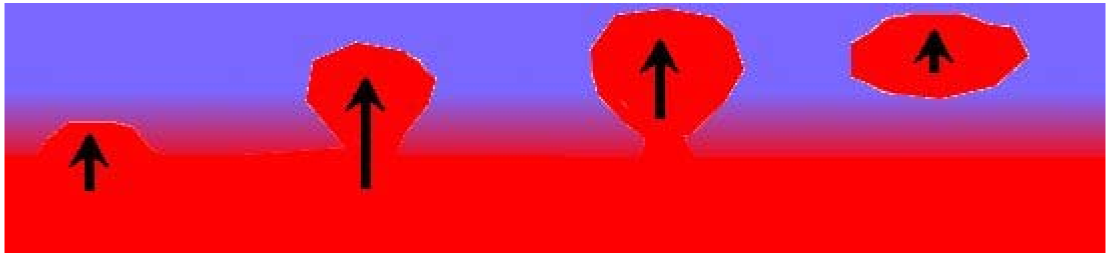

Perspiration
17 watts
Conduction
11 watts
23°C ambient
temperature
34°C skin
temperature
Convection
?
Basal production
rate ≈ 90 watts
Radiation
133 watts

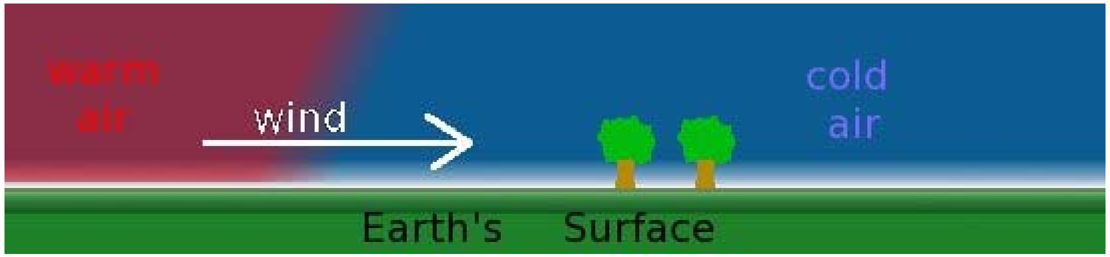


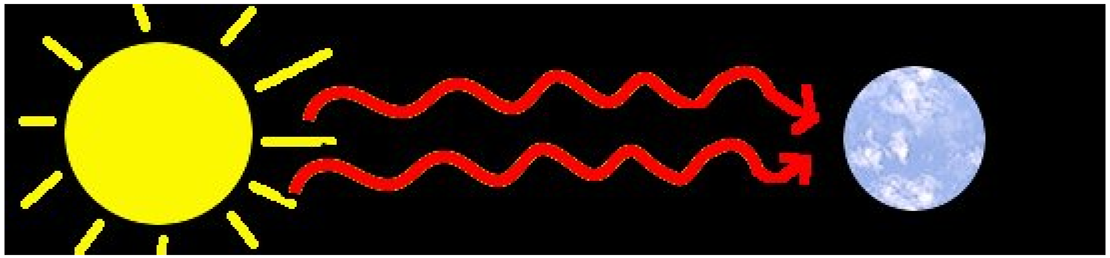


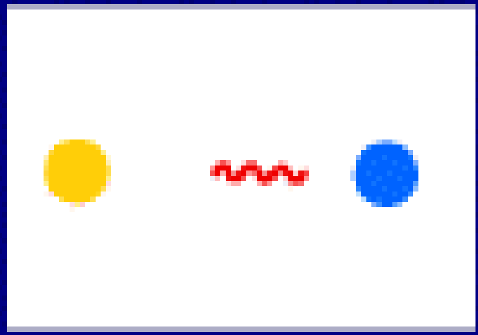
Heat can be transferred by:

Conduction
Convection
Advection
Radiation

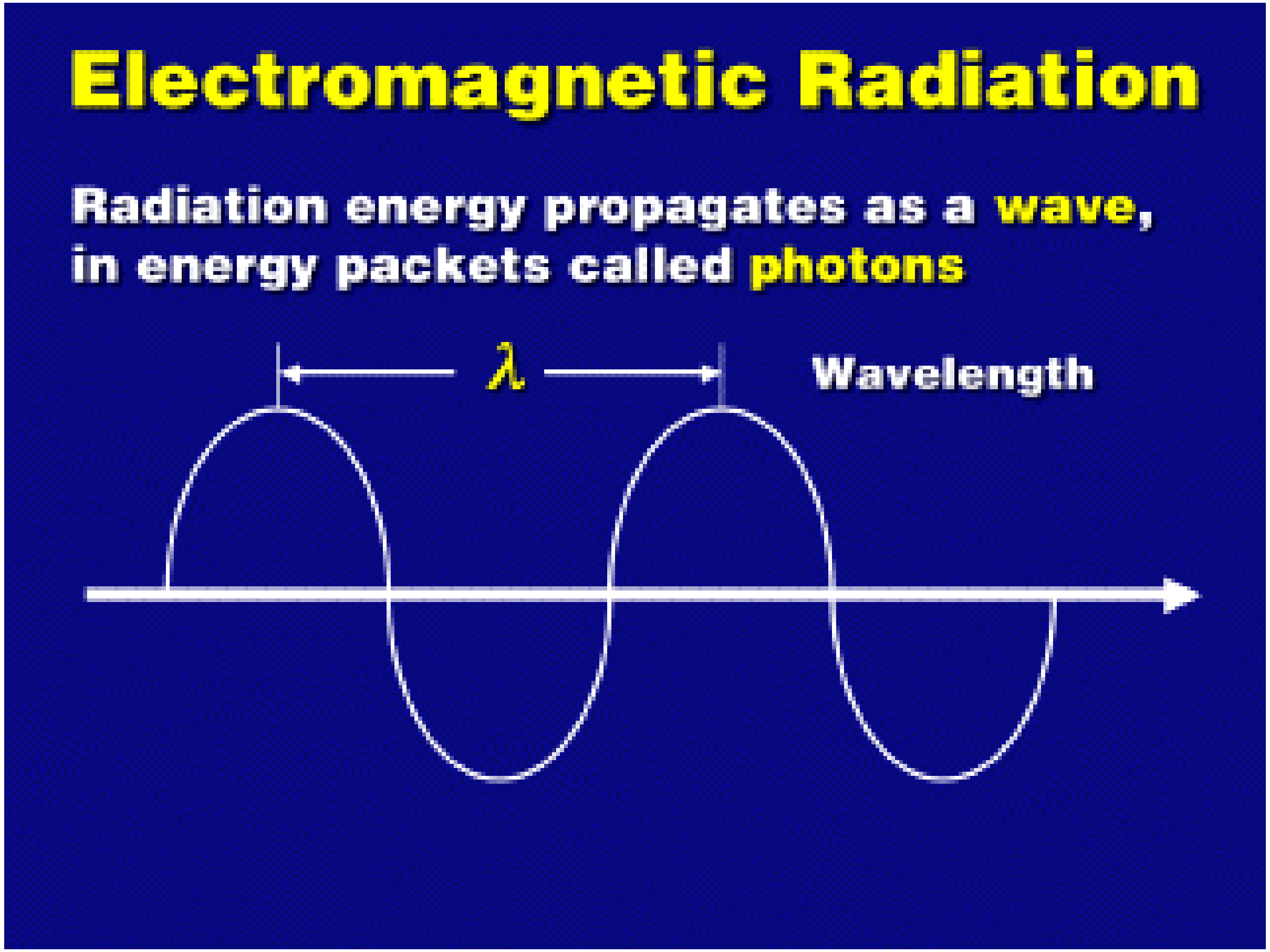


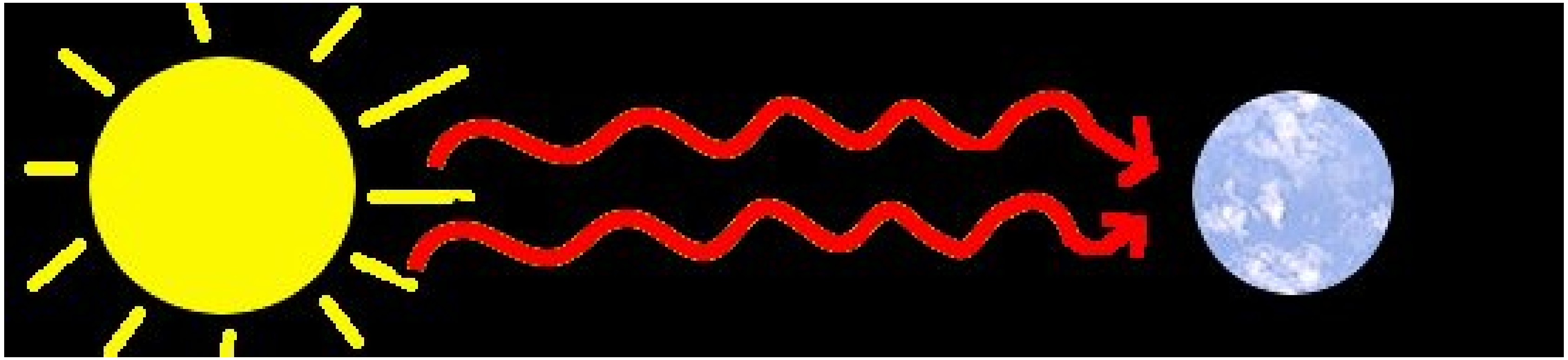






- **Radiation:** energy propagating through space in the form of electromagnetic radiation
  - → Moves as fast as speed of light
  - → Can travel through a vacuum


Electromagnetic Radiation
Radiation energy propagates as a wave,
in energy packets called photons
λ
Wavelength



Radiation allows heat to be transfered through wave energy. These waves are called ***Electromagnetic Waves***, because the energy travels in a combination of electric and magnetic waves. This energy is released when these waves are absorbed by an object. For example, energy traveling from the sun to your skin, you can feel your skin getting warmer as energy is absorbed.

Shorter wavelengths carry more energy than longer wavelengths.

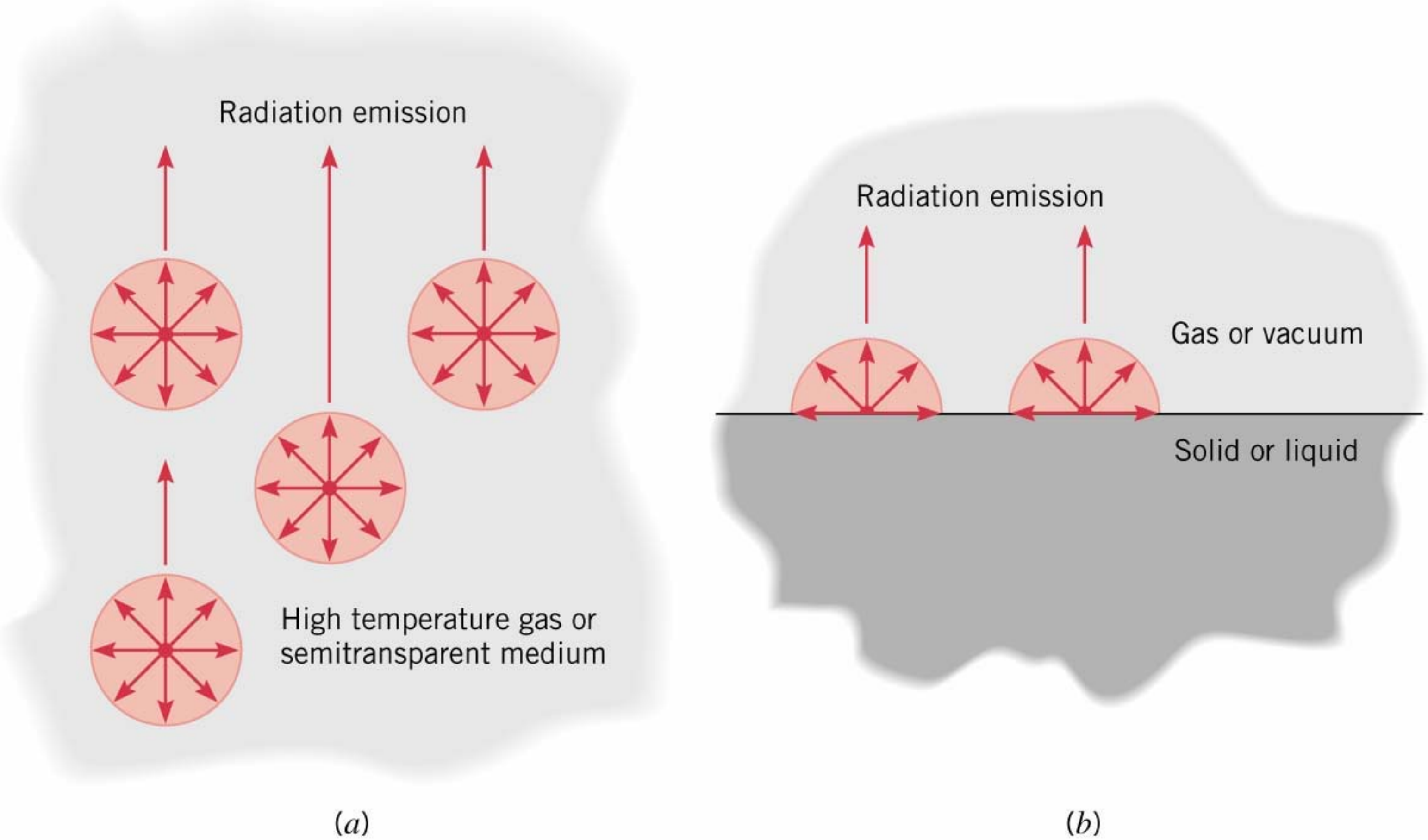


FIGURE 12.2 The emission process. (*a*) As a volumetric phenomenon. (*b*) Surface phenomenon.

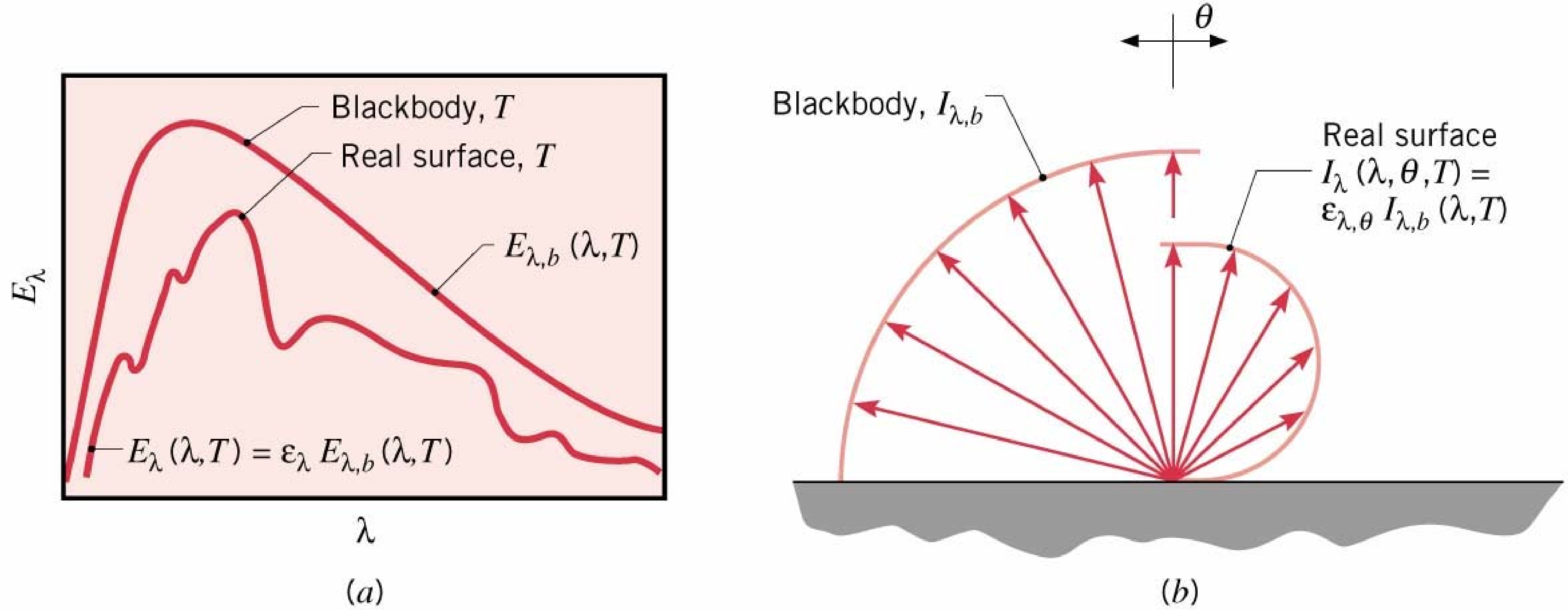
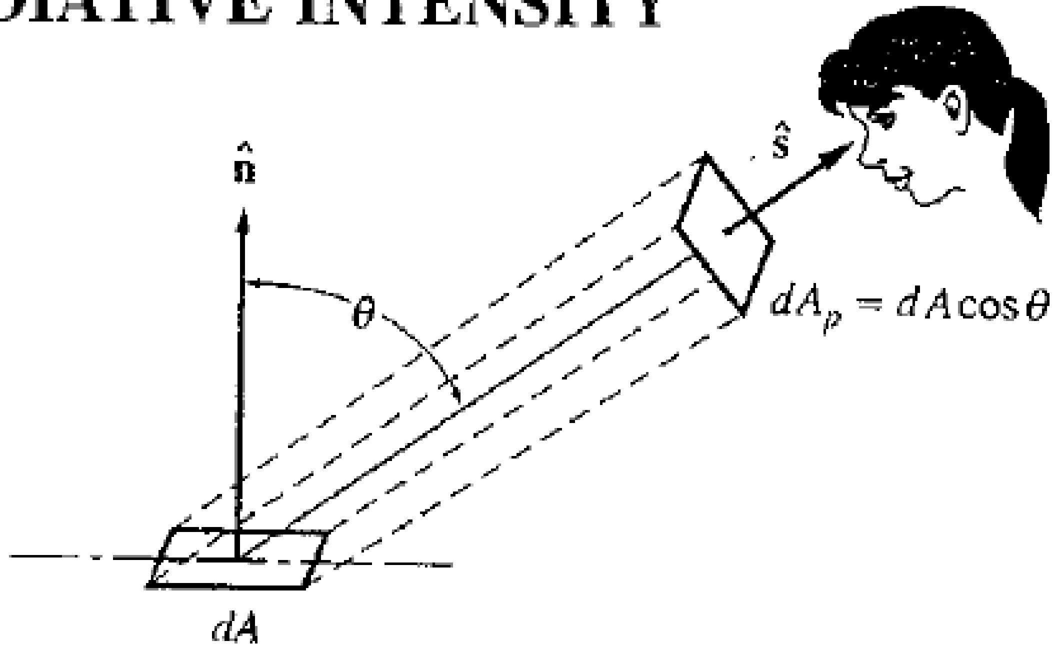


FIGURE 12.16 **Comparison of blackbody and real surface emission. (a) Spectral distribution. (b) Directional distribution.**

RADIATIVE INTENSITY



spectral intensity, $I_\lambda \equiv$
radiative energy flow/time/area
normal to rays/solid angle/wavelength

total intensity, $I \equiv$
radiative energy flow/time/area normal to rays/solid angle

$$I(\mathbf{r}, \hat{\mathbf{s}}) = \int_0^\infty I_\lambda(\mathbf{r}, \hat{\mathbf{s}}, \lambda)\, d\lambda.$$

Radiation Intensity

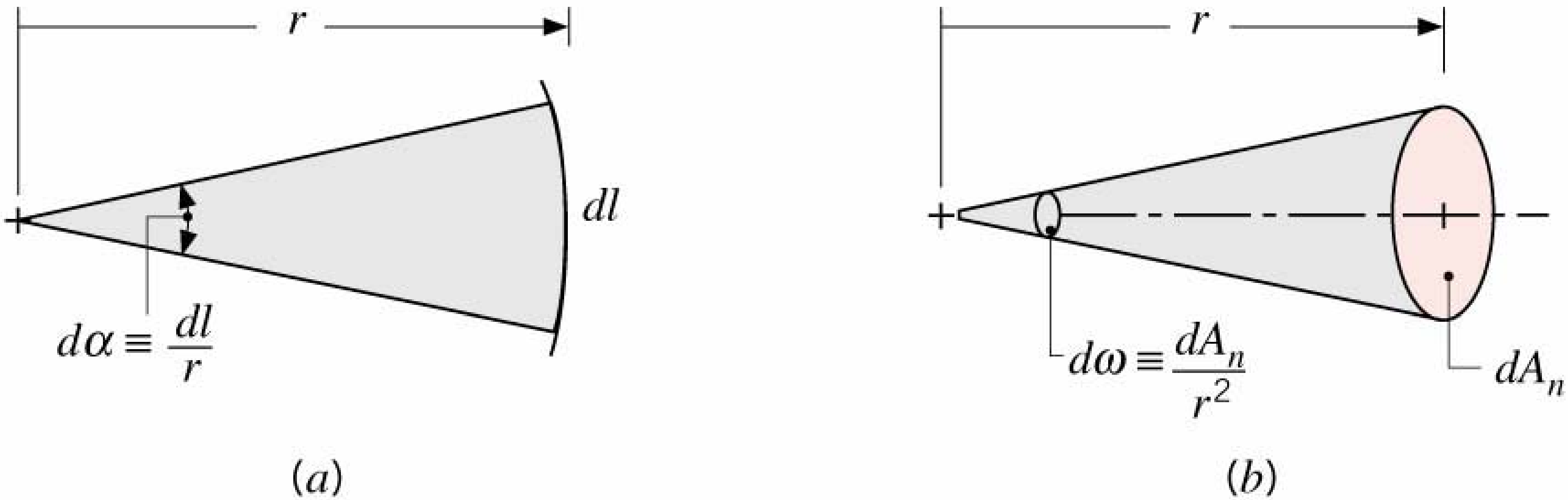


FIGURE 12.6 Definition of (*a*) plane and (*b*) solid angles.

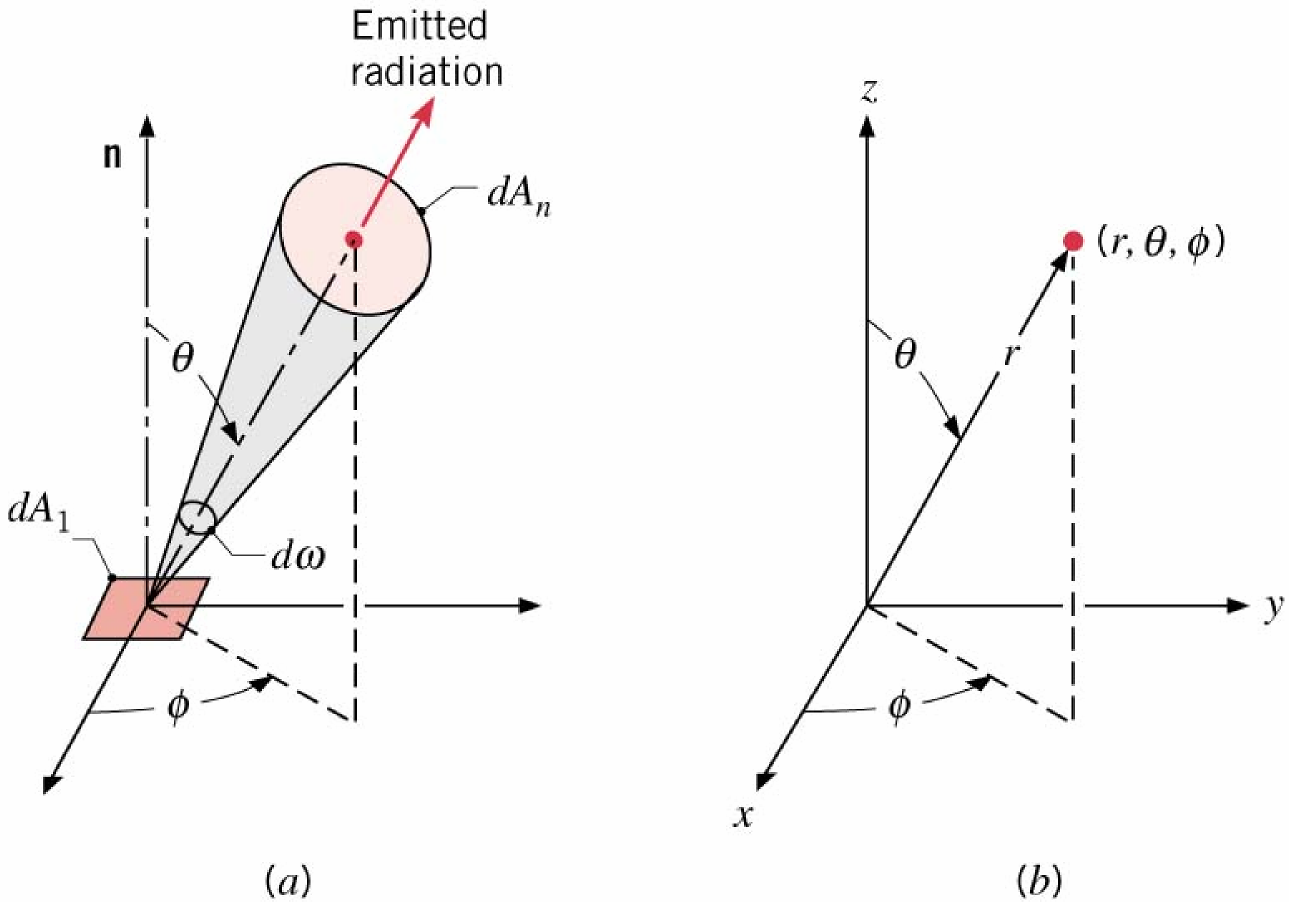


FIGURE 12.5 Directional nature of radiation. (a) Emission of radiation from a differential area dA_1 into a solid angle $d\omega$ subtended by dA_n at a point on dA_1. (b) The spherical coordinate system.

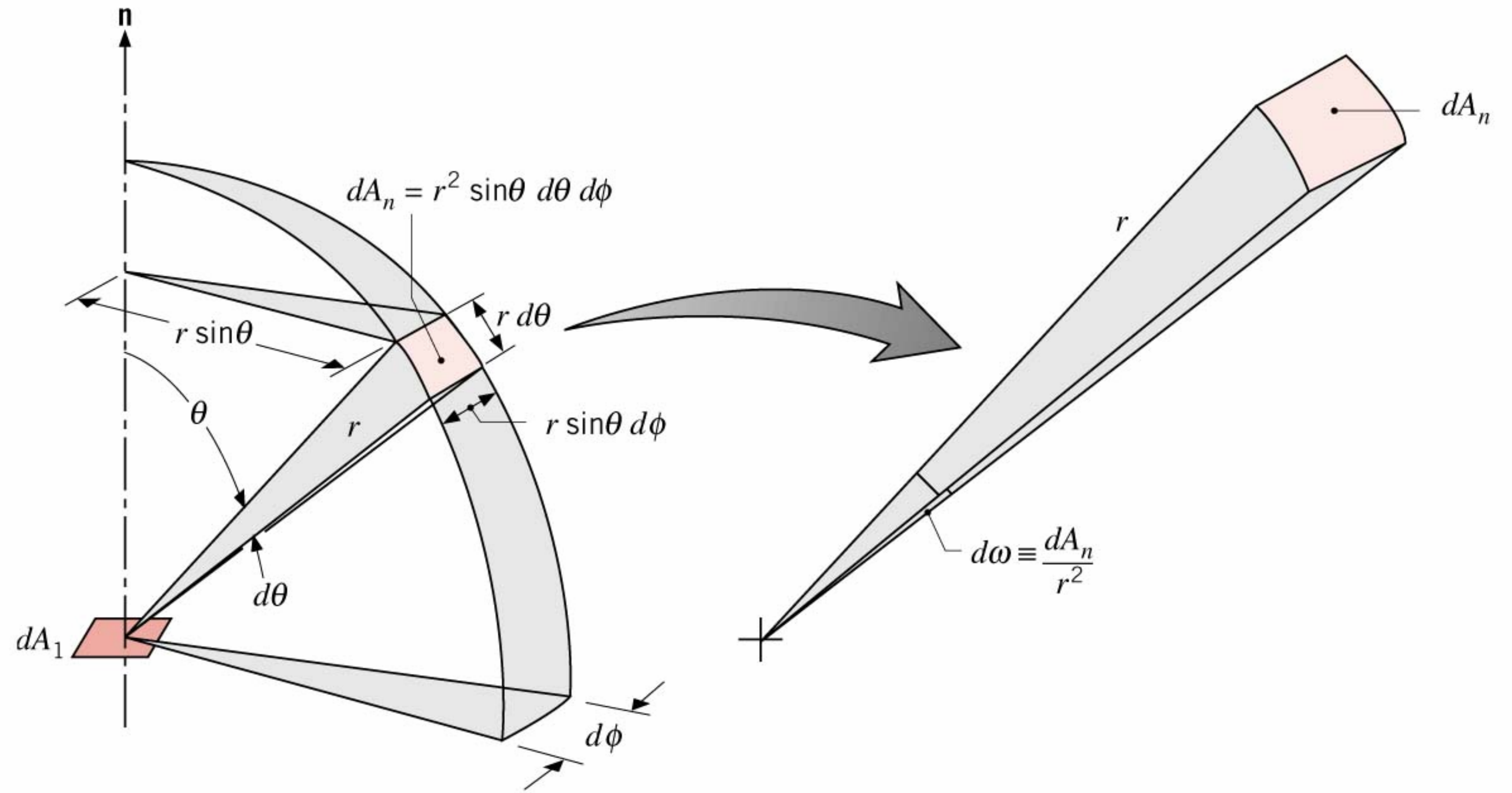


FIGURE 12.7 The solid angle subtended by dA_n at a point on dA_1 in the spherical coordinate system.

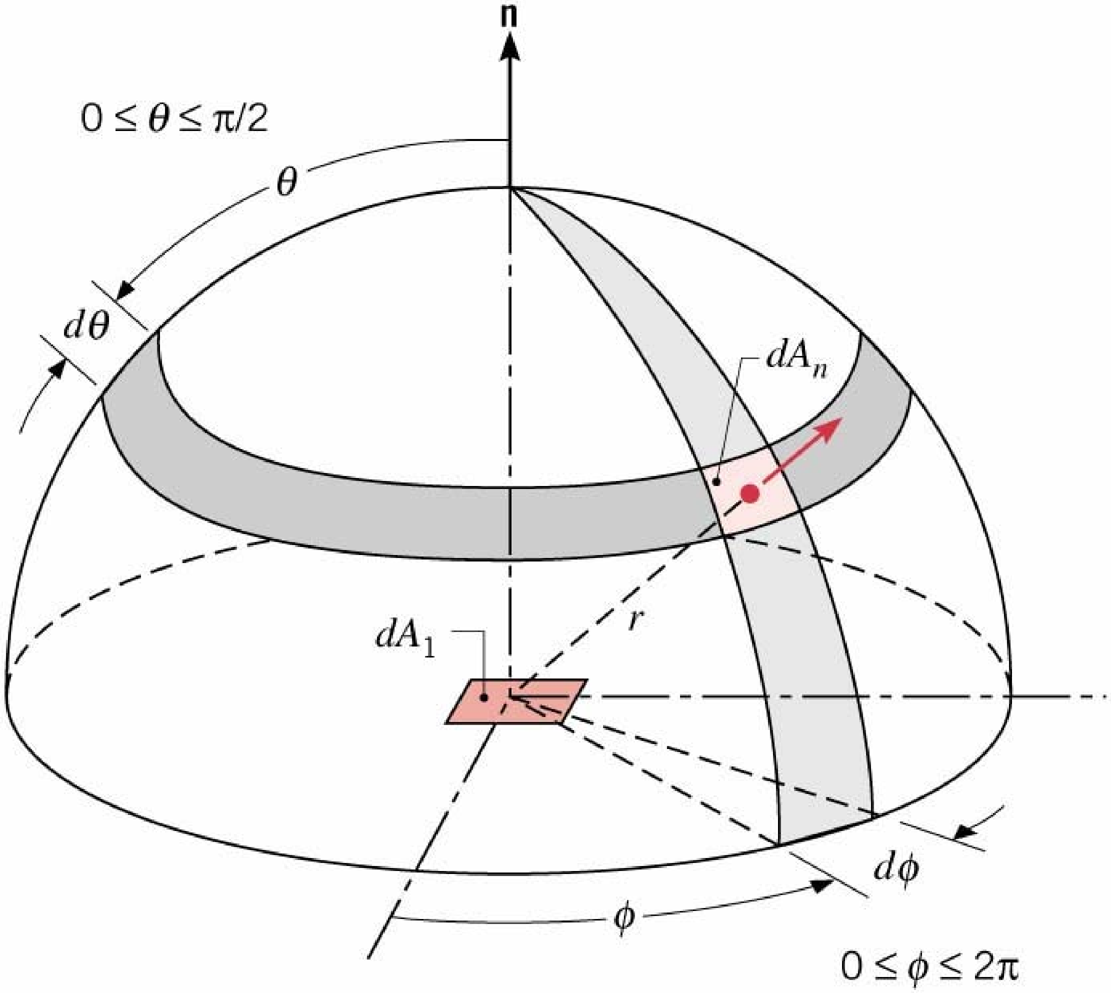
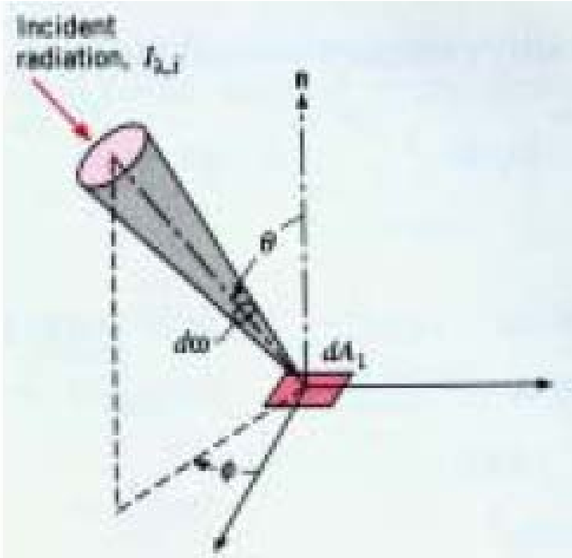


FIGURE 12.9
Emission from a differential element of area dA_1 into a hypothetical hemisphere centered at a point on dA_1.

Irradiation



Spectral irradiation, $G_\lambda(\lambda)$: $(\mathrm{W/m^2 \cdot \mu m})$

$$G_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

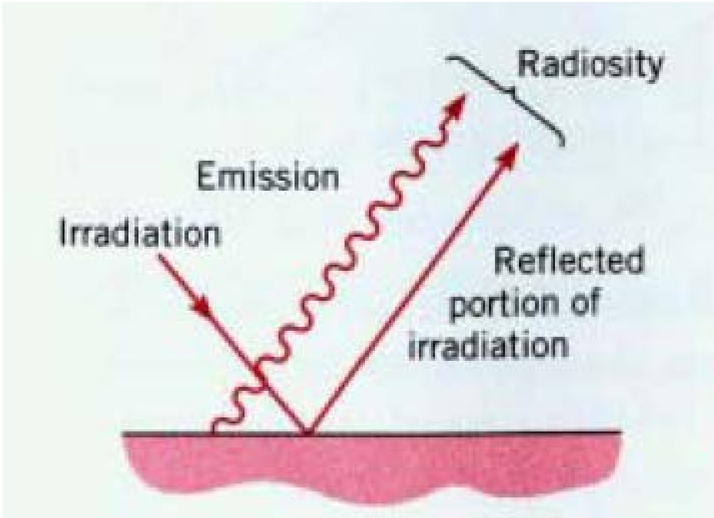
Total irradiation, G:

$$G = \int_0^\infty G_\lambda(\lambda) d\lambda$$

Radiosity

Radiosity is all the radiant energy leaving a surface, including emitted and reflected energy. (Fig. 12.11)

Surface radiosity.



Spectral radiosity, $J_\lambda(\lambda)$: $(\mathrm{W/m^2 \cdot \mu m})$

$$J_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

Total radiosity, J:

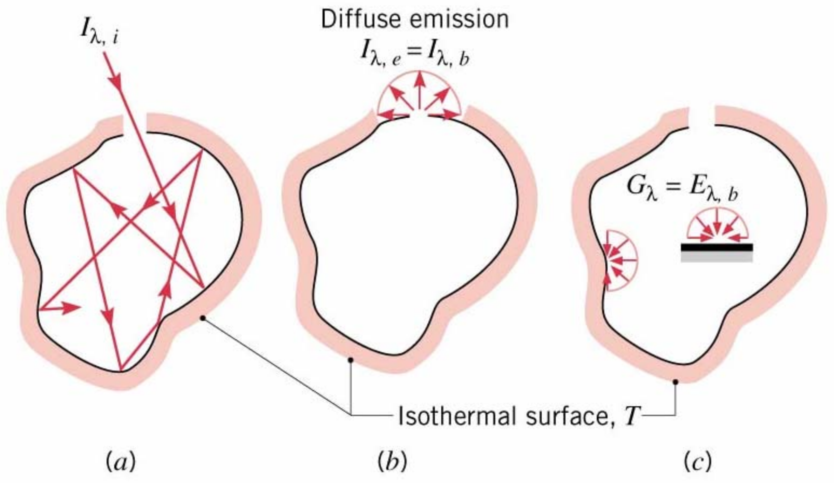
$$J = \int_0^\infty J_\lambda(\lambda) d\lambda$$

Blackbody Radiation

A ***blackbody*** is an ideal surface having the following properties:

1. A blackbody absorbs *all* incident radiation, regardless of wavelength and direction.
2. For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody.
3. Although the radiation emitted by a blackbody is a function of wavelength and temper**Blackbody Radiation**ent of direction. That is, the blackbody is a *diffuse* emitter.

No surface has precisely the properties of a blackbody. A cavity is a closest approximation.



2.3 Blackbody radiation

A blackbody has the following characteristics:

- it is the physically ideal perfect radiator and perfect absorber, i.e.:
- all incident radiation is completely absorbed (hence "black");
- at all wavelengths and in all directions maximum possible emission occurs (radiance is independent of direction i.e. isotropic radiation).

A hypothetic body that completely absorbs all wavelengths of thermal radiation incident on it. Such bodies do not reflect light, and therefore appear black if their temperatures are low enough so as not to be self-luminous. All blackbodies heated to a given temperature emit thermal radiation with the same spectrum, as required by arguments of classical physics involving thermal equilibrium. However, the distribution of blackbody radiation as a function of wavelength, known as the Planck law, cannot be predicted using classical physics. This fact was the first motivating force behind the development of quantum mechanics.

Introduction to Blackbody radiation
Wien Distribution
Wien Displacement Law
Stefan-Boltzmann Law

The Planck Distribution

Planck Law

$$\boxed{I_{\lambda,b}(\lambda,T)=\frac{2hc_0^2}{\lambda^5[\exp(hc_0/\lambda kT)-1]}}$$

Since a blackbody is a diffuse emitter, it follows that

$$\boxed{E_{\lambda,b}(\lambda,T)=\pi I_{\lambda,b}(\lambda,T)=\frac{C_1}{\lambda^5[exp(C_2/\lambda T)-1]}}$$

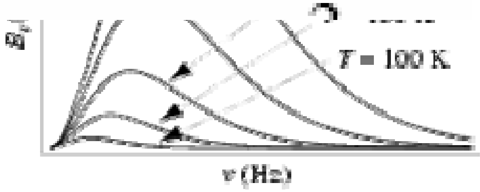
where h is Planck's constant, c is the speed of light, and k is Boltzmann's constant (Rybicki and Lightman 1979, p. 21).

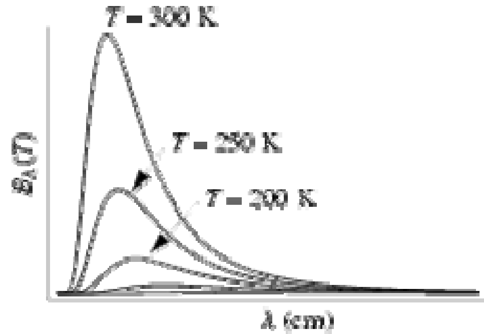
$$h = 6.6262\times10^{-27}\ \text{erg s} = 6.6262\times10^{-34}\ \text{J s}$$

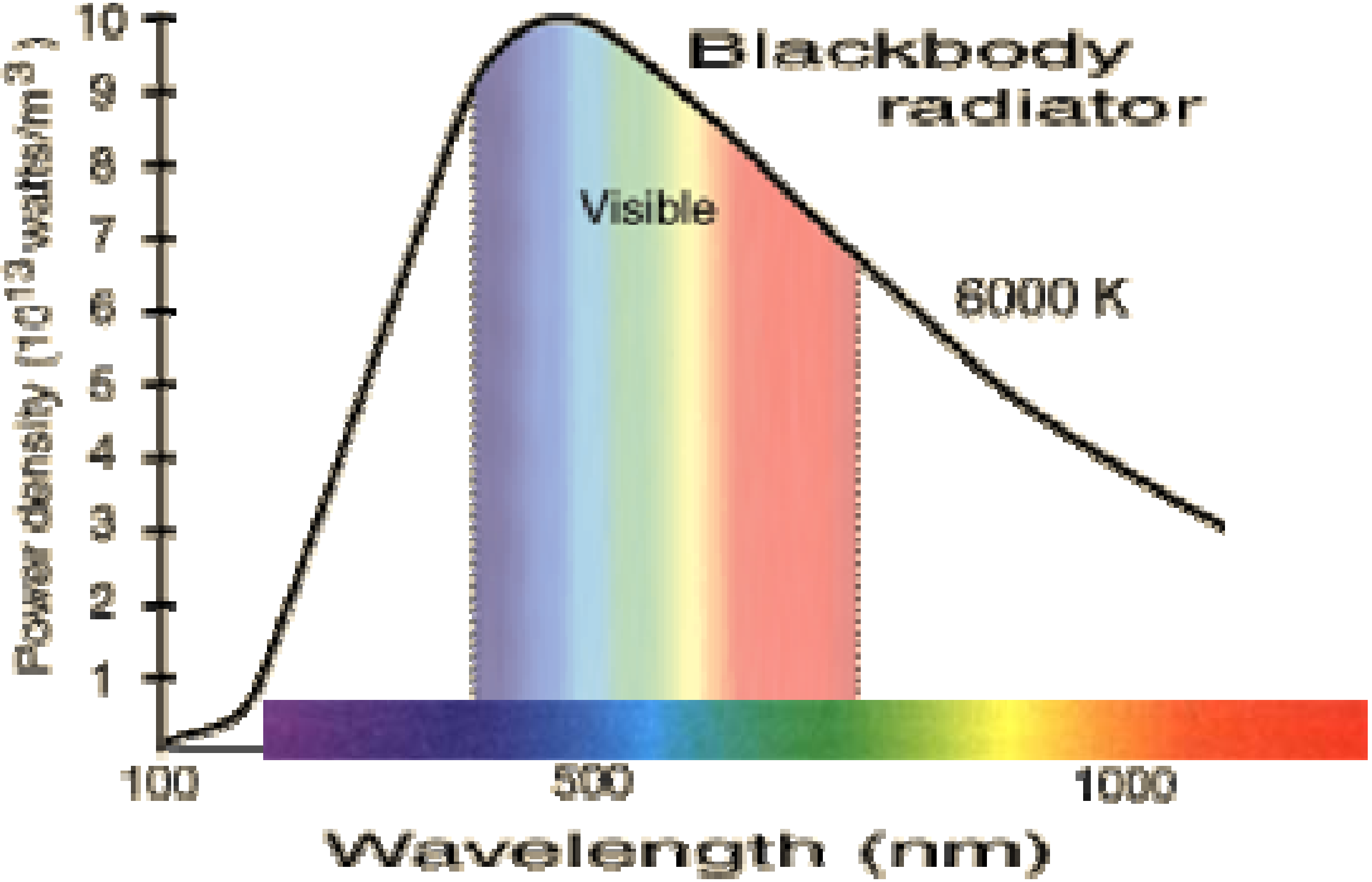
$$c \equiv 2.99792458\times10^{8}\ \text{m s}^{-1} = 2.99792458\times10^{10}\ \text{cm s}^{-1}$$

$$k = 1.3807\times10^{-16}\ \text{erg K}^{-1} = 1.3807\times10^{-23}\ \text{J K}^{-1}.$$

The Planck Distribution





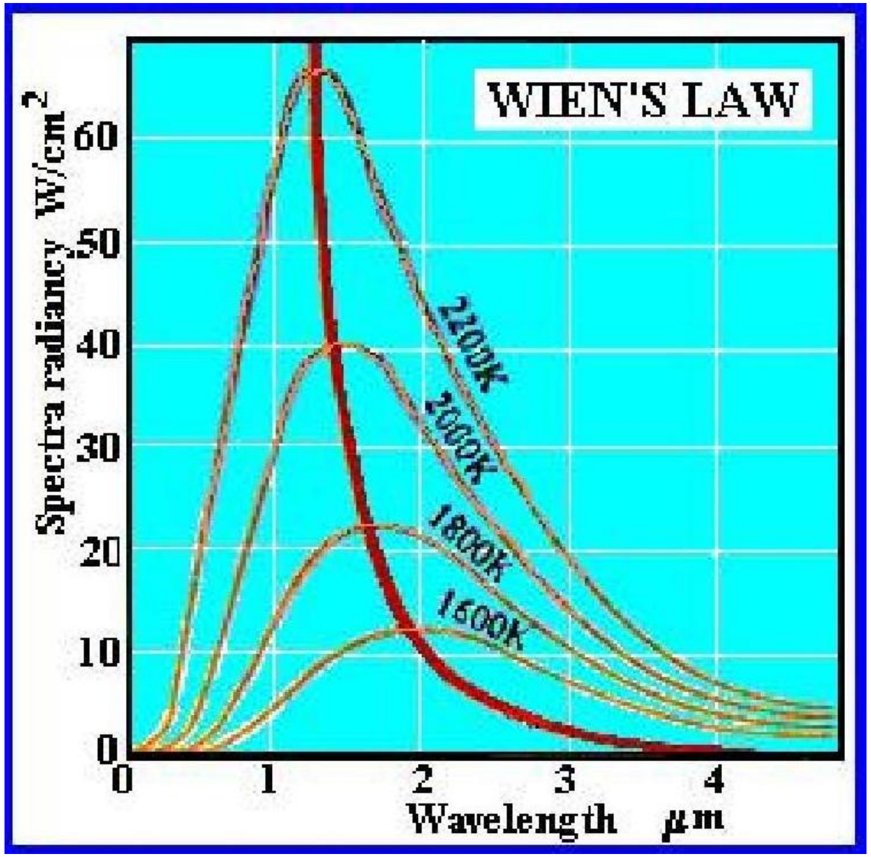


Wien's Displacement Law

Differentiating (12.26) w.r.t. λ to obtain

$$\lambda_{\max} T = C_3$$

This relation locates $\lambda_{\max}$ of blackbody emission at a certain T.



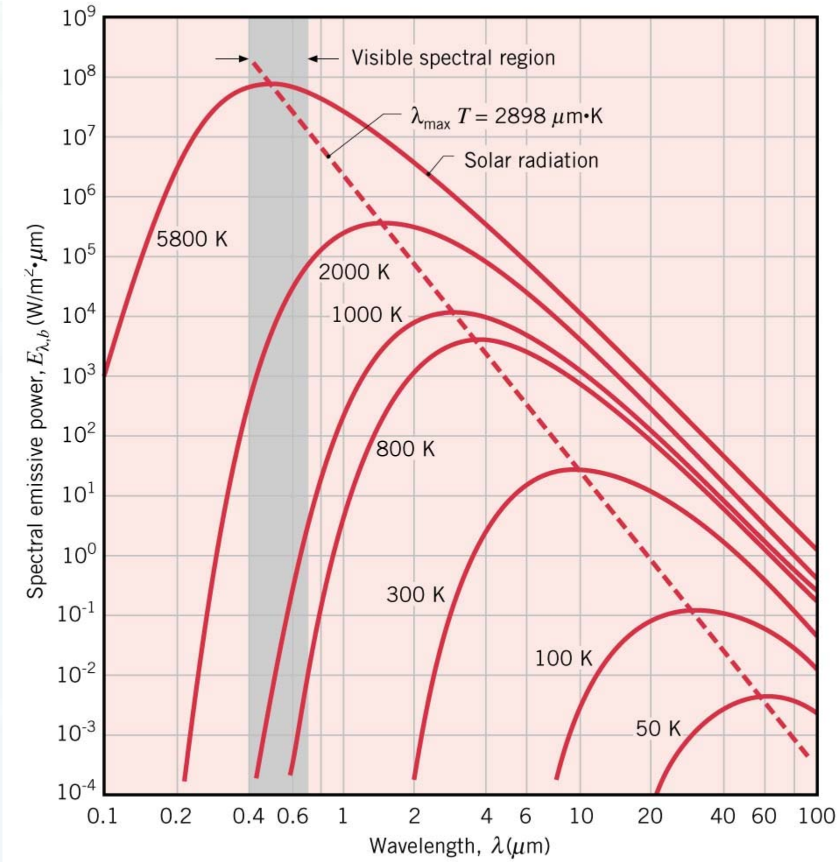


FIGURE 12.13 Spectral blackbody emissive power.

The Stefan-Boltzmann Law

The total emissive power of a blackbody:

$$\boxed{E_b(T) = \int_0^\infty \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} d\lambda = \sigma T^4}$$

where σ is the Stefan-Boltzmann constant.

Since a blackbody is diffuse, $I_b = E_b/\pi$

Total Blackbody Emissive Power

The total emissive power of a blackbody may be determined from equations (1.11) and (1.13) as

$$E_b(T) = \int_0^\infty E_{b\lambda}(T,\lambda)d\lambda = C_1 n^2 T^4 \int_0^\infty \frac{d(n\lambda T)}{(n\lambda T)^5\left[e^{C_2/(n\lambda T)} - 1\right]}$$
$$= \left[\frac{C_1}{C_2^4}\int_0^\infty \frac{\xi^3 d\xi}{e^\xi - 1}\right] n^2 T^4, \quad (n = \text{const}). \tag{1.19}$$

The integral in this expression may be evaluated by complex integration, and is tabulated in many good integral tables:

$$E_b(T) = n^2\sigma T^4, \quad \sigma = \frac{\pi^4 C_1}{15 C_2^4} = 5.670 \times 10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2\,K^4}}, \tag{1.20}$$

where σ is known as the *Stefan-Boltzmann constant.**

Stefan-Boltzmann Law

q = σ T4 A (1)

where

q = heat transfer per unit time (W)

σ = 5.6703 10-8 (W/m2K4) - ***The Stefan-Boltzmann Constant***

T = absolute temperature Kelvin (K)

A = area of the emitting body (m2)

The Stefan-Boltzmann Constant in Imperial Units

σ = 5.6703 10-8 (W/m2K4)

*= 0.1714 10-8 (Btu/(h ft2 oR4**))***

= 0.119 10-10 (Btu/(h in2 oR4))

Band Emission

It is often necessary to calculate the emissive power contained within a finite wavelength band, say between λ_1 and λ_2. Then

$$\int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda = \frac{C_1}{C_2^4} \int_{C_2/n\lambda_2 T}^{C_2/n\lambda_1 T} \frac{\xi^3 d\xi}{e^\xi - 1} n^2 T^4. \tag{1.21}$$

It is not possible to evaluate the integral in equation (1.21) analytically. Therefore, it is customary to express equation (1.21) in terms of the *fraction of blackbody emissive power* contained between 0 and $n\lambda T$,

$$f(n\lambda T) = \frac{\int_0^\lambda E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} = \int_0^{n\lambda T} \left(\frac{E_{b\lambda}}{n^3 \sigma T^5}\right) d(n\lambda T) = \frac{15}{\pi^4} \int_{C_2/n\lambda T}^{\infty} \frac{\xi^3 d\xi}{e^\xi - 1}, \tag{1.22}$$

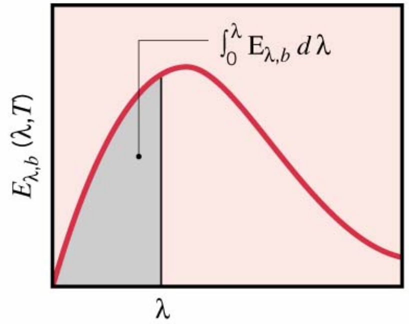
so that

$$\int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda = [f(n\lambda_2 T) - f(n\lambda_1 T)]\, n^2 \sigma T^4. \tag{1.23}$$

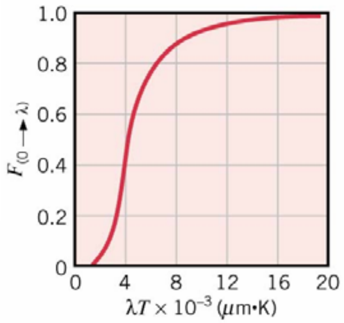
Band Emission

The fraction of the total emission from a blackbody that is in a certain wavelength interval of *band*:

$$F_{(0\to\lambda)} \equiv \frac{\int_0^{\lambda} E_{\lambda,b} d\lambda}{\int_0^{\infty} E_{\lambda,b} d\lambda} = \frac{\int_0^{\lambda} E_{\lambda,b} d\lambda}{\sigma T^4}$$



$$= \int_0^{\lambda T} \frac{E_{\lambda,b}}{\sigma T^5} d(\lambda T) = f(\lambda T)$$

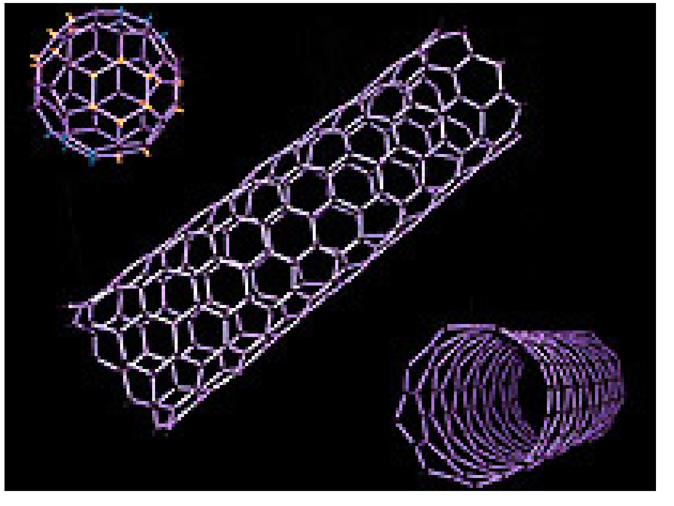


$$F_{(\lambda_1\to\lambda_2)} = \frac{\int_0^{\lambda_2} E_{\lambda,b} d\lambda - \int_0^{\lambda_1} E_{\lambda,b} d\lambda}{\sigma T^4} = F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}$$

吸收99.9%光 美研製人造最黑物質

The "darkest ever" substance known to science has been made in a US laboratory.

怎麼樣才能算是黑呢？這個物質必須要能吸收所有波長、所有角度的可見光波，同時一點也不能反射出來。目前最接近這個目標的，是美國紐約 Rensselaer Polytechnic Institute 研發的一種一顆原子厚的圓筒狀碳奈米管。將許多這種碳奈米管排在一起，就形成了一種吸光力特別強，又不反光的材質，對製造更有效率的太陽能板有很大的助益，或更棒的，讓 Kuro 更黑。John Pendry 教授說：「他們做出了科學界已知最黑的材料。



Carbon nanotubes are a basic building block of nanotechnology

Surface Emission

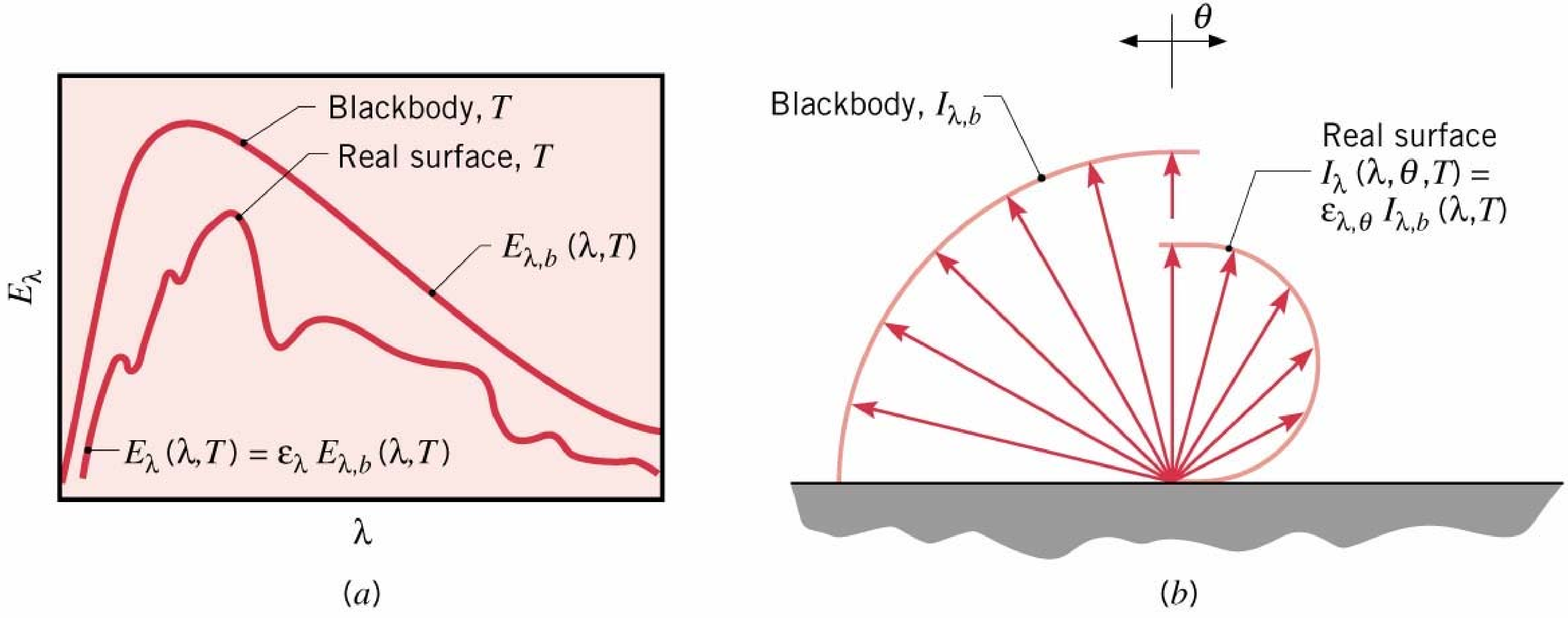


FIGURE 12.16 Comparison of blackbody and real surface emission. (*a*) Spectral distribution. (*b*) Directional distribution.

Total, directional emissivity $\varepsilon_\theta(\theta, \phi, T)$:

$$\varepsilon_\theta(\theta, \phi, T) \equiv \frac{I_e(\theta, \phi, T)}{I_b(T)}$$

Spectral, hemispherical emissivity $\varepsilon_\lambda(\lambda, T)$:

$$\varepsilon_\lambda(\lambda, T) \equiv \frac{E_\lambda(\lambda, T)}{E_{\lambda,b}(\lambda, T)}$$

Total, hemispherical emissivity $\varepsilon(T)$:

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{\lambda,b}(\lambda, T) d\lambda}{E_b(T)}$$

Total, hemispherical emissivity $\varepsilon(T)$:

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{\lambda,b}(\lambda, T) d\lambda}{E_b(T)}$$

Usually, $\varepsilon \approx \varepsilon_n$; $\varepsilon_\lambda \approx \varepsilon_{\lambda,n}$

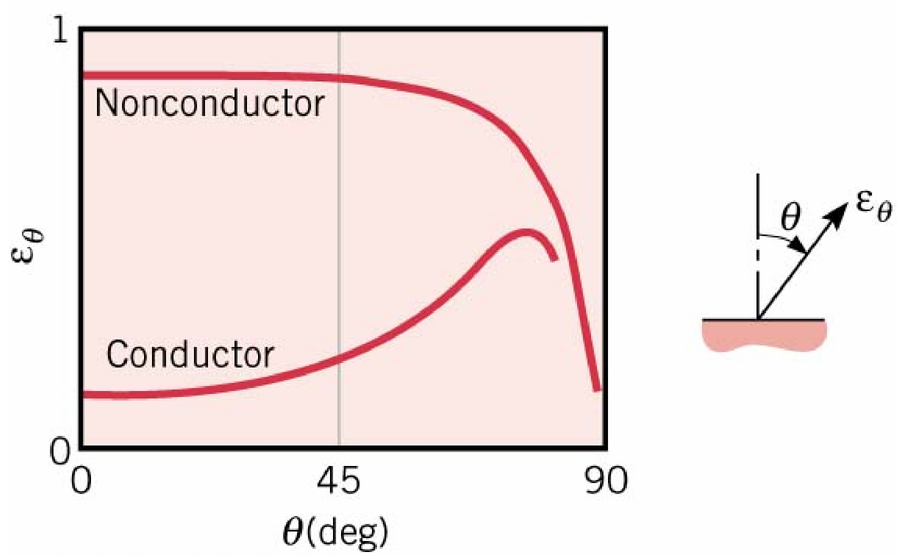


FIGURE 12.17
Representative directional distributions of the total, directional emissivity.

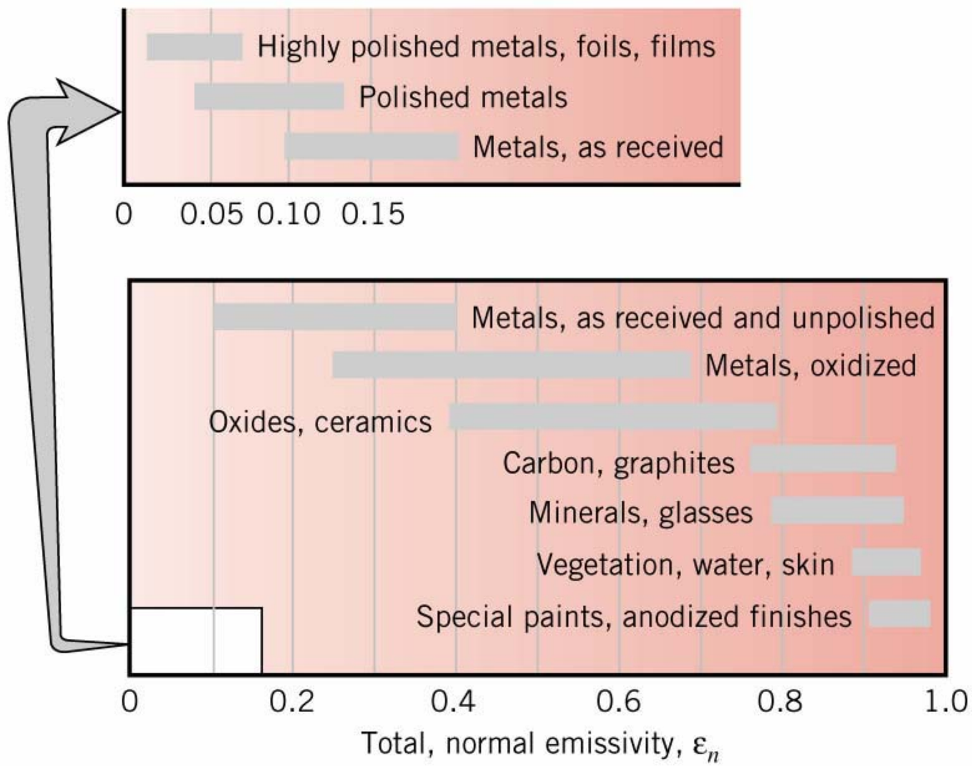


FIGURE 12.20 Representative values of the total, normal emissivity ε_n.

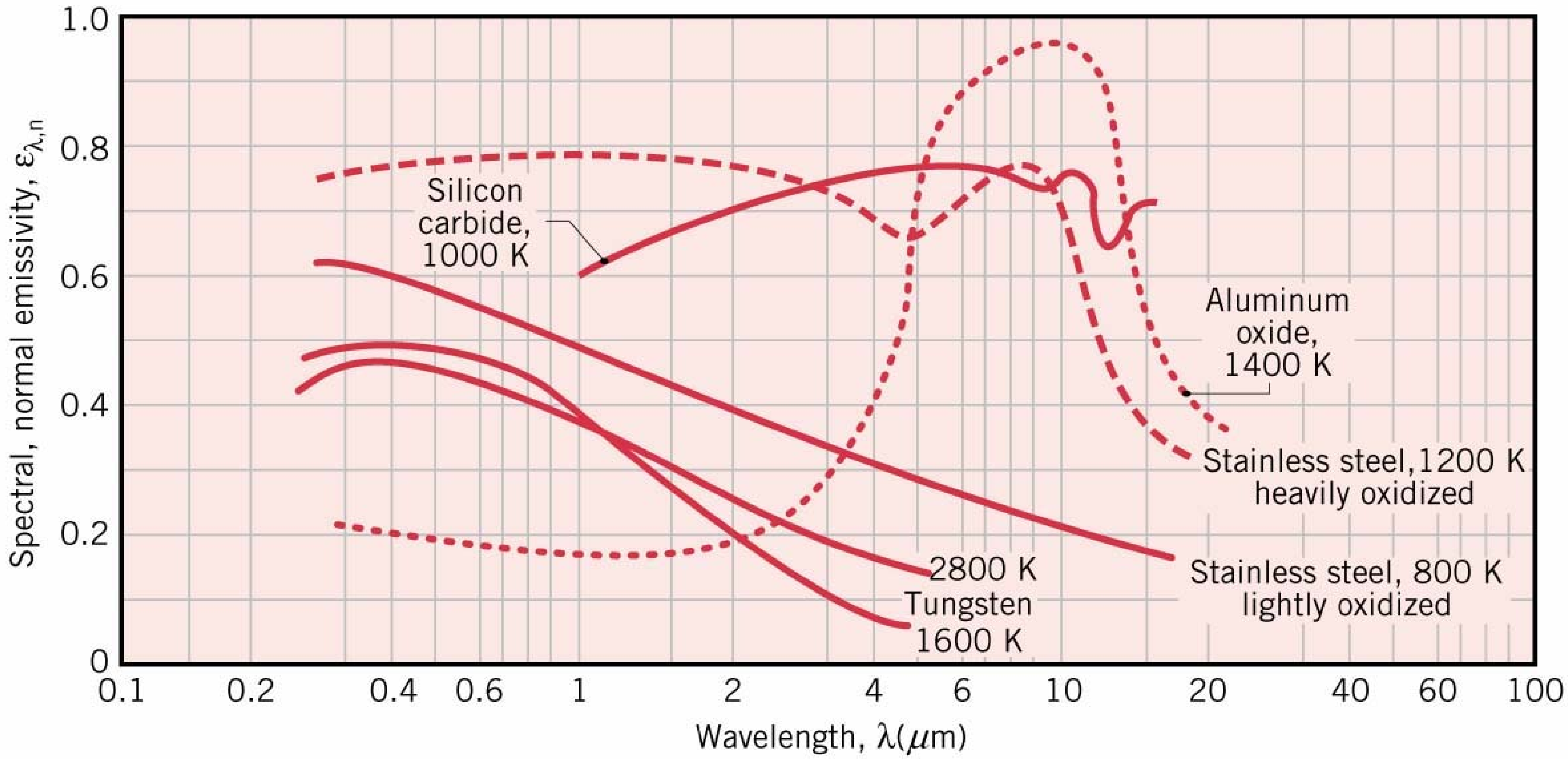


FIGURE 12.18 Spectral dependence of the spectral, normal emissivity $\varepsilon_{\lambda, n}$ of selected materials.

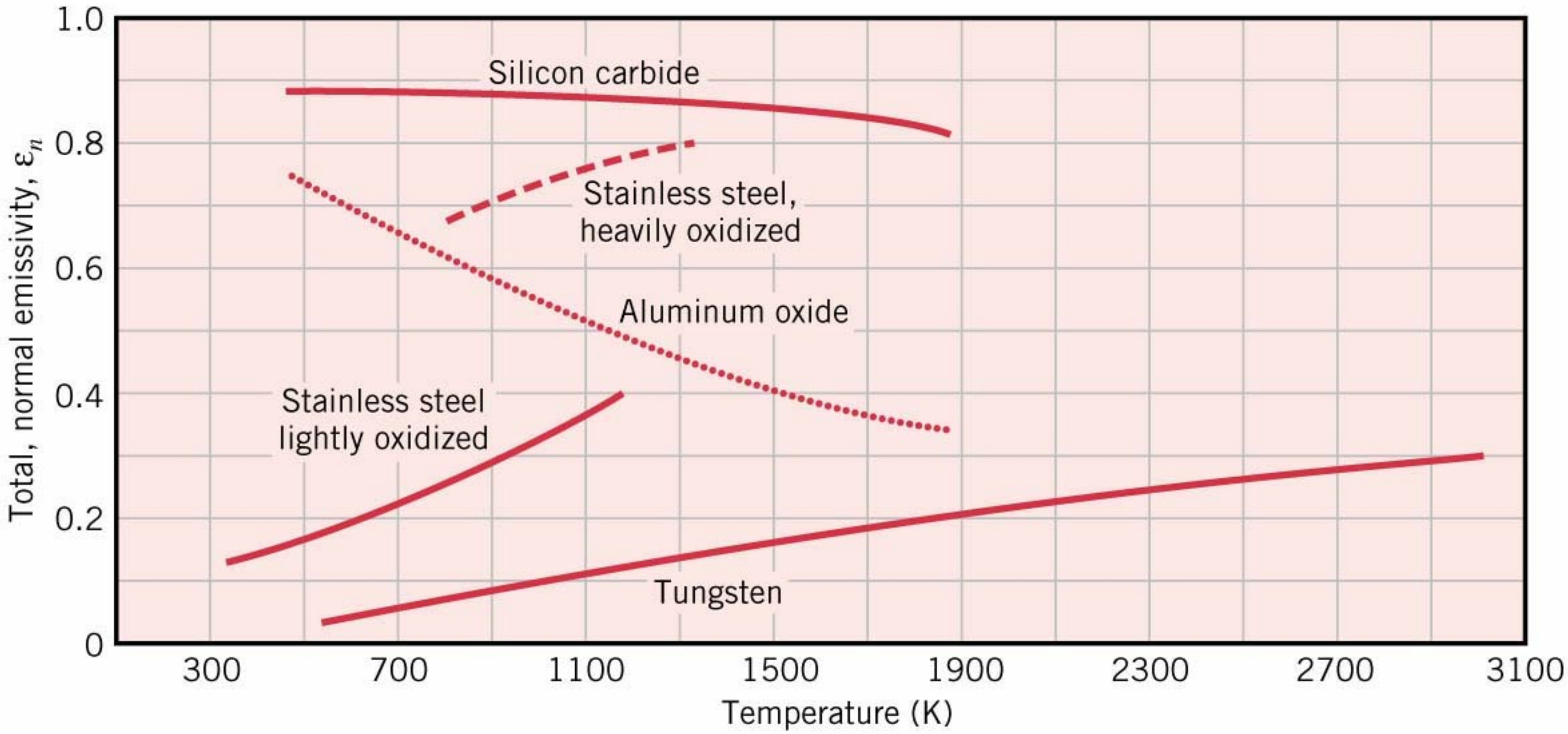


FIGURE 12.19 Temperature dependence of the total, normal emissivity ε_n of selected materials.

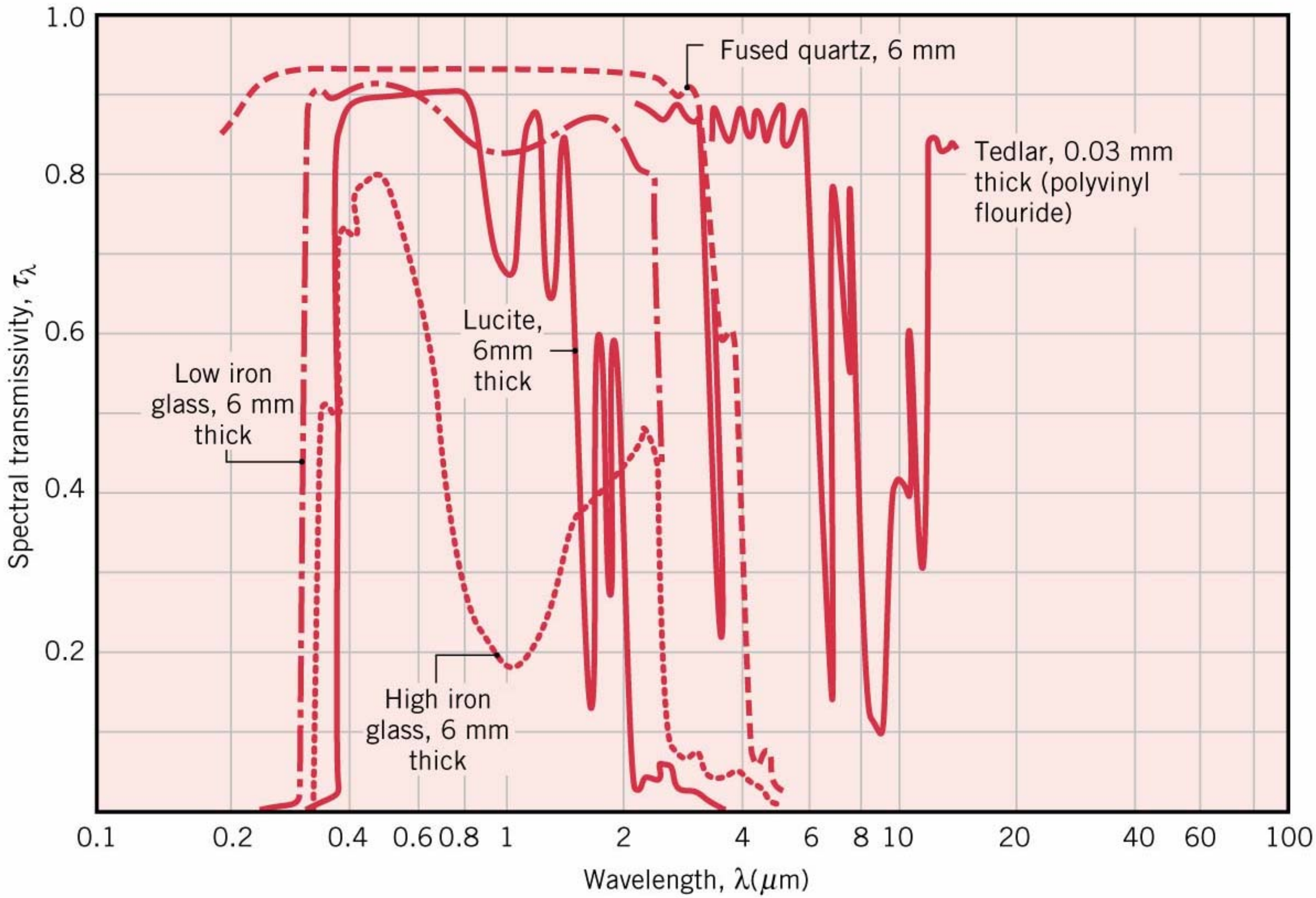


FIGURE 12.24 Spectral dependence of the spectral transmissivities τ_λ of selected semitransparent materials.

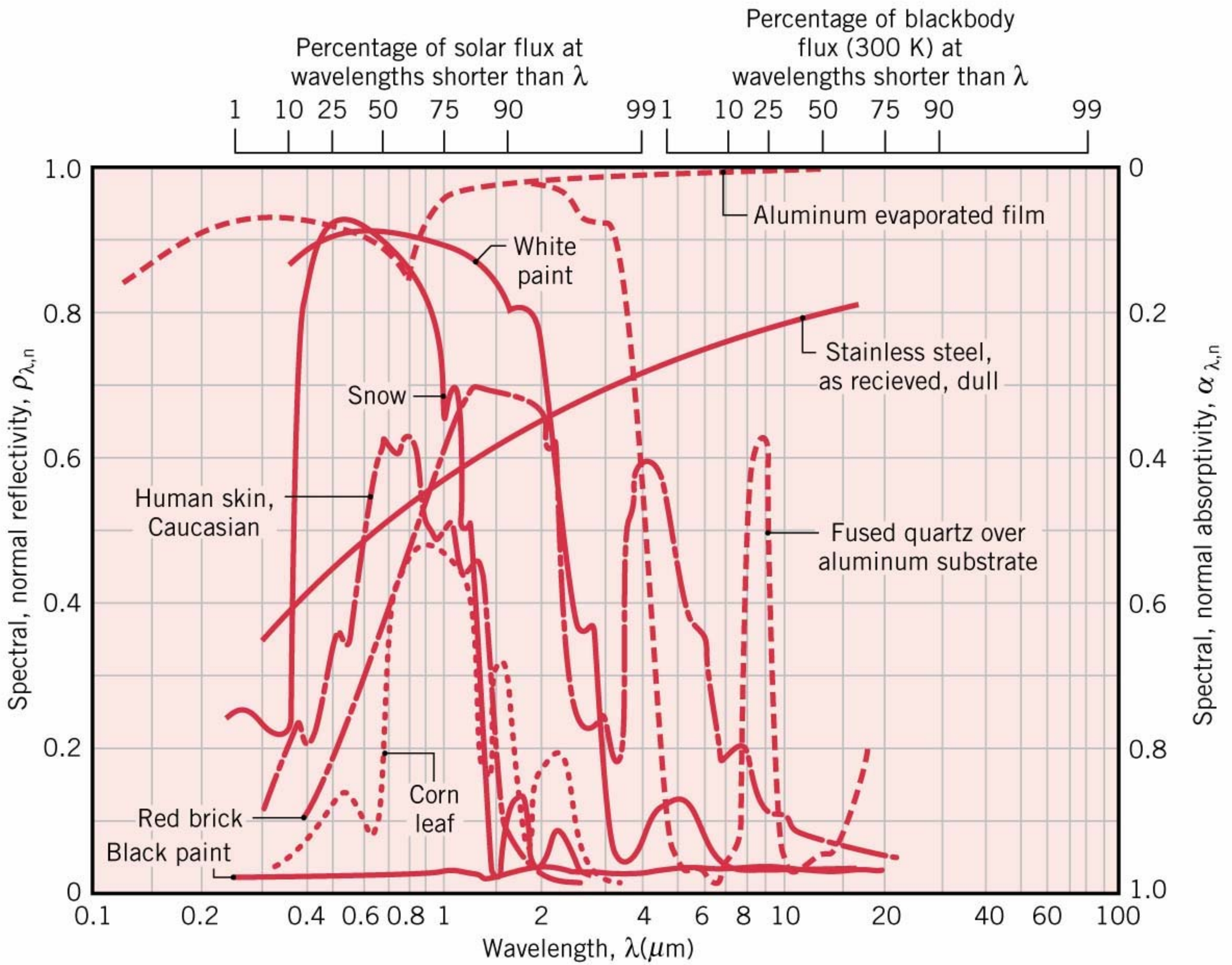
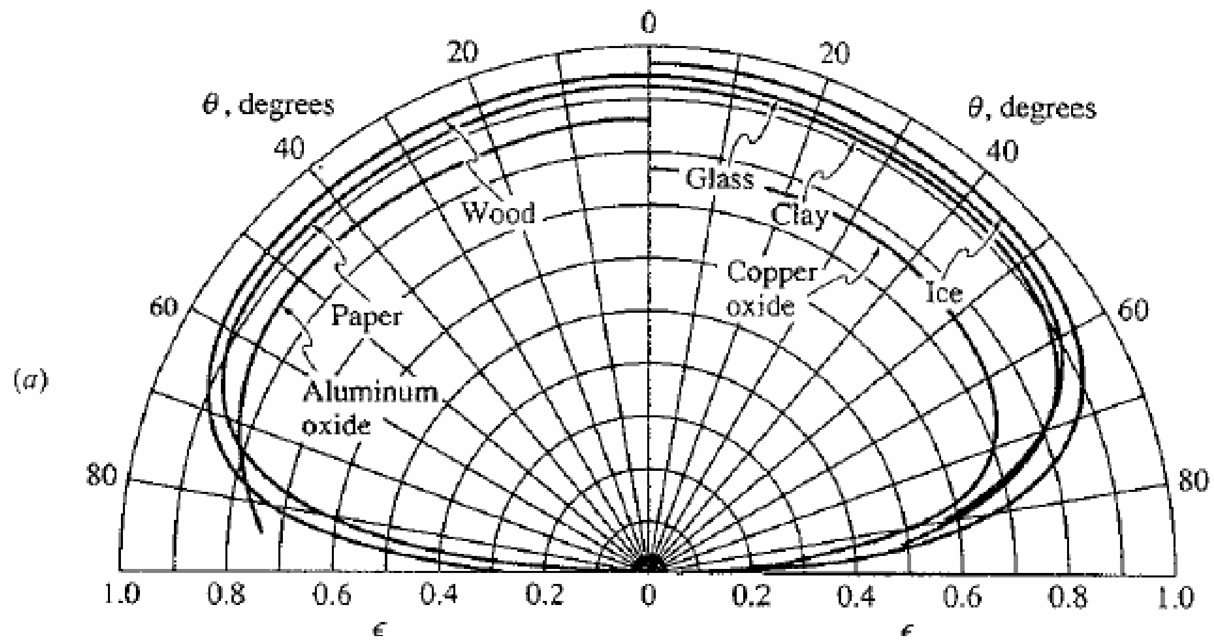
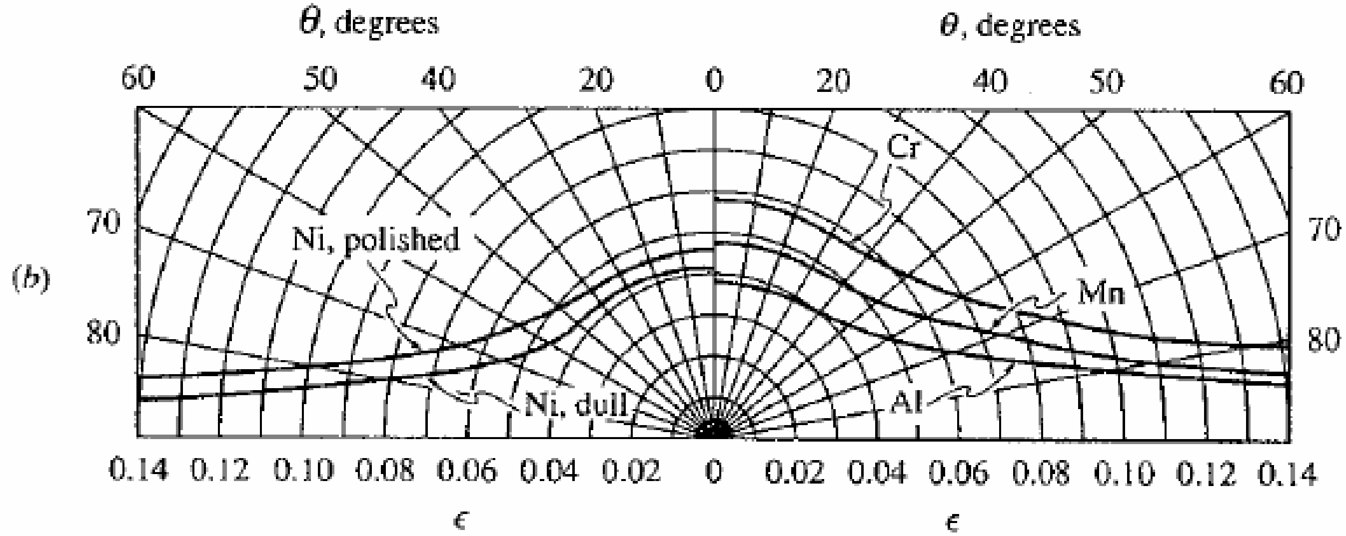


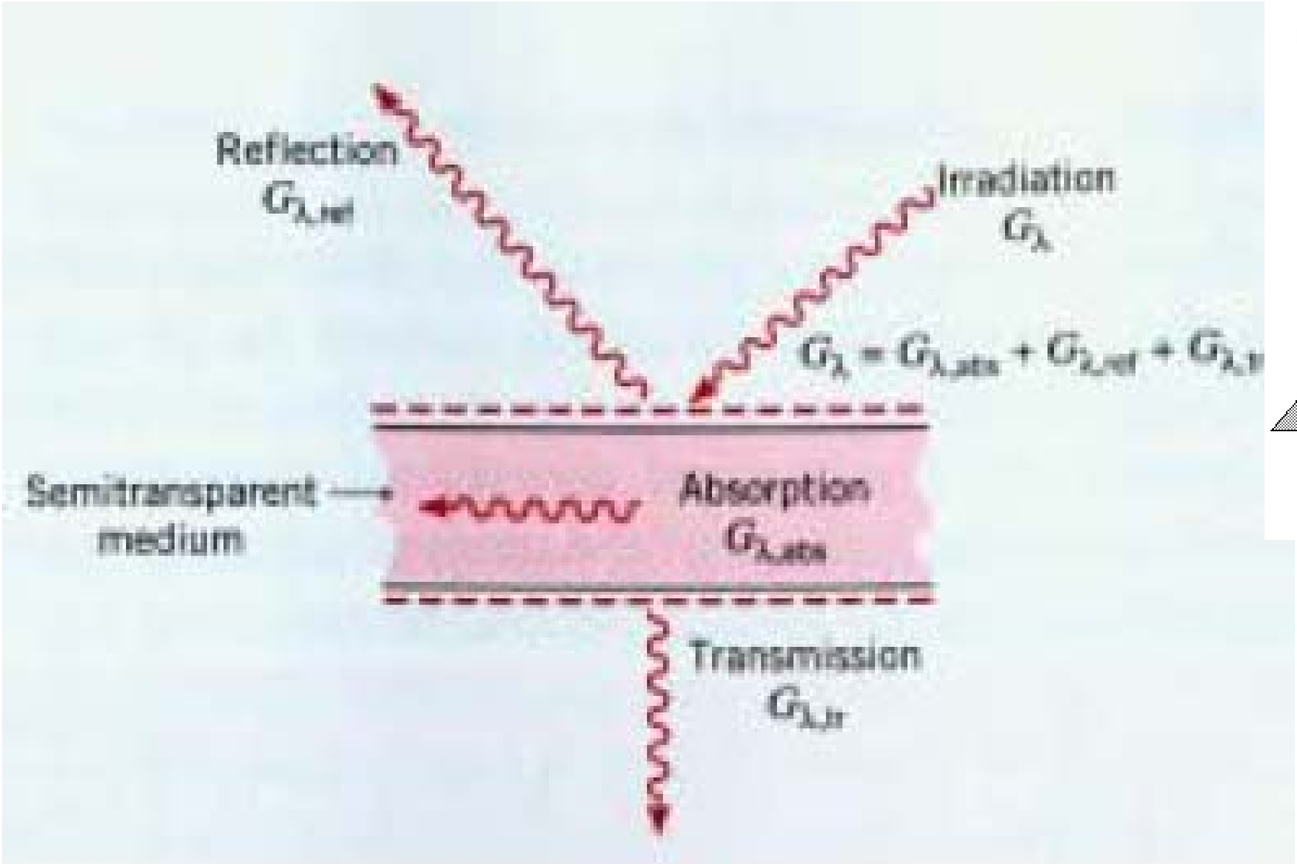
FIGURE 12.23 Spectral dependence of the spectral, normal absorptivity $\alpha_{\lambda,\, n}$ and reflectivity $\rho_{\lambda,\, n}$ of selected opaque materials.

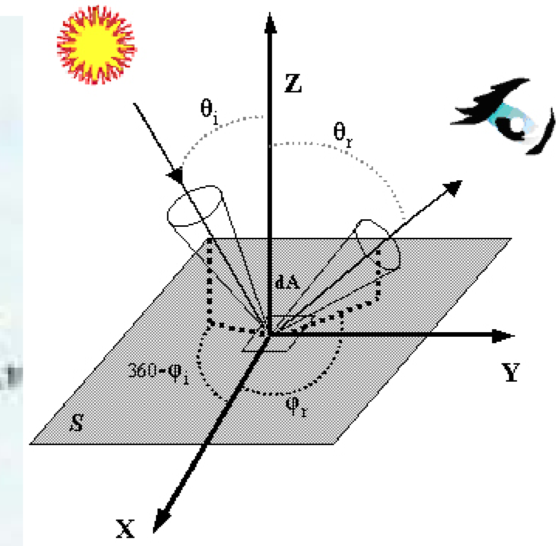


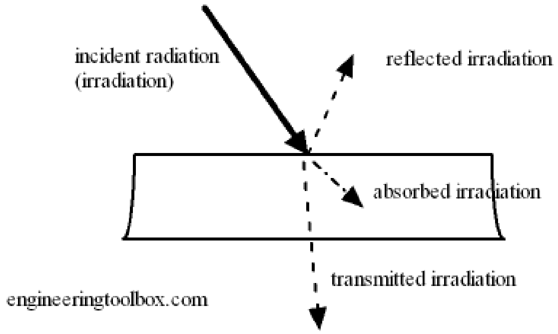


Directional variation of surface emissivities (*a*) for several nonmetals and (*b*) for several metals [9].

Surface Absorption, Reflection, and Transmission







$$G_\lambda = G_{\lambda,\text{ref}} + G_{\lambda,\text{abs}} + G_{\lambda,\text{tr}}$$

Spectral, directional absorptivity

$$\alpha_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,\mathrm{abs}}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

Spectral, hemispherical absorptivity

$$\alpha_{\lambda}(\lambda) \equiv \frac{G_{\lambda,\mathrm{abs}}(\lambda)}{G_{\lambda}(\lambda)}$$

Total, hemispherical absorptivity

$$\alpha \equiv \frac{G_{\mathrm{abs}}}{G}$$

Spectral, hemispherical transmissivity

$$\tau_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{tr}}(\lambda)}{G_\lambda(\lambda)}$$

Total transmissivity $\tau \equiv \dfrac{G_{\mathrm{tr}}}{G}$

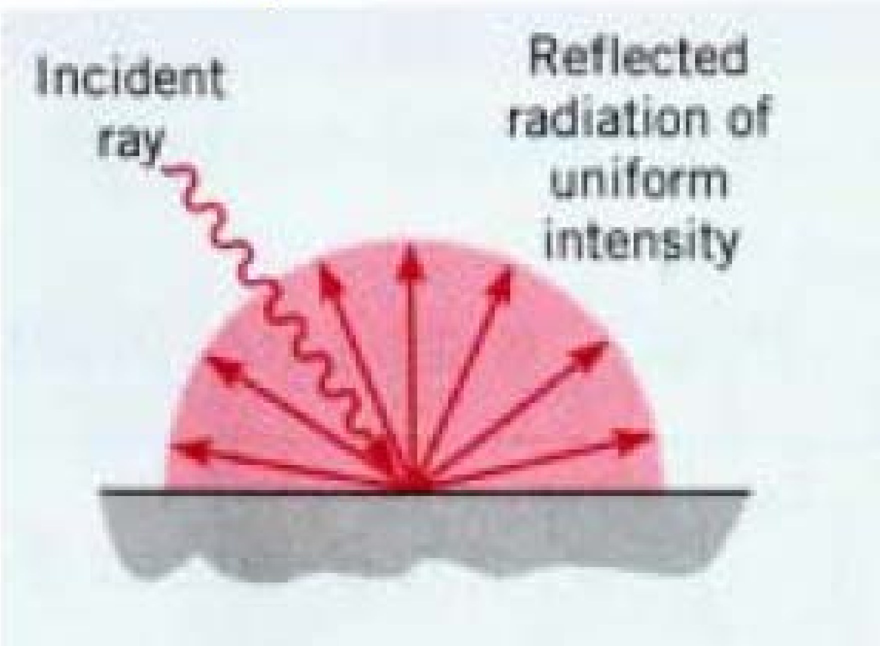
spectral normal absorptivities;spectral transmissivities

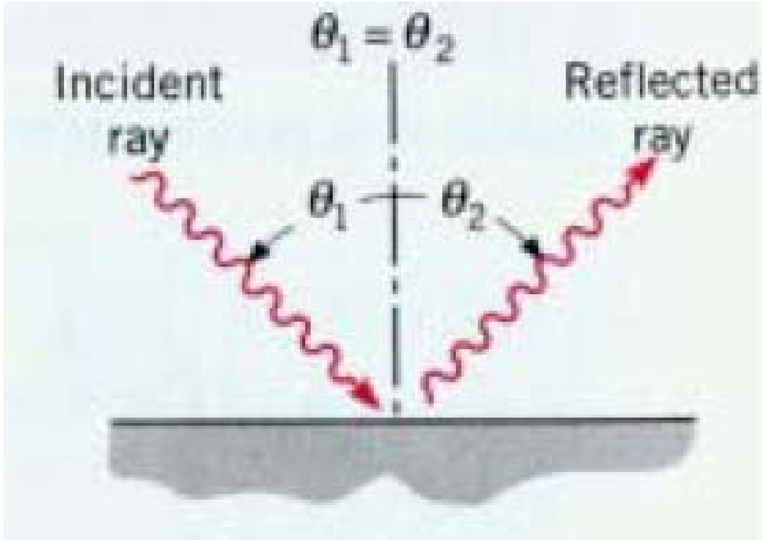
$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$$
$$\rho + \alpha + \tau = 1$$

For *opaque* medium: $\rho_\lambda + \alpha_\lambda = 1$

$$\rho + \alpha = 1$$

Definitions for *reflectivities*





Diffuse and specular reflection.

Kirchhoff's Law

Considering the thermal equilibrium between a body in an enclosure under steady-state conditions the following relations, called **Kirchhoff's law**,

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)=\alpha_{\lambda,\theta}(\lambda,\theta,\phi,T)$$

$\varepsilon_{\lambda}(\lambda,T)=\alpha_{\lambda}(\lambda,T)$, if the irradiation is diffuse;

or the surface is diffuse

$\varepsilon(T)=\alpha(T)$, if the irradiation is emission from a black-body at the same temperature as the surface;

or the surface is gray and diffuse; or ...

The Gray Surface

$$\varepsilon_\lambda = \frac{\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\lambda,\theta} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi}$$

$$= \frac{\int_0^{2\pi}\int_0^{\pi/2} \alpha_{\lambda,\theta} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi} = \alpha_\lambda$$

$$\varepsilon_\lambda(\lambda, T) = \alpha_\lambda(\lambda, T)$$

if 1. the irradiation is diffuse, or
2. the surface is diffuse. (Fig. 12.17)

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda, T) d\lambda}{E_b(T)} = \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda) d\lambda}{G} = \alpha$$

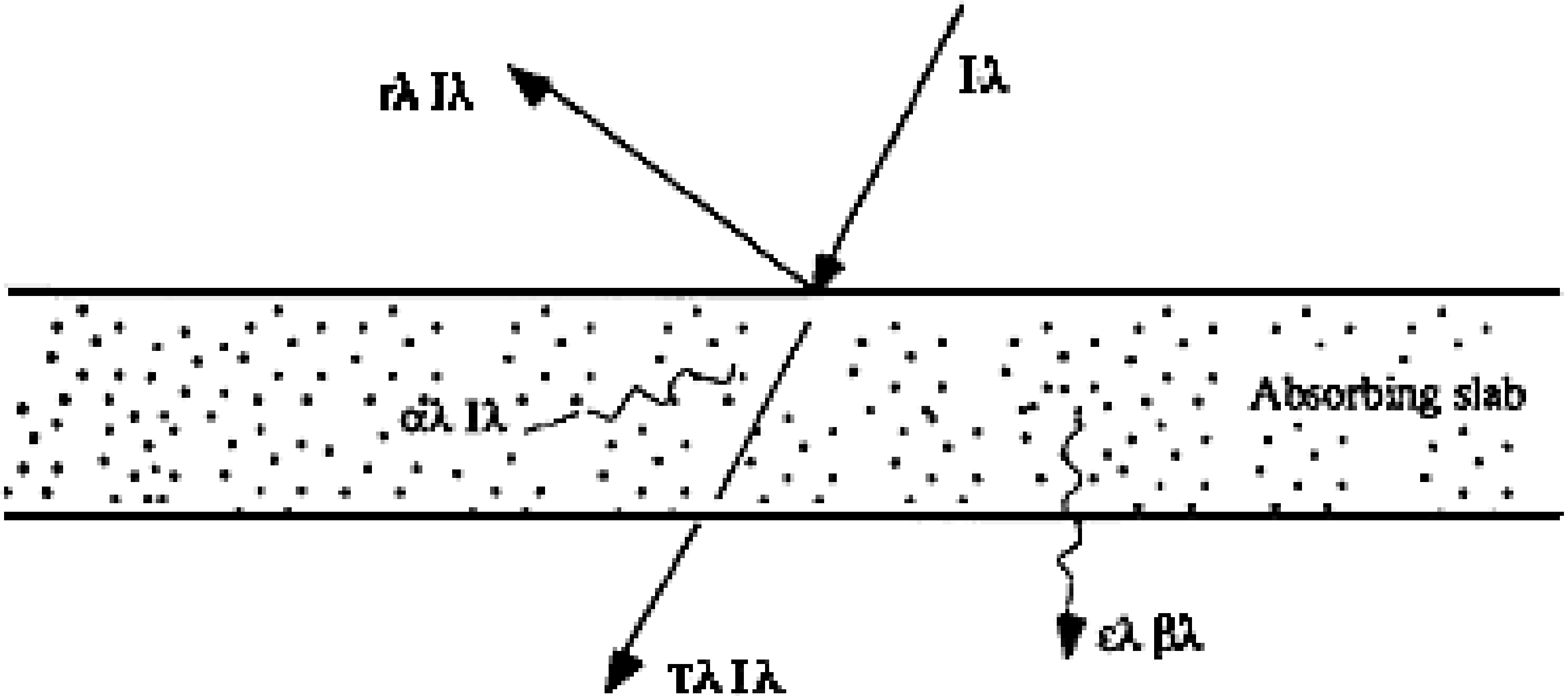
Stefan-Boltzmann Law

The energy radiated by a blackbody radiator per second per unit area is proportional to the fourth power of the absolute temperature and is given by

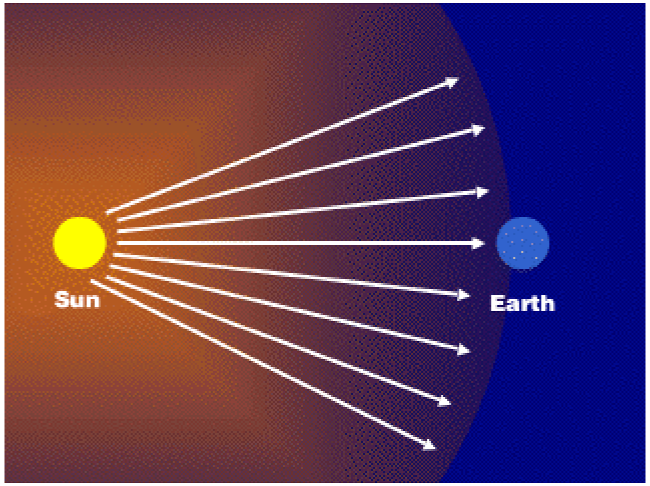
$$\frac{P}{A} = \sigma T^4 \; j/m^2 s$$ Stefan-Boltzmann Law

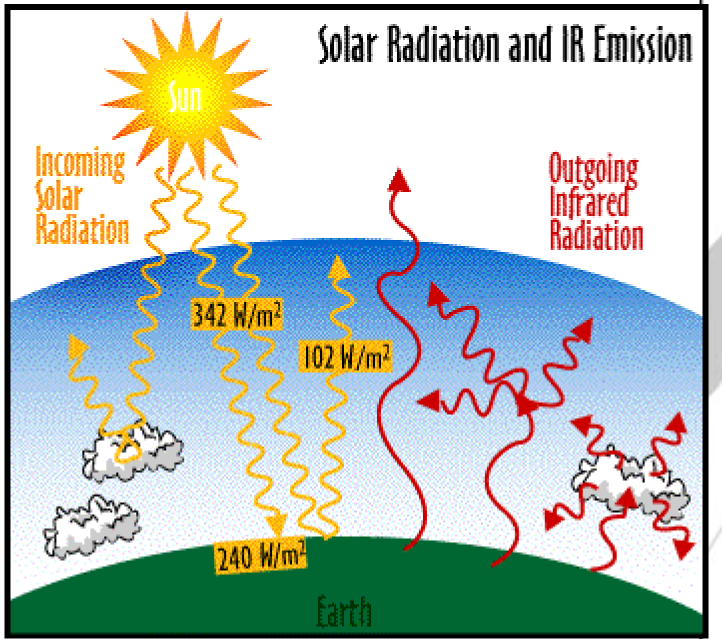
$$\sigma = 5.6703 x 10^{-8} \; watt / m^2 K^4$$

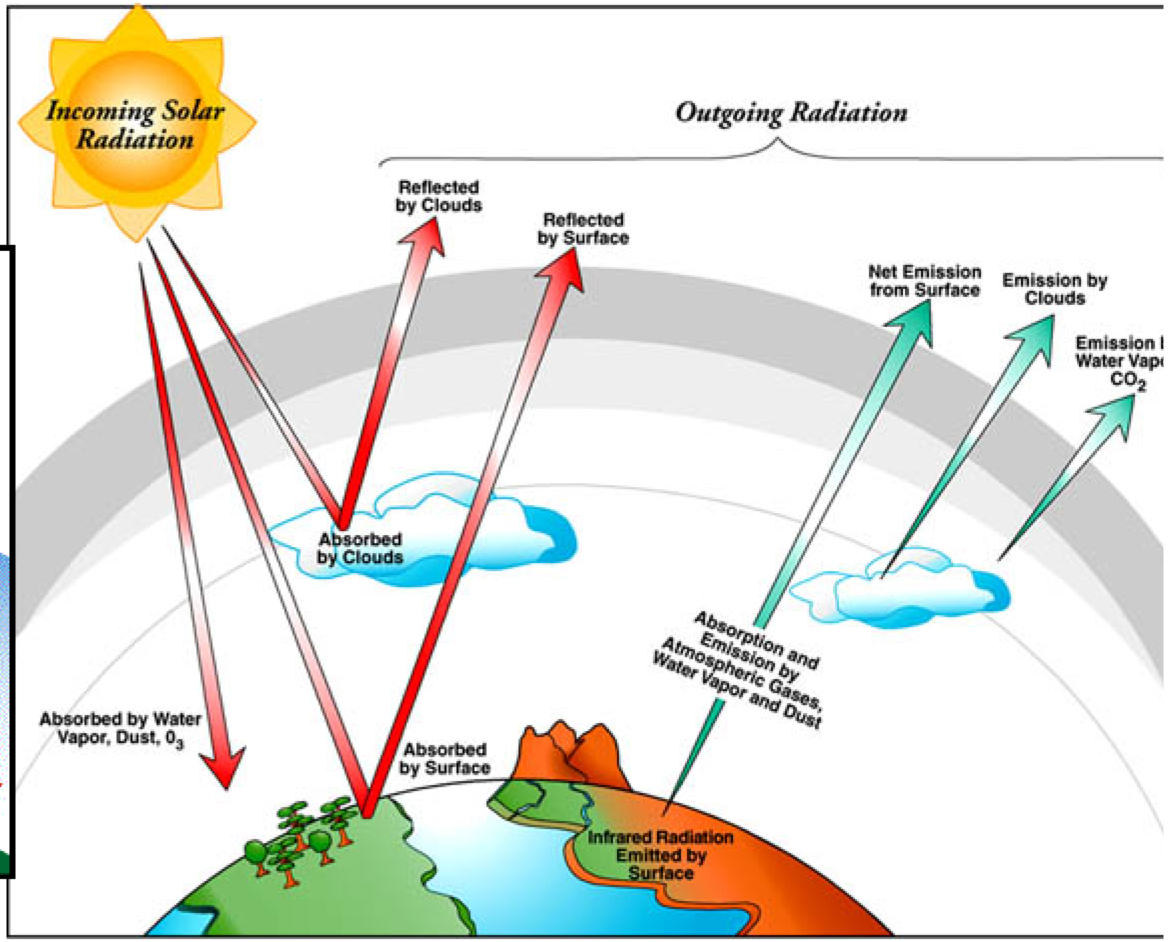
For hot objects other than ideal radiators, the law is expressed in the form:



rλ Iλ
Iλ
αλ Iλ
Absorbing slab
τλ Iλ
ελ βλ

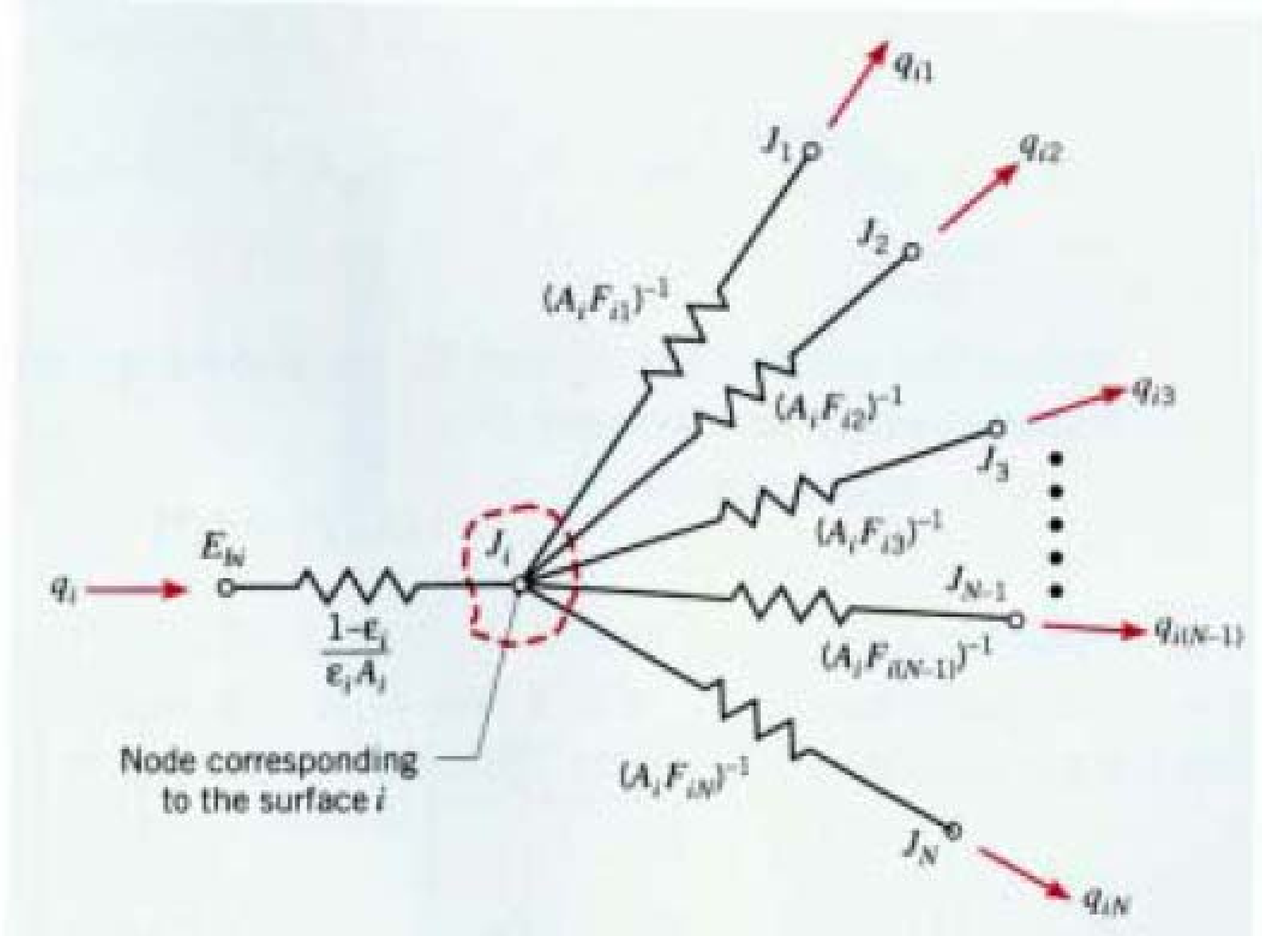

Sun
Earth


Solar Radiation and IR Emission
Sun
Incoming Solar Radiation
Outgoing Infrared Radiation
342 W/m^2
102 W/m^2
240 W/m^2
Earth

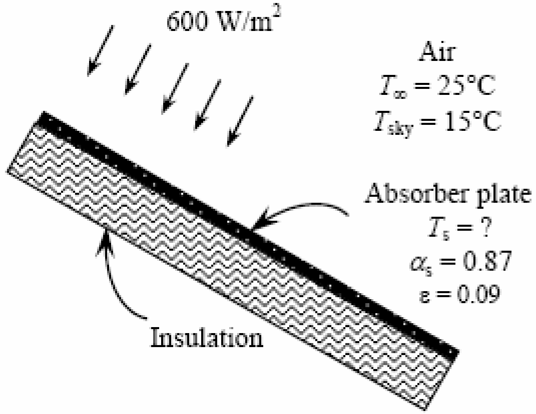

Incoming Solar Radiation
Outgoing Radiation
Reflected by Clouds
Reflected by Surface
Net Emission from Surface
Emission by Clouds
Absorbed by Clouds
Absorption and Emission by Atmospheric Gases, Water Vapor and Dust
Absorbed by Water Vapor, Dust, O_3
Absorbed by Surface
Infrared Radiation Emitted by Surface

13.3 Radiation Exchange Between Diffuse, Gray Surfaces in an Enclosure

Assumptions: isothermal surface; uniform radiosity and irradiation; opaque, diffuse, gray surface; nonparticipating medium



11-63 The absorber surface of a solar collector is exposed to solar and sky radiation. The equilibrium temperature of the absorber surface is to be determined if the backside of the plate is insulated.



Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.87$ and $\varepsilon = 0.09$.

Analysis The backside of the absorbing plate is insulated (instead of being attached to water tubes), and thus

$$\dot{q}_{net} = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s^{\ 4} - T_{sky}^{\ \ 4}) + h(T_s - T_{air})$$

$$(0.87)(600 \text{ W/m}^2) = (0.09)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)\left[(T_s)^4 - (288\text{ K})^4\right] + (10 \text{ W/m}^2\cdot\text{K})(T_s - 298\text{ K})$$

$$T_s = \mathbf{346\ K}$$

Thermal infrared thermography

Ideal for:

- General Thermography
- Printed Circuit Board (PCB) Analysis
- Electrical Predictive Maintenance
- Substation and Power Generation Analysis
- Motor Control Centers
- Uninterrupted Power Supplies (UPS)
- Mechanical Preventative Maintenance
- Flat Roof Inspections
- Energy Audits
- Building Envelope Analysis
- Mold and Moisture Remediation Surveys
- Quality Assurance
- Security
- Medical Thermography
- Veterinary Medicine
- Equine Thermography
- Educational Presentations
- Research and development

Electrical & machinery Predictive / Preventive Maintenance PM / PDM applications

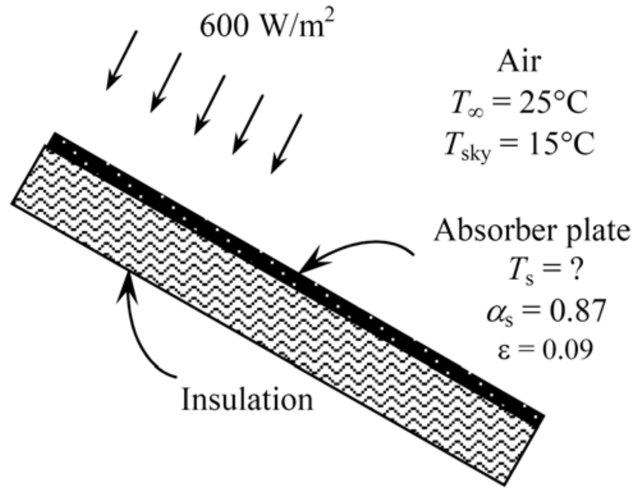
Research & Development, Non Destructive testing

Mechanical / Steam trap / distribution / Refractory / Boiler / Petrochemical applications

Energy audit / Heat loss / building efficiency surveying applications

Law enforcement / Military / Surveillance for ground-based / air borne or marine applications

11-63 The absorber surface of a solar collector is exposed to solar and sky radiation. The equilibrium temperature of the absorber surface is to be determined if the backside of the plate is insulated.



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$$\dot{q}_{net} = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s^{\,4} - T_{sky}^{\,4}) + h(T_s - T_{air})$$

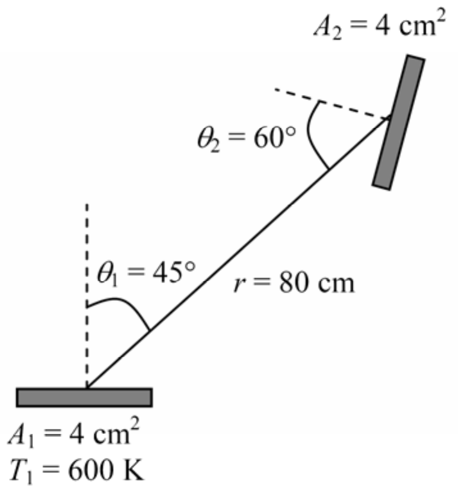
$$(0.87)(600\ \text{W/m}^2) = (0.09)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(T_s)^4 - (288\ \text{K})^4\right] + (10\ \text{W/m}^2\cdot\text{K})(T_s - 298\ \text{K})$$

$$T_s = \mathbf{346\ K}$$

11-31 A surface is subjected to radiation emitted by another surface. The solid angle subtended and the rate at which emitted radiation is received are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from Eq. 11-12 to be



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(4\text{ cm}^2)\cos 60°}{(80\text{ cm})^2} = \mathbf{3.125 \times 10^{-4}\ sr}$$

since the normal of A_2 makes 60° with the direction of viewing. Note that solid angle subtended by A_2 would be maximum if A_2 were positioned normal to the direction of viewing. Also, the point of viewing on A_1 is taken to be a point in the middle, but it can be any point since A_1 is assumed to be very small.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(800\text{ K})^4}{\pi} = 7393\text{ W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

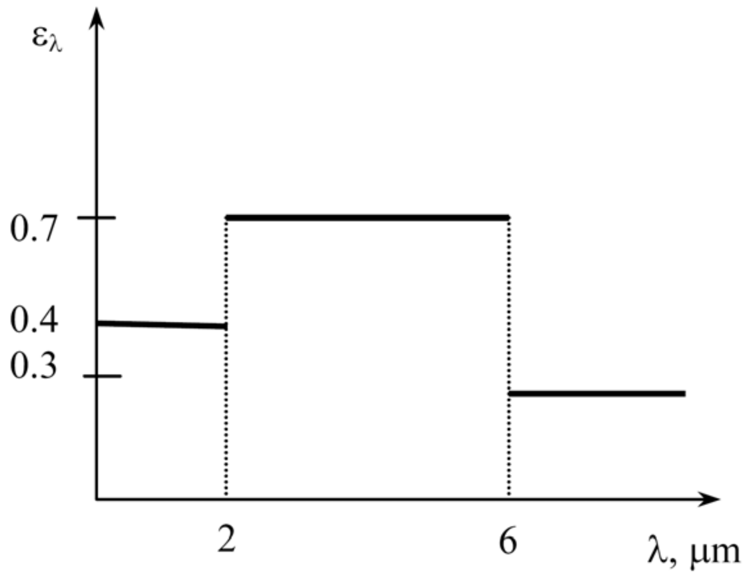
$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1} \\ &= (7393\text{ W/m}^2\cdot\text{sr})(4\times10^{-4}\cos 45°\text{ m}^2)(3.125\times10^{-4}\text{ sr}) \\ &= \mathbf{6.534\times10^{-4}\ W}\end{aligned}$$

Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 6.534×10^{-4} W.

If A_2 were directly above A_1 at a distance 80 cm, $\theta_1 = 0°$ and the rate of radiation energy emitted by A_1 becomes zero.

11-41 The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from



$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b_\lambda}(T)d\lambda}{\sigma T^4}$$
$$= \varepsilon_1 f_{0\text{-}\lambda_1} + \varepsilon_2 f_{\lambda_1\text{-}\lambda_2} + \varepsilon_3 f_{\lambda_2\text{-}\infty}$$
$$= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (2\ \mu\text{m})(1000\ \text{K}) = 2000\ \mu\text{mK} \longrightarrow \text{f}_{\lambda_1} = 0.066728$$

$$\lambda_2 T = (6\ \mu\text{m})(1000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow \text{f}_{\lambda_2} = 0.737818$$

$$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1} \text{ since } f_0 = 0 \text{ and } \text{f}_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} \text{ since } f_\infty = 1.$$

and,

$$\varepsilon = (0.4)0.066728 + (0.7)(0.737818 - 0.066728) + (0.3)(1 - 0.737818) = \mathbf{0.575}$$

Then the emissive power of the surface becomes

$$E = \varepsilon\sigma T^4 = 0.575(5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\ \text{K})^4 = \mathbf{32.6\ kW/m^2}$$

11-43 The variation of transmissivity of the glass window of a furnace at a specified temperature with wavelength is given. The fraction and the rate of radiation coming from the furnace and transmitted through the window are to be determined.

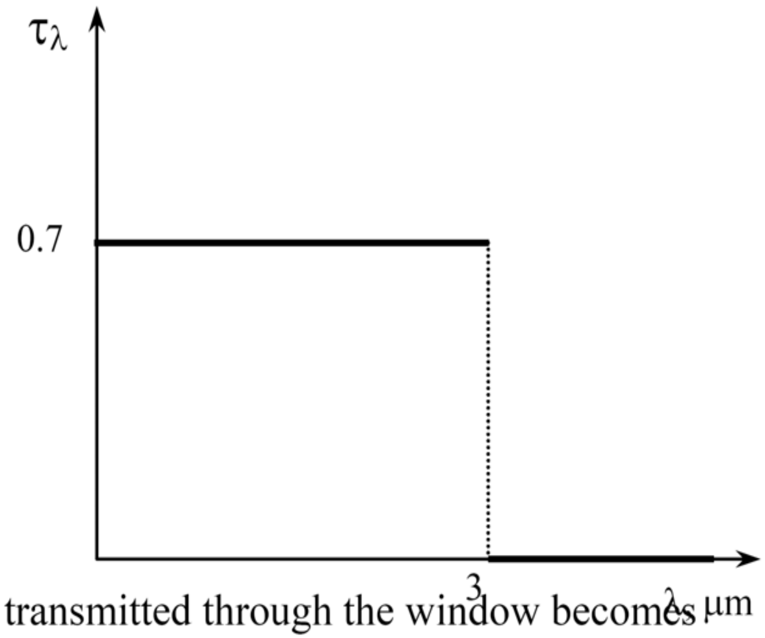
Assumptions The window glass behaves as a black body.

Analysis The fraction of radiation at wavelengths smaller than 3 μm is

$$\lambda T = (3\ \mu\text{m})(1200\ \text{K}) = 3600\ \mu\text{mK} \longrightarrow f_\lambda = 0.403607$$

The fraction of radiation coming from the furnace and transmitted through the window is

$$\begin{aligned}\tau(T) &= \tau_1 f_\lambda + \tau_2(1 - f_\lambda) \\ &= (0.7)(0.403607) + (0)(1 - 0.403607) \\ &= \mathbf{0.283}\end{aligned}$$



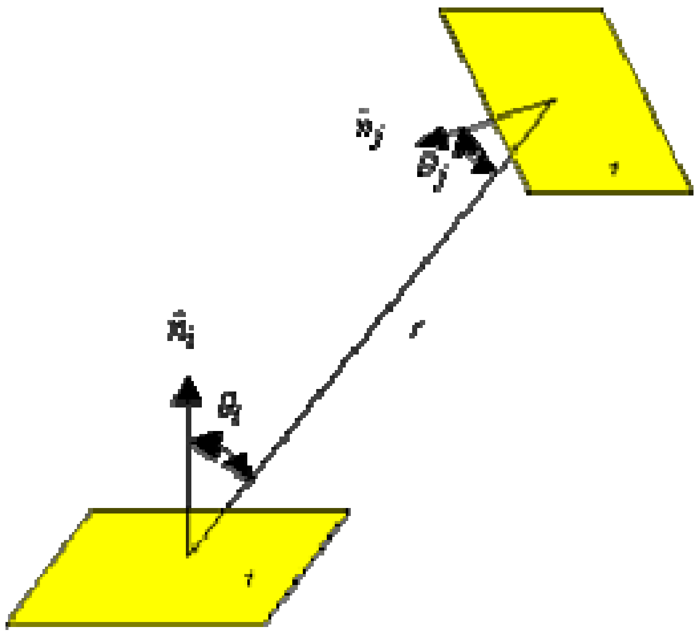
Then the rate of radiation coming from the furnace and transmitted through the window becomes

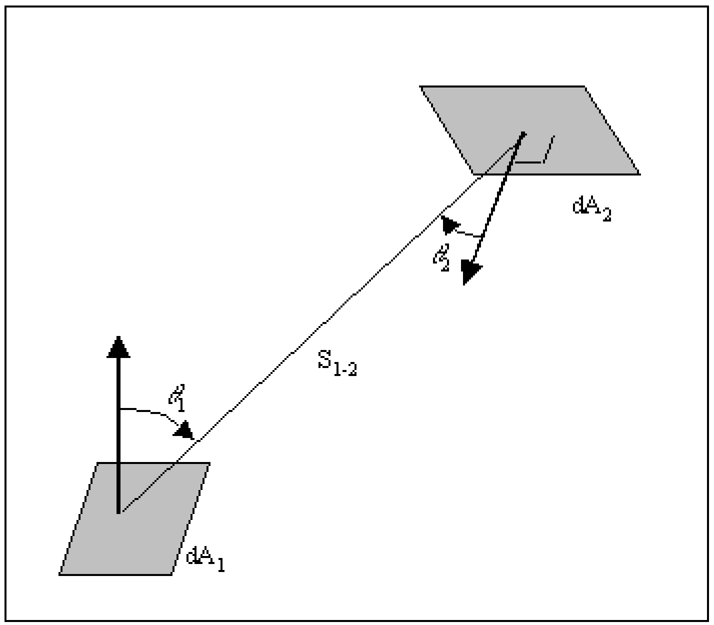
$$G_{tr} = \tau A \sigma T^4 = 0.283(0.25 \times 0.25\ \text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1200\ \text{K})^4 = \mathbf{2076\ W}$$

View Factor

View factor (or **shape factor**) F_{ij} : *fraction* of the radiation *leaving surface i* that is *intercepted by surface j*.

$$F_{ij} = \frac{q_{i \to j}}{A_i J_i} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$$





Fiew Factor Relations

$$A_i F_{ij} = A_j F_{ji}$$ --reciprocity relation

$$= \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$$

For an closure of N surfaces,

$$\sum_{j=1}^{N} F_{ij} = 1$$ --summation rule

13.2 Blackbody Radiation Exchange

For a blackbody surface, there is no reflection and radiosity equals emissive power, $J_i = E_{bi}$.

$$q_{i\to j} = (A_i J_i)F_{ij} = A_i F_{ij} E_{bi}$$

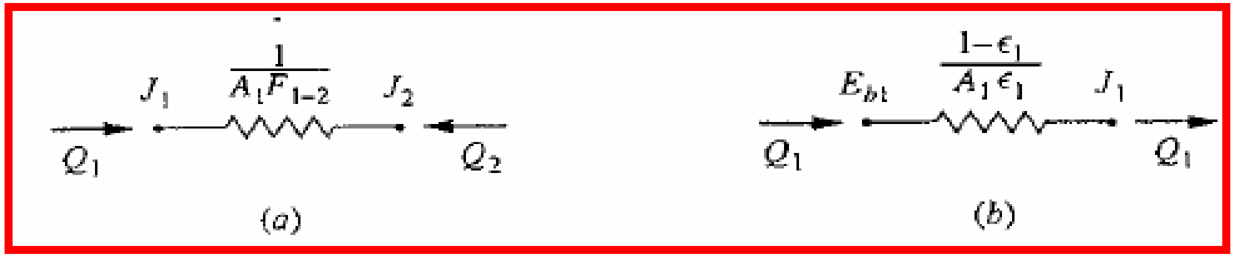
$$q_{j\to i} = A_j F_{ji} E_{bj}$$

The net rate of radiation exchange is

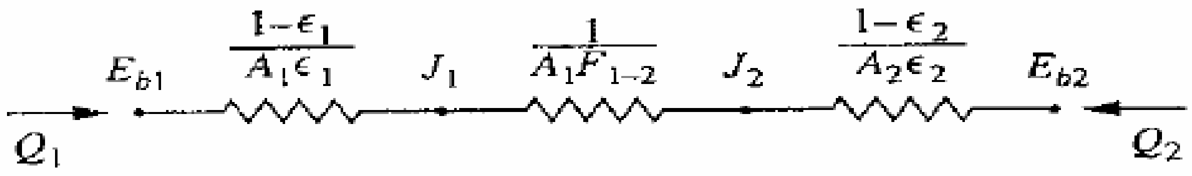
$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj} = A_i F_{ij} \sigma (T_i^4 - T_j^4)$$

For an enclosure of N surfaces, the net radiation from surface i is

$$\boxed{q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4)}$$



(a) Space resistance, (b) surface resistance,



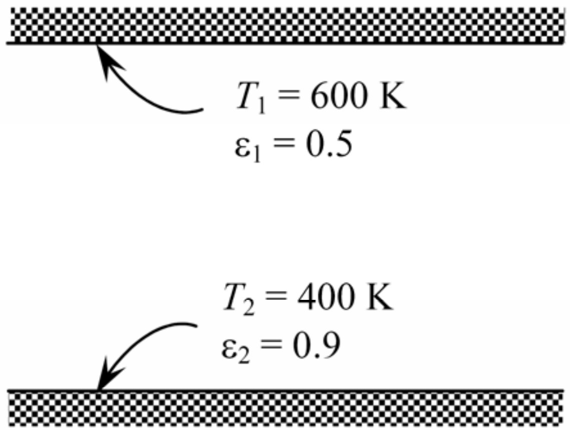
(c) total resistance.

Electric network analogy for infinite parallel plates:

$$\boxed{Q_1 = \frac{E_{b1} - J_1}{\dfrac{1-\epsilon_1}{A_1\epsilon_1}} = \frac{J_2 - E_{b2}}{\dfrac{1-\epsilon_2}{A_2\epsilon_2}} = -Q_2,}$$

$$\boxed{Q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\epsilon_1}{A_1\epsilon_1} + \dfrac{1}{A_1F_{1-2}} + \dfrac{1-\epsilon_2}{A_2\epsilon_2}} = -Q_2,}$$

12-26 Two very large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities ε of the plates are given to be 0.5 and 0.9.

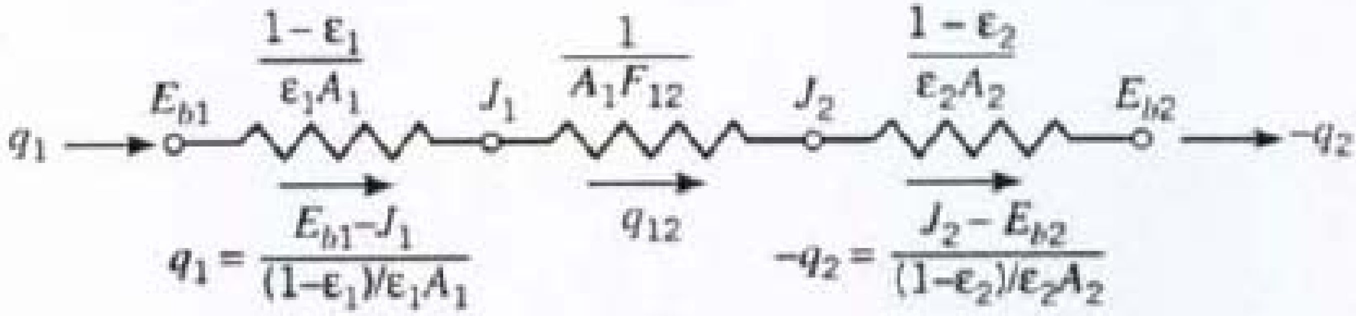
Analysis The net rate of radiation heat transfer between the two surfaces per unit area of the plates is determined directly from

$$\frac{\dot{Q}_{12}}{A_s} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(600\ \text{K})^4 - (400\ \text{K})^4]}{\dfrac{1}{0.5} + \dfrac{1}{0.9} - 1} = \mathbf{2795\ W/m^2}$$

$$q_1 = -q_2 = q_{12}$$

A2, T2, ε2
q12
A1, T1, ε1

(a)

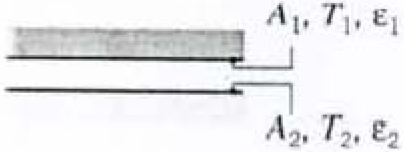


1 − ε1 / ε1A1
1 / A1F12
1 − ε2 / ε2A2
Eb1
J1
J2
Eb2
q1
−q2
q12
q1 = (Eb1 − J1) / ((1−ε1)/ε1A1)
−q2 = (J2 − Eb2) / ((1−ε2)/ε2A2)

(b)

TABLE Special Diffuse, Gray, Two-Surface Enclosures

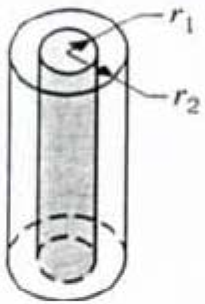
Large (Infinite) Parallel Planes



$$A_1 = A_2 = A \qquad F_{12} = 1$$

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

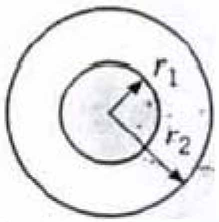
Long (Infinite) Concentric Cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2} \qquad F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

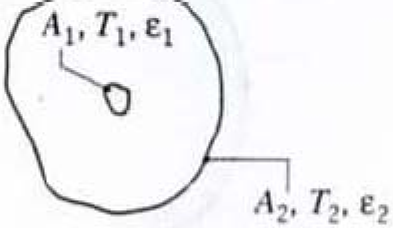
Concentric Spheres



$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \qquad F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$$

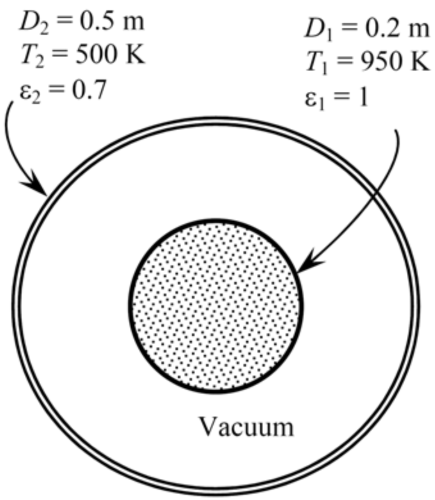
Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0 \qquad F_{12} = 1$$

$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

12-30 Two very long concentric cylinders are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.7$.

Analysis The net rate of radiation heat transfer between the two cylinders per unit length of the cylinders is determined from

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)} = \frac{[\pi(0.2\text{ m})(1\text{ m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(950\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1}{1} + \dfrac{1-0.7}{0.7}\left(\dfrac{2}{5}\right)}$$

$$= 22{,}870\text{ W} = \mathbf{22.87\ kW}$$

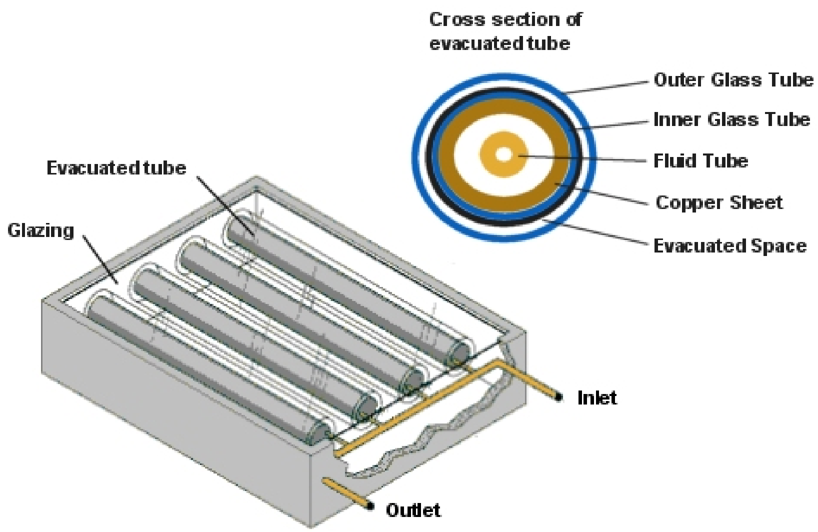


TABLE 13.1 View Factors for Two-Dimensional Geometries [4]

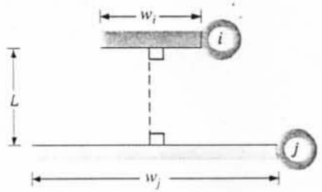
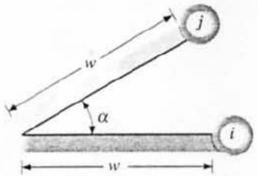
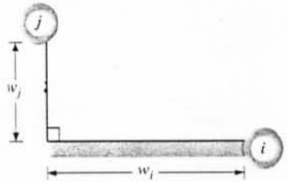
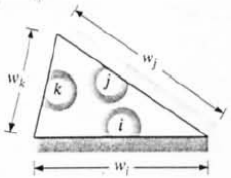
Geometry	Relation
Parallel Plates with Midlines Connected by Perpendicular 	$F_{ij} = \dfrac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$ $W_i = w_i/L,\ W_j = w_j/L$
Inclined Parallel Plates of Equal Width and a Common Edge 	$F_{ij} = 1 - \sin\left(\dfrac{\alpha}{2}\right)$
Perpendicular Plates with a Common Edge 	$F_{ij} = \dfrac{1 + (w_j/w_i) - [1 + (w_j/w_i)^2]^{1/2}}{2}$
Three-Sided Enclosure 	$F_{ij} = \dfrac{w_i + w_j - w_k}{2w_i}$

TABLE 13.1 *Continued*

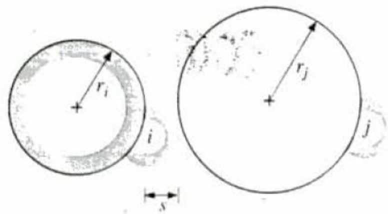
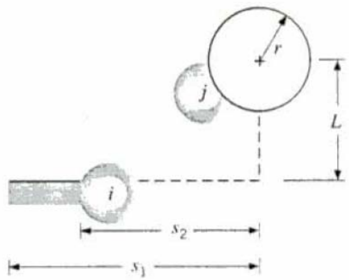
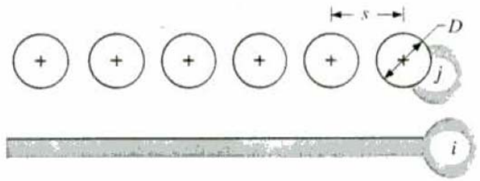
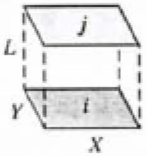
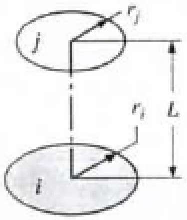
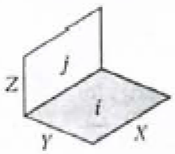
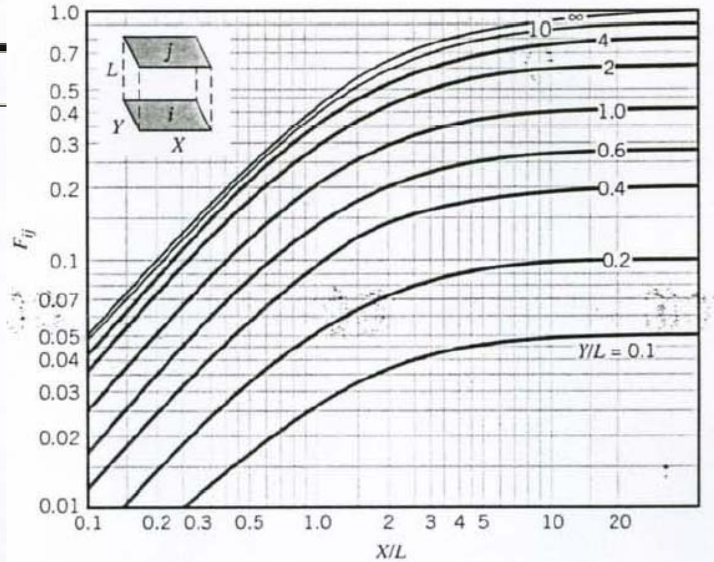
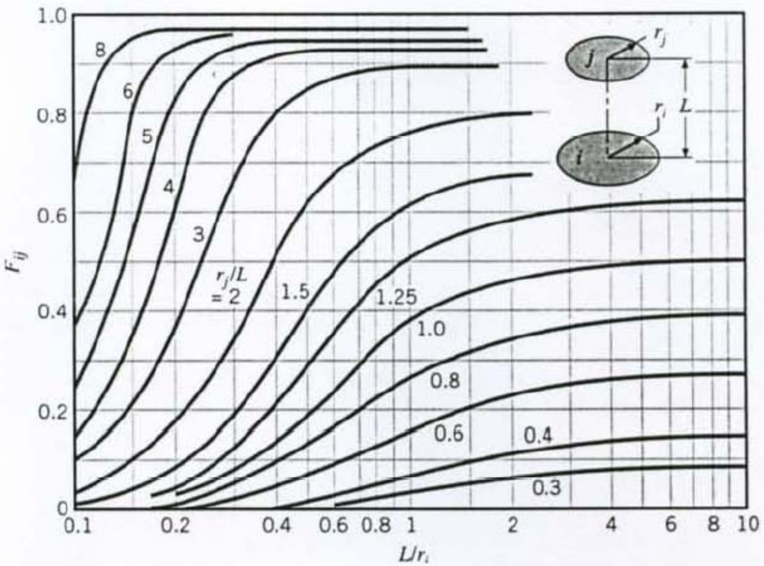
Geometry	Relation
Parallel Cylinders of Different Radii 	$F_{ij} = \dfrac{1}{2\pi}\Big\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\dfrac{R}{C}\right) - \left(\dfrac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\dfrac{R}{C}\right) + \left(\dfrac{1}{C}\right)\right]\Big\}$ $R = r_j/r_i,\ S = s/r_i$ $C = 1 + R + S$
Cylinder and Parallel Rectangle 	$F_{ij} = \dfrac{r}{s_1 - s_2}\left[\tan^{-1}\dfrac{s_1}{L} - \tan^{-1}\dfrac{s_2}{L}\right]$
Infinite Plane and Row of Cylinders 	$F_{ij} = 1 - \left[1 - \left(\dfrac{D}{s}\right)^2\right]^{1/2} + \left(\dfrac{D}{s}\right)\tan^{-1}\left(\dfrac{s^2 - D^2}{D^2}\right)^{1/2}$

TABLE 13.2 View Factors for Three-Dimensional Geometries [4]

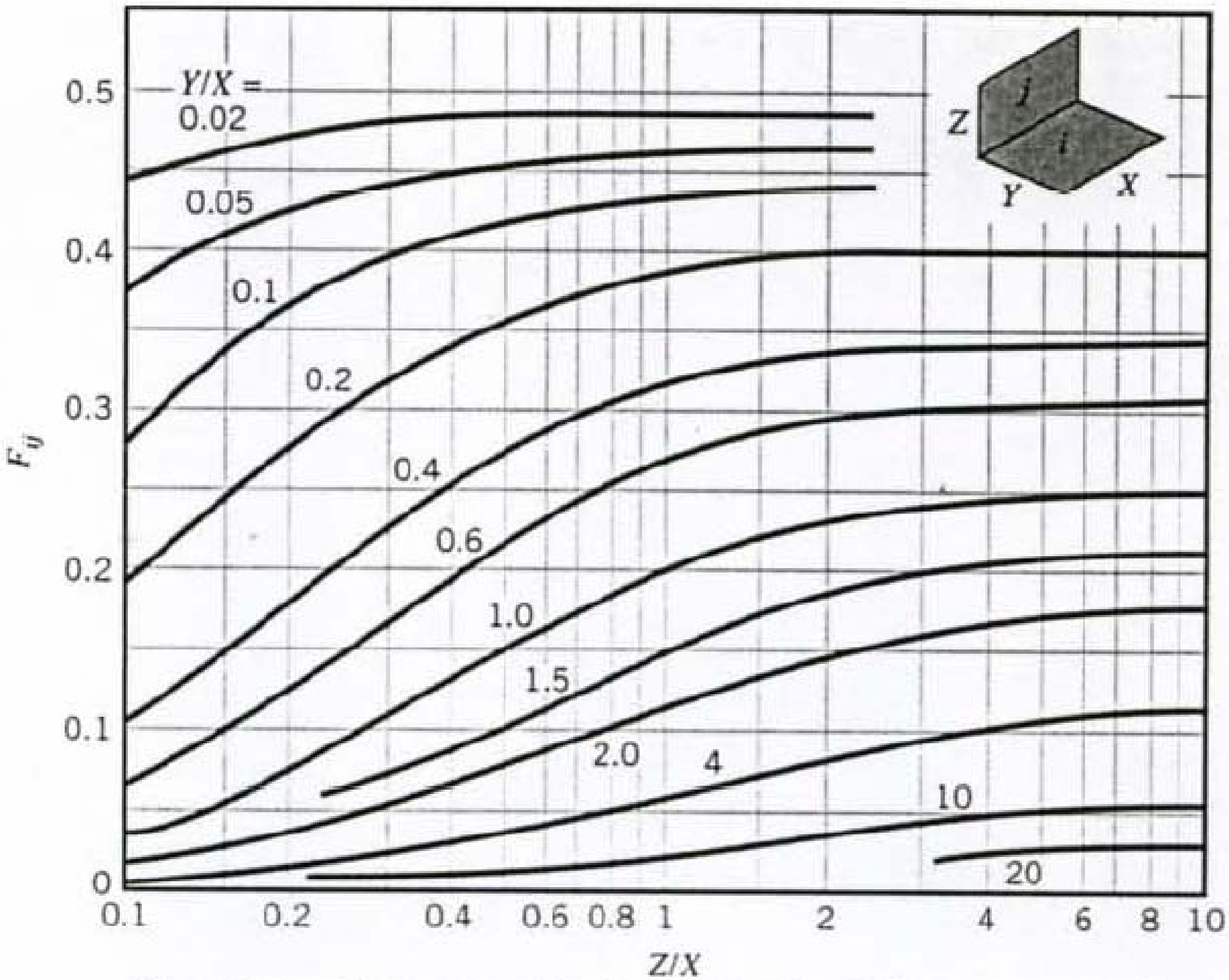
Geometry	Relation
Aligned Parallel Rectangles (Figure 13.4) 	$\bar{X} = X/L,\ \bar{Y} = Y/L$ $F_{ij} = \frac{2}{\pi \bar{X} \bar{Y}}\left\{ \ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$
Coaxial Parallel Disks (Figure 13.5) 	$R_i = r_i/L,\ R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$
Perpendicular Rectangles with a Common Edge (Figure 13.6) 	$H = Z/X,\ W = Y/X$ $F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$



View factor for aligned parallel rectangles.



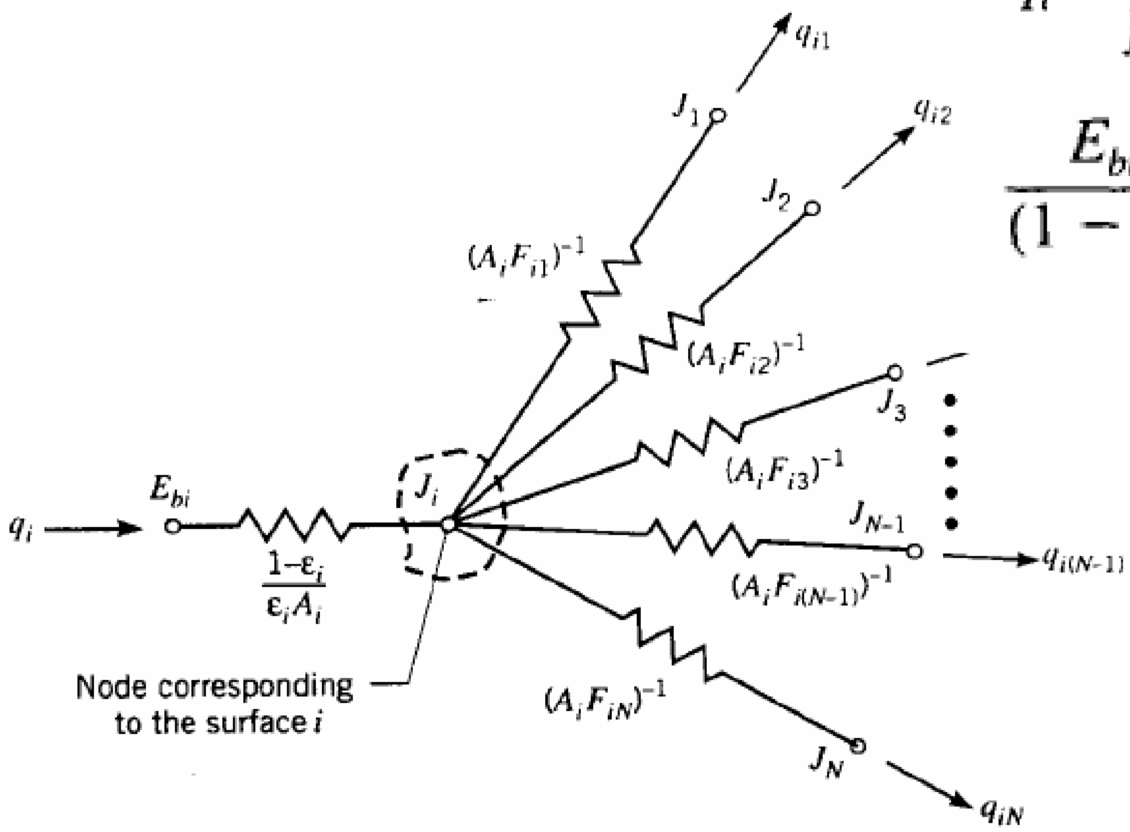
View factor for coaxial parallel disks.



View factor for perpendicular rectangles with a common edge.

$$q_i = \sum_{j=1}^{N} A_i F_{ij}(J_i - J_j) = \sum_{j=1}^{N} q_{ij}$$

$$\frac{E_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}}$$



Network representation of radiative exchange between surface i and the remaining surfaces of an enclosure.

$$\begin{bmatrix} a_{11}J_1 + \cdots + a_{1i}J_i + \cdots + a_{1N}J_N = C_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{i1}J_1 + \cdots + a_{ii}J_i + \cdots + a_{iN}J_N = C_i \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{N1}J_1 + \cdots + a_{Ni}J_i + \cdots + a_{NN}J_N = C_N \end{bmatrix}$$

$$[A][J] = [C]$$

$$[A] = \begin{bmatrix} a_{11} \cdots a_{1i} \cdots a_{1N} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ a_{i1} \cdots a_{ii} \cdots a_{iN} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ a_{N1} \cdots a_{Ni} \cdots a_{NN} \end{bmatrix} \quad [J] = \begin{bmatrix} J_1 \\ \cdot \\ \cdot \\ J_i \\ \cdot \\ \cdot \\ J_N \end{bmatrix} \quad [C] = \begin{bmatrix} C_1 \\ \cdot \\ \cdot \\ C_i \\ \cdot \\ \cdot \\ C_N \end{bmatrix}$$

$$\begin{bmatrix} J_1 = b_{11}C_1 + \cdots + b_{1i}C_i + \cdots + b_{1N}C_N \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ J_i = b_{i1}C_1 + \cdots + b_{ii}C_i + \cdots + b_{iN}C_N \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ J_N = b_{N1}C_1 + \cdots + b_{Ni}C_i + \cdots + b_{NN}C_N \end{bmatrix}$$

$$[J] = [A]^{-1}[C]$$

$$[A]^{-1} = \begin{bmatrix} b_{11} \cdots b_{1i} \cdots b_{1N} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ b_{i1} \cdots b_{ii} \cdots b_{iN} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ b_{N1} \cdots b_{Ni} \cdots b_{NN} \end{bmatrix}$$

12-53 Two very large plates are maintained at uniform temperatures. The number of thin aluminum sheets that will reduce the net rate of radiation heat transfer between the two plates to one-fifth is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

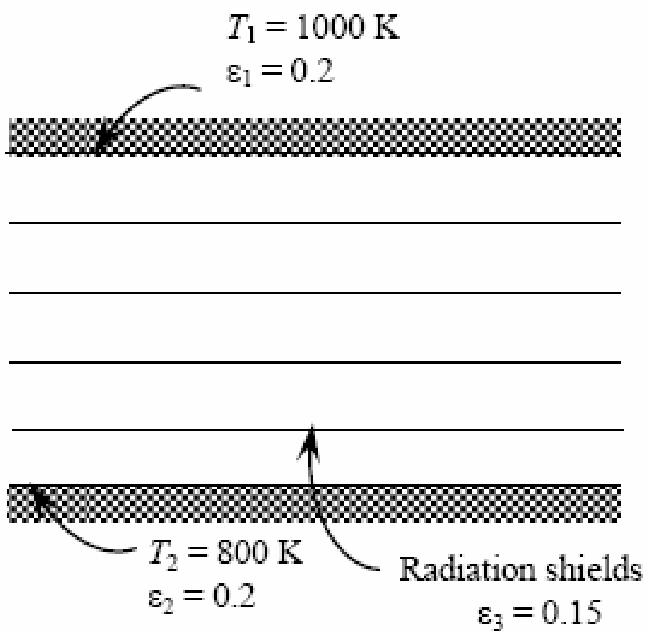
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.2$, and $\varepsilon_3 = 0.15$.

Analysis The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,\textit{no}\ \text{shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\ \text{K})^4 - (800\ \text{K})^4]}{\left(\dfrac{1}{0.2} + \dfrac{1}{0.2} - 1\right)}$$

$$= 3720\ \text{W/m}^2$$



The number of sheets that need to be inserted in order to reduce the net rate of heat transfer between the two plates to onefifth can be determined from

$$\dot{Q}_{12,\text{shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$\frac{1}{5}(3720\ \text{W/m}^2) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\ \text{K})^4 - (800\ \text{K})^4]}{\left(\dfrac{1}{0.2} + \dfrac{1}{0.2} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{0.15} + \dfrac{1}{0.15} - 1\right)} \longrightarrow N_{\text{shield}} = 2.92 \cong \mathbf{3}$$

9-67 The absorber plate and the glass cover of a flat-plate solar collector are maintained at specified temperatures. The rate of heat loss from the absorber plate by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat loss by radiation is negligible. **4** The air pressure in the enclusure is 1 atm.

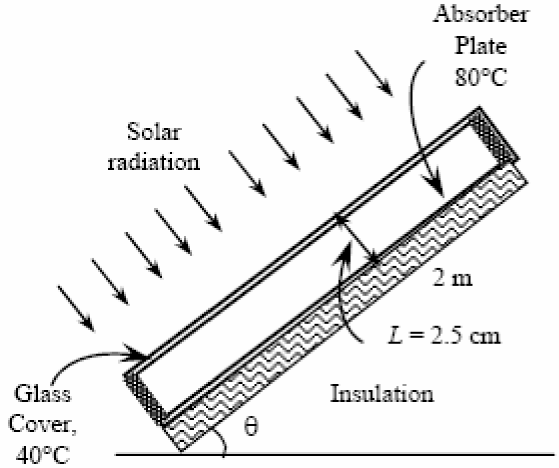
Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (80+40)/2 = 60$°C are (Table A-15)

$$k = 0.02808 \text{ W/m.°C}$$

$$\upsilon = 1.896\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7202$$

$$\beta = \frac{1}{T_f} = \frac{1}{(60+273)\text{K}} = 0.003003 \text{ K}^{-1}$$



Analysis For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $L_c = L = 0.025$ m Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003003 \text{ K}^{-1})(80-40 \text{ K})(0.025 \text{ m})^3}{(1.896\times10^{-5} \text{ m}^2\text{/s})^2}(0.7202) = 3.689\times10^4$$

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}}\right]^+ + \left[\frac{\text{Ra}^{1/3}}{18}-1\right]^+$$

$$= 1+1.44\left[1-\frac{1708}{3.689\times10^4}\right]^+ + \left[\frac{(3.689\times10^4)^{1/3}}{18}-1\right]^+ = 3.223$$

Then

$$A_s = H \times W = (2\text{ m})(3\text{ m}) = 6\text{ m}^2$$

$$\dot{Q} = kNuA_s \frac{T_1 - T_2}{L} = (0.02808\text{ W/m.°C})(3.223)(6\text{ m}^2)\frac{(80-40)\text{°C}}{0.025\text{ m}} = \mathbf{869\ W}$$

For $\theta = 20°$, we obtain

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}\cos\theta}\right]^+ \left[1 - \frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}\cos\theta}\right] + \left[\frac{(\text{Ra}\cos\theta)^{1/3}}{18} - 1\right]^+$$

$$= 1 + 1.44\left[1 - \frac{1708}{(3.689\times10^4)\cos(20)}\right]^+ \left[1 - \frac{1708[\sin(1.8\times20)]^{1.6}}{(3.689\times10^4)\cos(20)}\right] + \left[\frac{[(3.689\times10^4)\cos(20)]^{1/3}}{18} - 1\right]^+ = 3.152$$

$$\dot{Q} = kNuA_s \frac{T_1 - T_2}{L} = (0.02808\text{ W/m.°C})(3.152)(6\text{ m}^2)\frac{(80-40)\text{°C}}{0.025\text{ m}} = \mathbf{850\ W}$$

For $\theta = 90°$, we have vertical rectangular enclosure. The Nusselt number for this geometry and orientation can be determined from ($Ra = 3.689\times10^4$ - same as that for horizontal case)

$$Nu = 0.42Ra^{1/4}\Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(3.689\times10^4)^{1/4}(0.7202)^{0.012}\left(\frac{2\text{ m}}{0.025\text{ m}}\right)^{-0.3} = 1.557$$

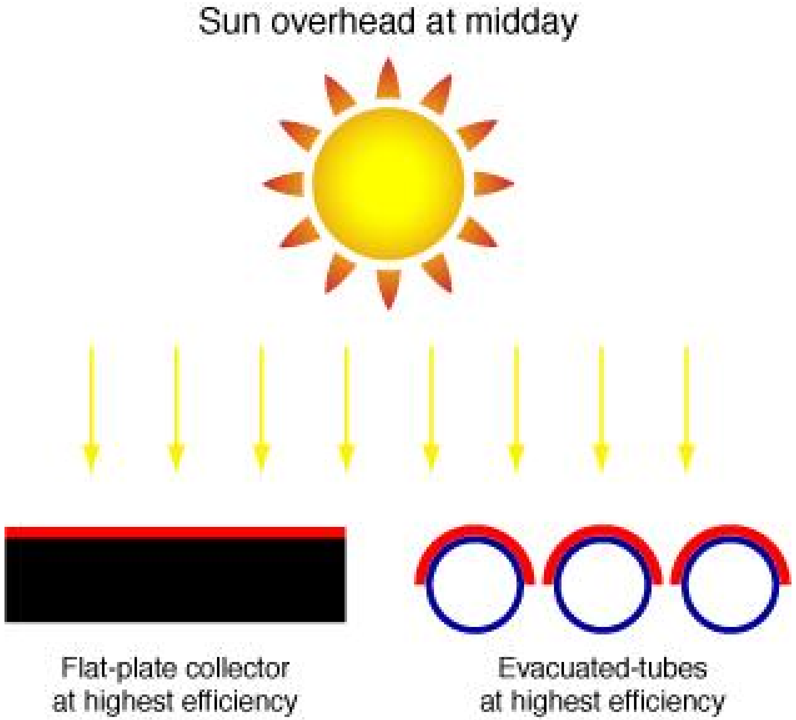
$$\dot{Q} = kNuA_s \frac{T_1 - T_2}{L} = (0.02808\text{ W/m.°C})(1.557)(6\text{ m}^2)\frac{(80-40)\text{°C}}{0.025\text{ m}} = \mathbf{420\ W}$$

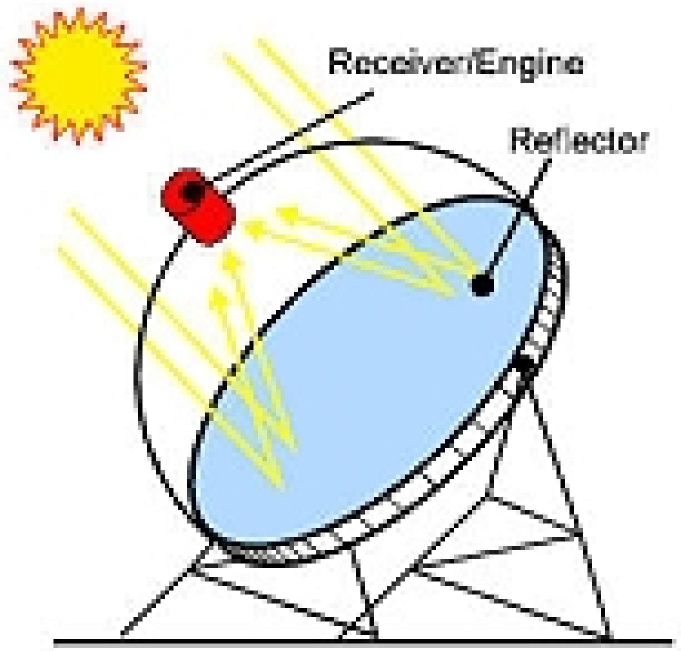
Discussion Caution is advised for the vertical case since the condition $H/L < 40$ is not satisfied.

METHODS OF SOLAR COLLECTION AND THERMAL CONVERSION

4

Part (I)





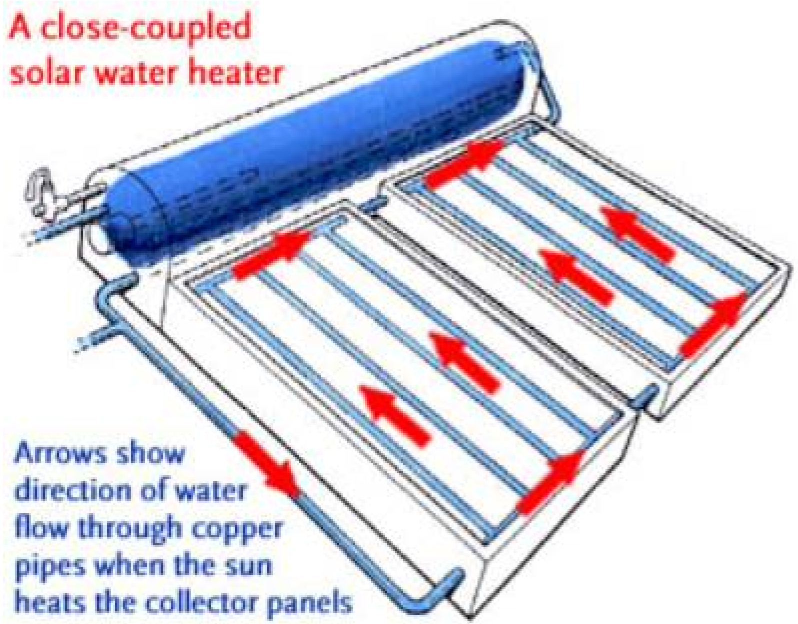
- Converting the sun's radiant energy to heat is the most common and well-developed solar conversion technology today.

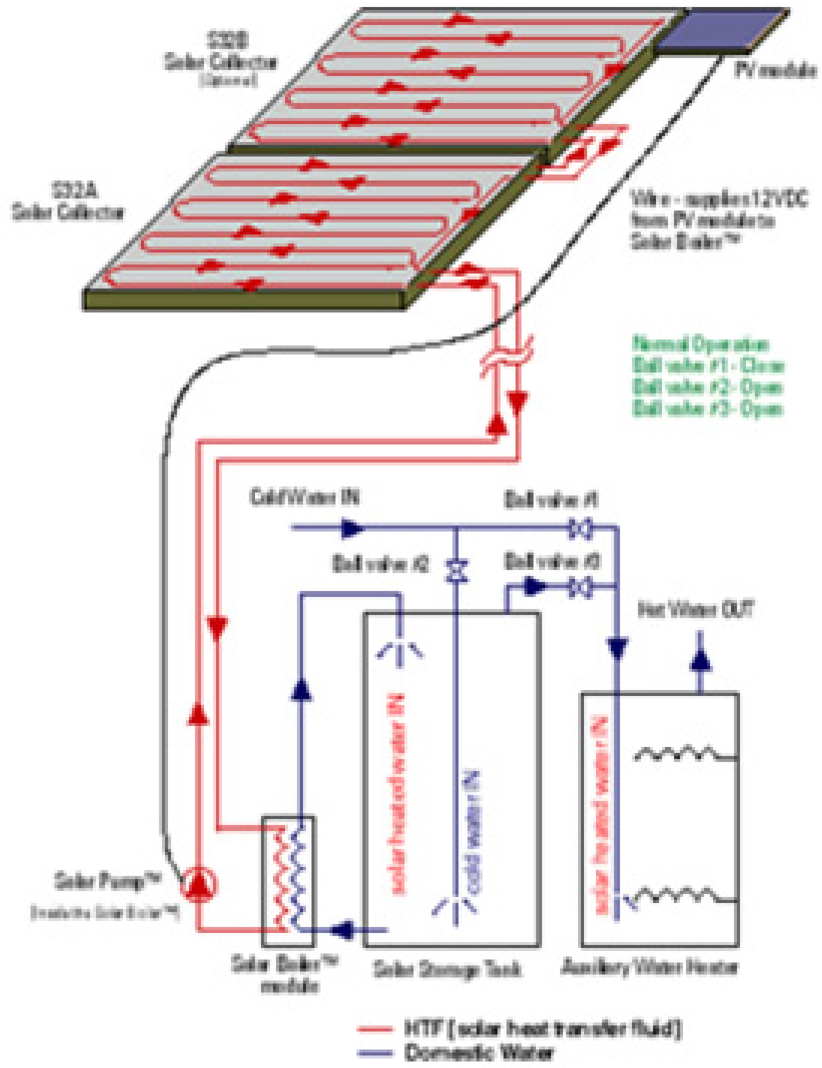
- This chapter analyzes in detail different methods for converting solar energy at temperatures ranging from moderately above ambient to above 3000 K.

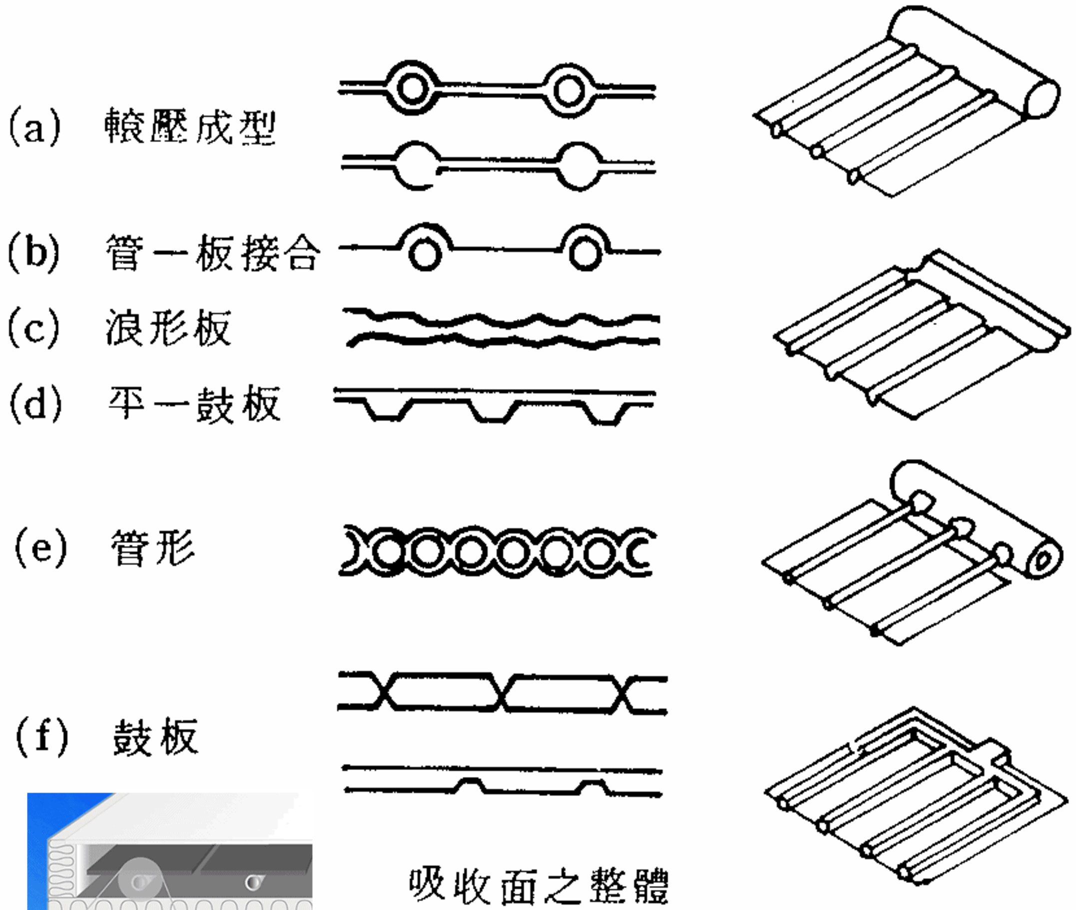
- The thermal and optical performance of several solar-thermal collectors is described.

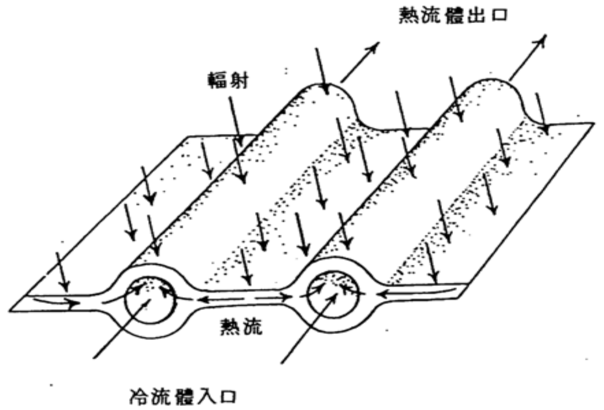
- They range from air- and liquid-cooled nonconcentrating, flat-plate types to compound-curvature, continuously tracking types with concentration ratios up to 3000 more.

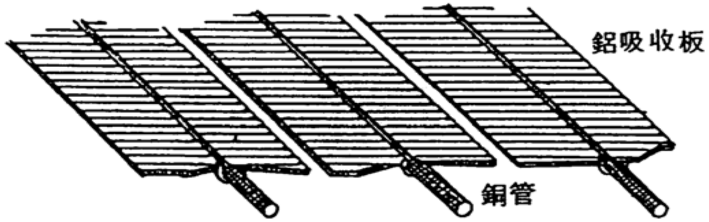
4.1 Energy Balance for a Flat-Plate Collector(用於加熱熱水)

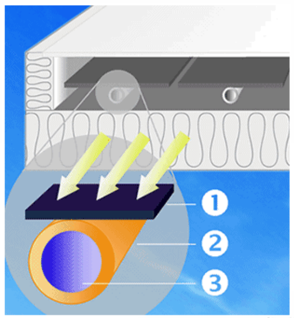


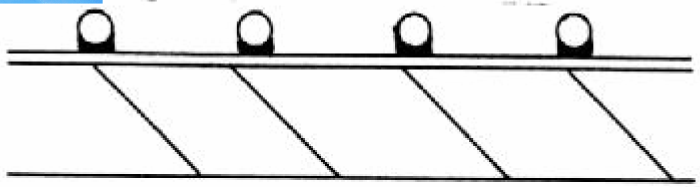




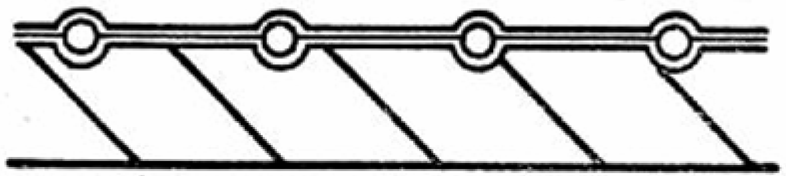








WATER TUBES BONDED TO ABSORBER



BONDED TUBE-IN-SHEET TYPE ABSORBER

4.1 ENERGY BALANCE FOR A FLAT-PLATE COLLECTOR

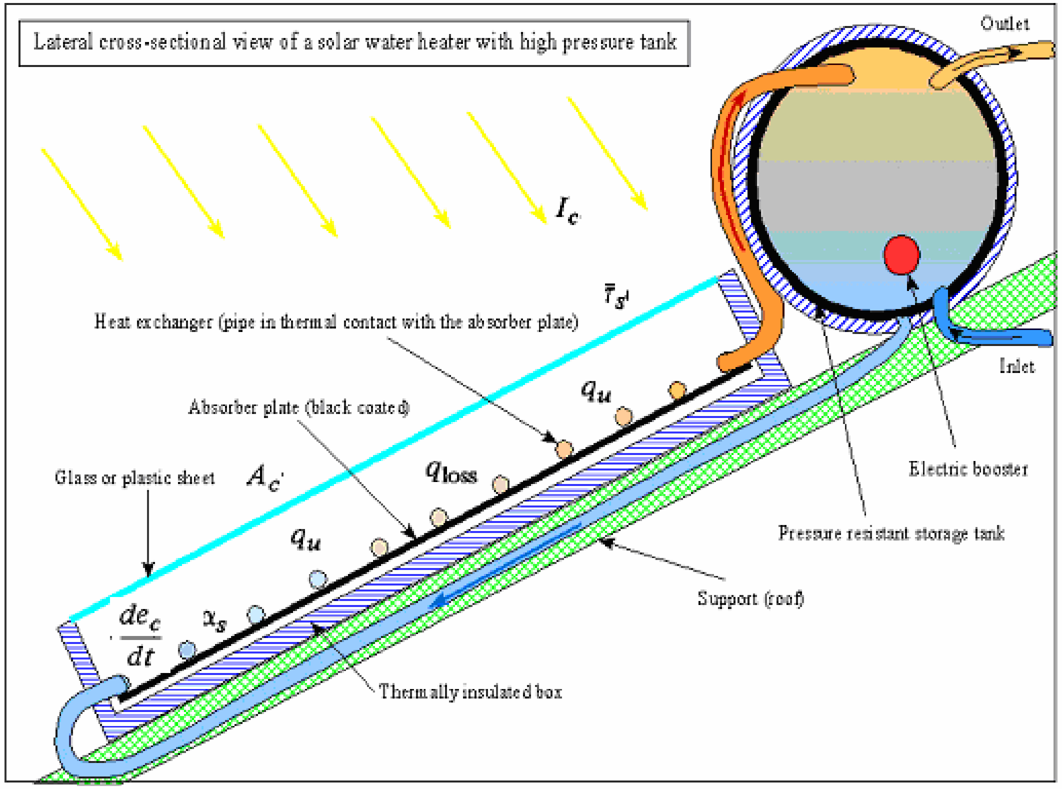


FIGURE 4.1 Schematic diagram of solar collector with two covers.

For a flat-plate collector of area A_c this energy balance is

$$I_c A_c \bar{\tau}_s \alpha_s = q_u + q_{\text{loss}} + \frac{de_c}{dt}$$

q_{loss} = rate of heat transfer (or heat loss) from the collector-absorber plate to the surroundings

I_c = solar irradiation on a collector surface
$\bar{\tau}_s$ = effective solar transmittance of the collector cover(s)
α_s = solar absorptance of the collector-absorber plate surface
q_u = rate of heat transfer from the collector-absorber plate to the working fluid
q_{loss} = rate of heat transfer (or heat loss) from the collector-absorber plate to the surroundings
de_c/dt = rate of internal energy storage in the collector

- The instantaneous efficiency of a collector η_c

$$\boxed{\eta_c = \frac{q_u}{A_c I_c} = \frac{\text{the useful energy delivered}}{\text{the total incoming solar energy}}} \tag{4.2}$$

- for the average efficiency

$$\boxed{\bar{\eta}_c = \frac{\displaystyle\int_0^t q_u \, dt}{\displaystyle\int_0^t A_c I_c \, dt}} \tag{4.3}$$

t is the time period over which the performance is averaged.

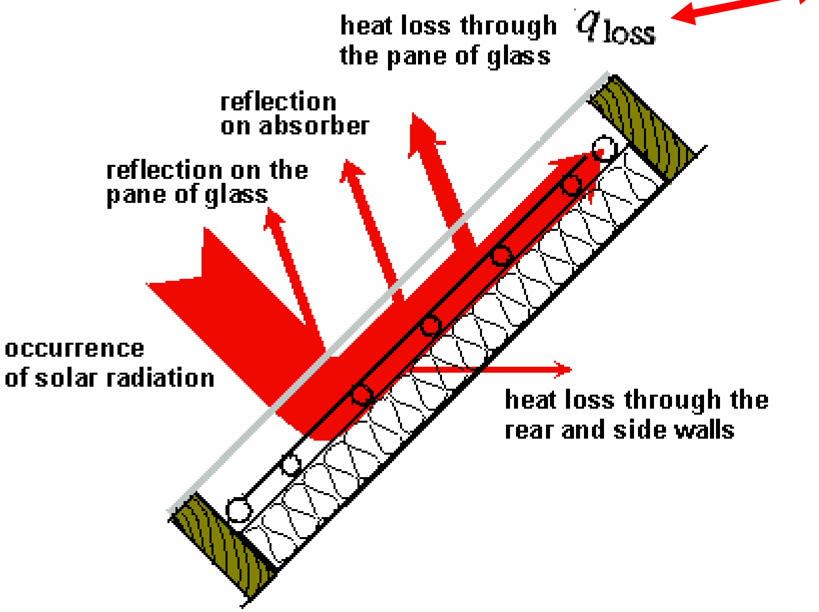
In a standard performance test this period is on the order of 15 or 20 min

4.1.1 Collector Heat-Loss Conductance

In order to obtain an understanding of the parameters determining the thermal efficiency of a solar collector, it is important to develop the concept of *collector heat-loss conductance.*

$$I_c A_c \bar{\tau}_s \alpha_s = q_u + q_{\text{loss}} + \frac{de_c}{dt}$$

heat loss q_{loss} = rate of heat transfer from the collector-absorber plate to the surroundings



$$q_{\text{loss}} = U_c A_c (T_c - T_a)$$

U_c the collector heat-loss conductance

T_c: average temperature

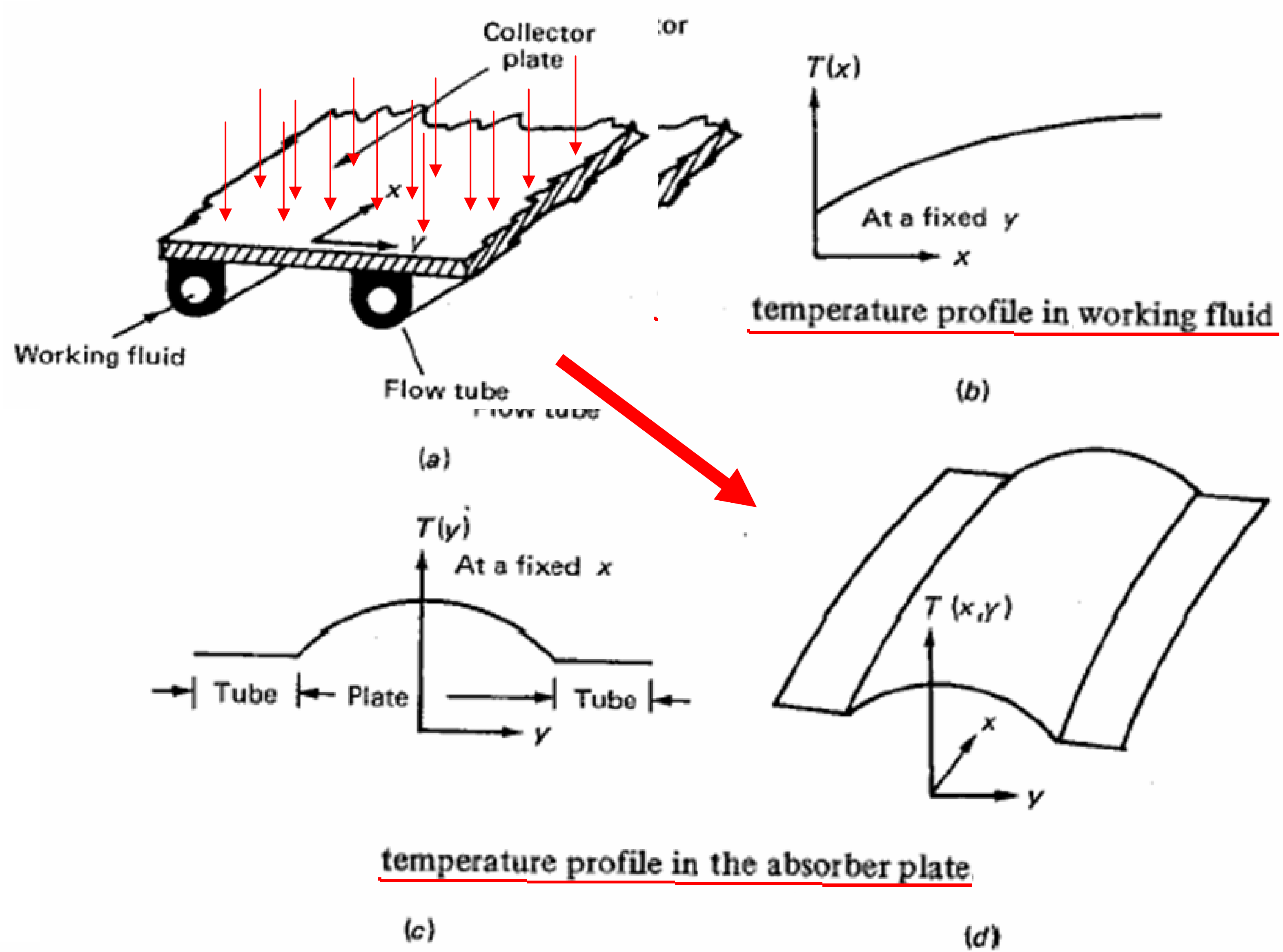
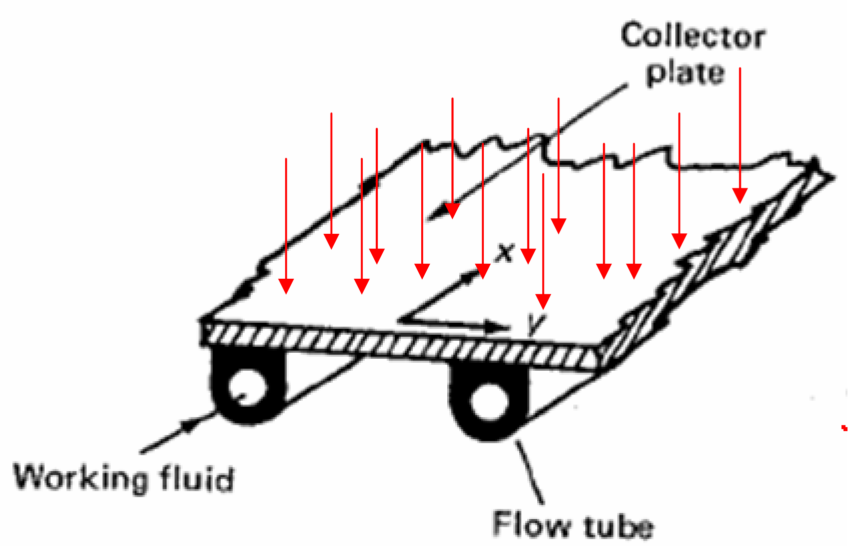
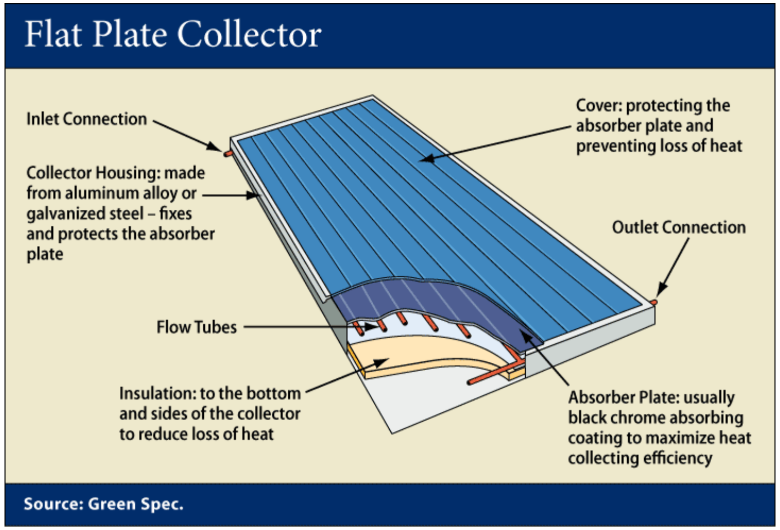


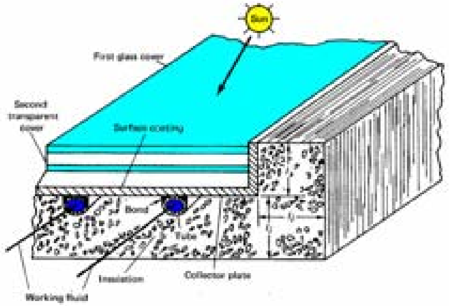
FIGURE 4.3 Qualitative temperature distribution in the absorber plate of a flat-plate collector. (*a*) Schematic diagram of absorber; (*b*) temperature profile in the direction of the flow of the working fluid; (*c*) temperature profile at given *x*; (*d*) temperature distribution in the absorber plate.

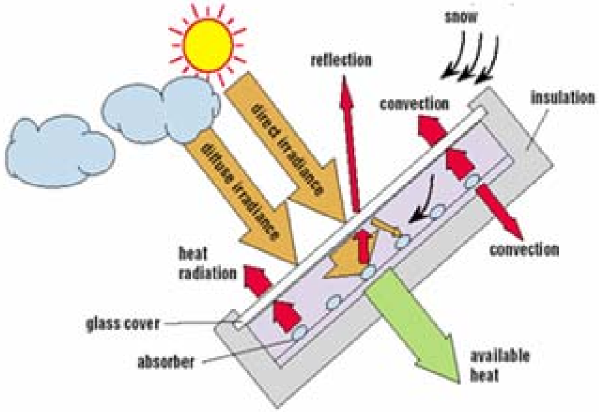
★ The thermal analysis of a flat-plate collector

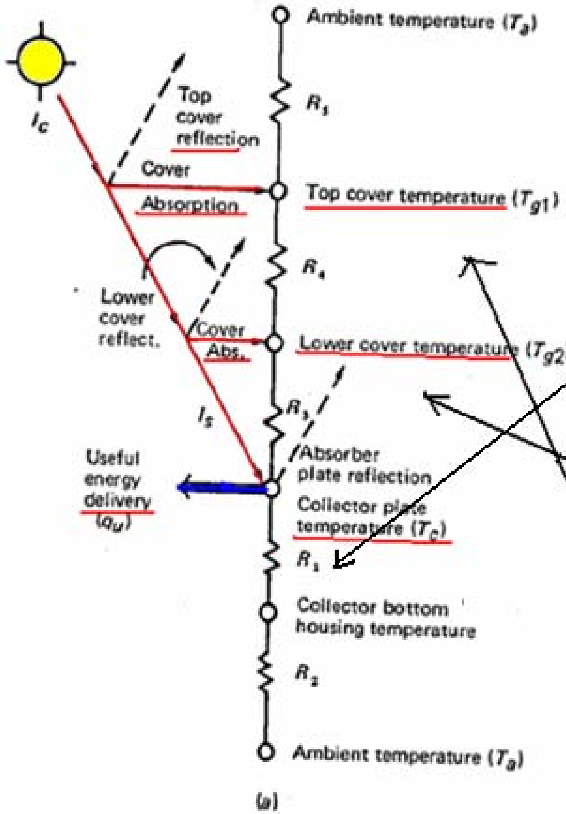
1. The collector is thermally in steady state.
2. The temperature drop between the top and bottom of the absorber plate is negligible.
3. Heat flow is one-dimensional through the covers as well as through the back insulation.
4. The headers connecting the tubes cover only a small area of the collector and provide uniform flow to the tubes.
5. The sky can be treated as though it were a black-body source for infrared radiation at an equivalent sky temperature.
6. The irradiation on the collector plate is uniform.











Thermal circuits

$$q''_x = -k\frac{dT}{dx} = -k\frac{T_2 - T_1}{L}$$

$$q_x = q''_x A_c$$

- for a collector of area $l_1 l_2$ and thickness l_3, and with a layer of insulation l_i thick at the bottom and along the sides, the effective back loss would be approximately

$$q_{\text{back loss}} = \frac{A_c k_i}{l_i}(T_c - T_a)\left[1 + \frac{(2l_3 + l_i)(l_1 + l_2)}{l_1 l_2}\right] \qquad (4.6)$$

(底板加四邊)

- the rate of heat transfer per unit surface area of collector between the absorber plate and the second glass cover is

$$q_{\text{top loss}} = A_c \bar{h}_{c2}(T_c - T_{g2}) + \frac{\sigma A_c (T_c{}^4 - T_{g2}{}^4)}{1/\epsilon_{p,i} + 1/\epsilon_{g2,i} - 1} \qquad (4.7)$$

$$= (\bar{h}_{c2} + h_{r2})A_c(T_c - T_{g2}) = \frac{T_c - T_{g2}}{R_3} A_c$$

$$h_{r2} = \frac{\sigma(T_c + T_{g2})(T_c{}^2 + T_{g2}{}^2)}{(1/\epsilon_{p,i} + 1/\epsilon_{g2,i} - 1)}$$

- A similar derivation for the rate of heat transfer between the two cover plates gives

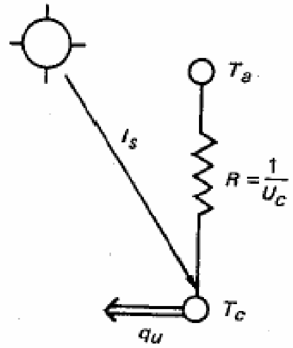
$$q_{\text{top loss}} = (\bar{h}_{c1} + h_{r1})A_c(T_{g2} - T_{g1}) = \frac{T_{g2} - T_{g1}}{R_4} A_c \qquad (4.9a)$$

$$\text{where } h_{r1} = \frac{\sigma(T_{g1} + T_{g2})(T_{g1}{}^2 + T_{g2}{}^2)}{(1/\epsilon_{g1,i} + 1/\epsilon_{g2,i} - 1)} \qquad (4.9b)$$

- the convection heat exchange occurs between T_{g1} and the ambient air at T_{air} gives

$$q_{\text{top loss}} = (h_{c,\infty} + h_{r,\infty})(T_{g1} - T_{\text{air}})A_c = \frac{T_{g1} - T_{\text{air}}}{R_5} A_c \qquad (4.10)$$

where $h_{r,\infty} = \epsilon_{g1,i}\sigma(T_{g1} + T_{\text{sky}})(T_{g1}{}^2 + T_{\text{sky}}^2)\left[\frac{(T_{g1} - T_{\text{sky}})}{(T_{g1} - T_{\text{air}})}\right]$



- The total heat-loss conductance U_c can then be expressed in the form

$$U_c = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4 + R_5} \qquad (4.11)$$

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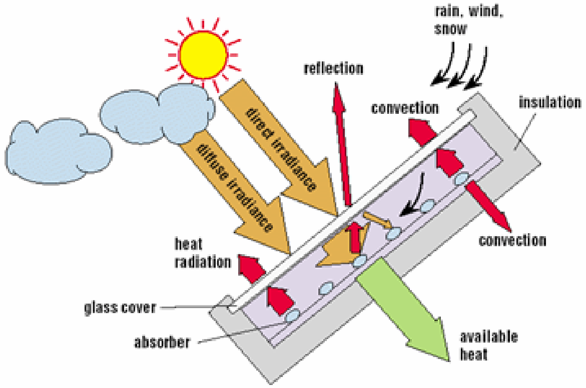
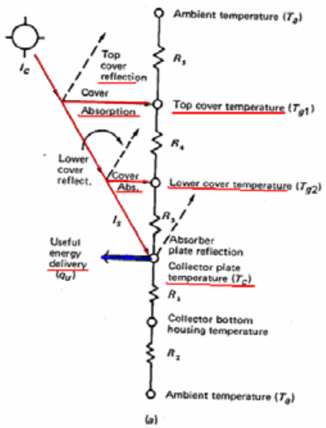
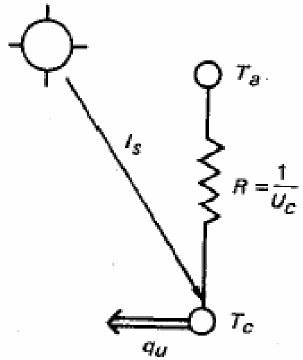
$$q_{\text{loss}} = U_c A_c (T_c - T_a)$$

U_c the collector heat-loss conductance
T_c: average temperature





Thermal circuits



● A simplified procedure for calculating U_c suggested by Hottel and Woertz (25)

$$\boxed{q_{\text{top loss}} = \frac{(T_c - T_a)A_c}{N/\{(C/T_c)[(T_c - T_a)/(N+f)]^{0.33}\} + 1/h_{c,\infty}} + \frac{\sigma(T_c{}^4 - T_a{}^4)A_c}{1/[\epsilon_{p,i} + 0.05N(1+\epsilon_{p,i})] + (2N+f-1)/\epsilon_{g,i} - N}} \tag{4.12}$$

where $f = (1 - 0.04h_{c,\infty} + 0.0005h_{c,\infty}{}^2)(1 + 0.091N)$ N = number of covers

$C = 365.9\ (1 - 0.00883\beta + 0.00013\beta^2)$ $h_{c,\infty} = 5.7 + 3.8V$ (m/sec)

$\epsilon_{g,i}$ = infrared emittance of the covers

The values of $q_{\text{top loss}}$ calculated from Eq. (4.12) agreed closely with the values obtained from Eq. (4.11) for 972 different observations encompassing the following conditions:

$$320 < T_c < 420 \text{ K}$$
$$260 < T_a < 310 \text{ K}$$
$$0.1 < \epsilon_{p,i} < 0.95$$
$$0 \leqslant V \leqslant 10 \text{ m/sec}$$
$$1 \leqslant N \leqslant 3$$
$$0 \leqslant \beta \leqslant 90$$

EXAMPLE 3.15

Calculate the rate of heat transfer between the surface of a flat-plate collector ($\epsilon_1 = 0.95$, $T_1 = 360$ K) and the glass cover ($\epsilon_2 = 0.90$, $T_2 = 340$ K) if the convection heat-transfer coefficient is 10 W/m^2 · K (1.76 Btu/hr · ft^2 · °F).

SOLUTION

Radiation and convection occur in parallel between T_1 and T_2.

the convection resistance from Eq. (3.43)

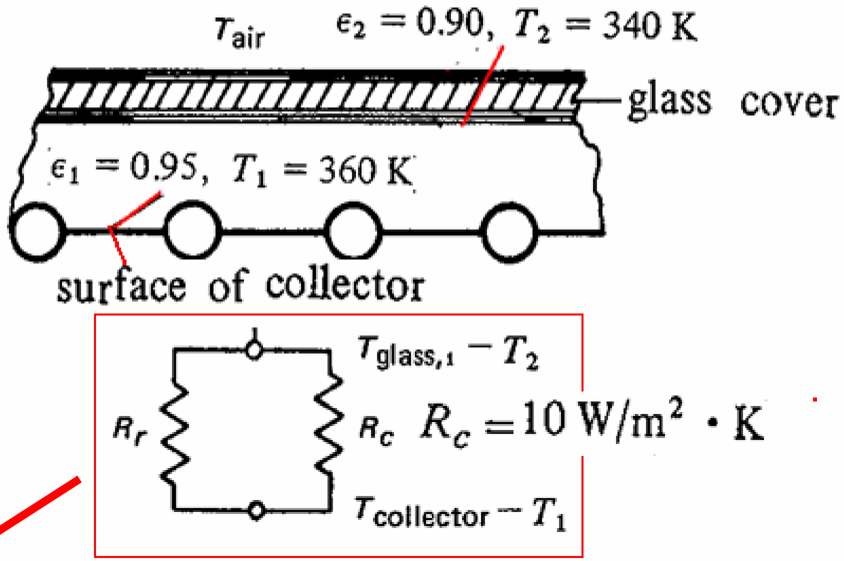
$$R_c = \frac{1}{Ah_c} = 0.1 \text{ m}^2 \cdot \text{K/W } (0.568 \text{ hr} \cdot {}^\circ\text{F/Btu})$$

and from Eq. (3.104) the radiation resistance

$$R_r = \frac{1/\epsilon_1 + 1/\epsilon_2 - 1}{\sigma(T_1 + T_2)(T_1{}^2 + T_2{}^2)}$$

$$= \frac{(1.11 + 1.052 - 1) \times 10^8}{5.67(340 + 360)(1.15 + 1.29) \times 10^5}$$

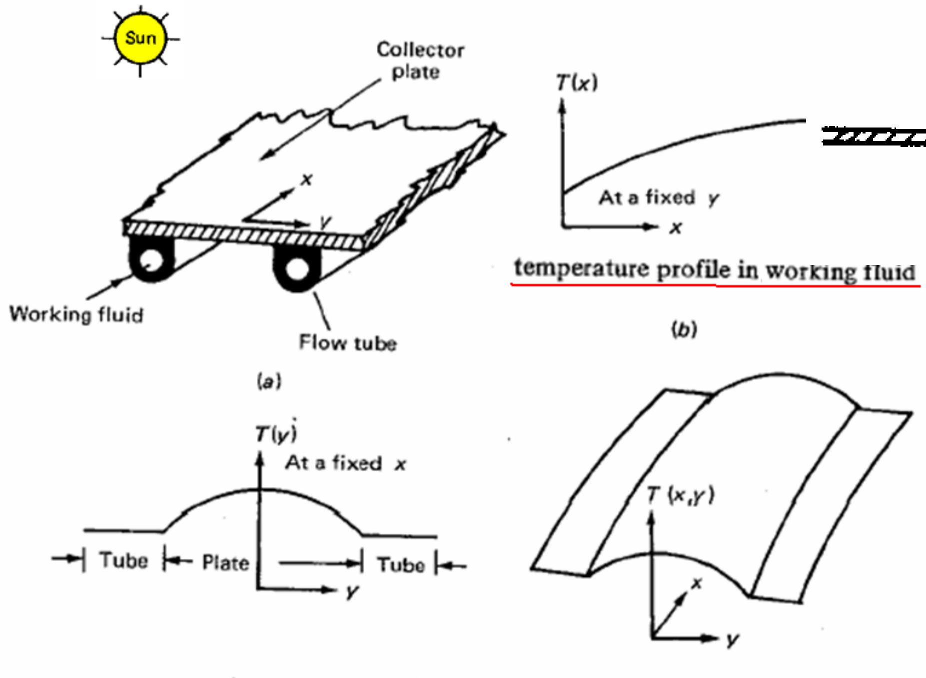
$$= 0.120 \text{ m}^2 \cdot \text{K/W } (0.680 \text{ hr} \cdot {}^\circ\text{F/Btu})$$



flat-plate collector

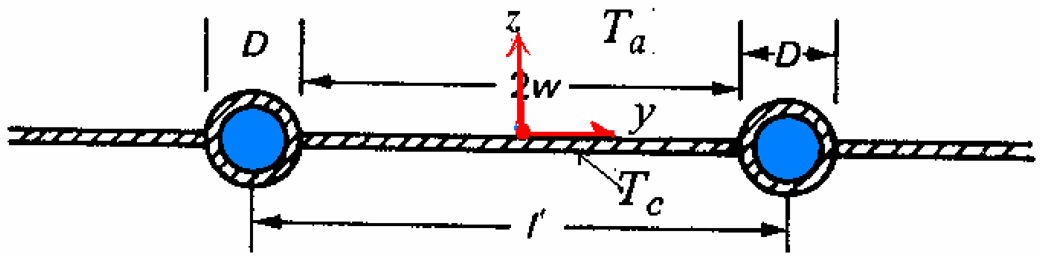
Thus $$\frac{q}{A} = \frac{(T_1 - T_2)(R_r + R_c)}{R_r R_c} = \frac{20/0.012}{0.1 + 0.120}$$

4.1.2 The thermal analysis of a flat-plate collector



temperature profile in working fluid

temperature profile in the absorber plate



solar absober plate

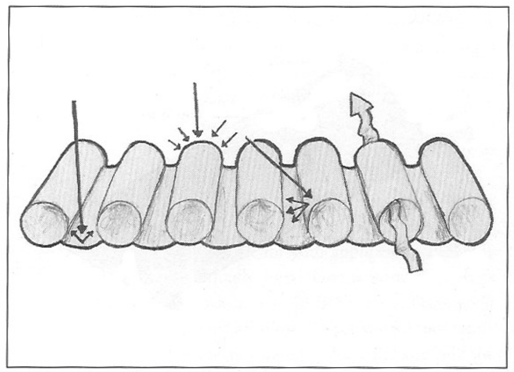


FIGURE 3.10
Web and tube configuration

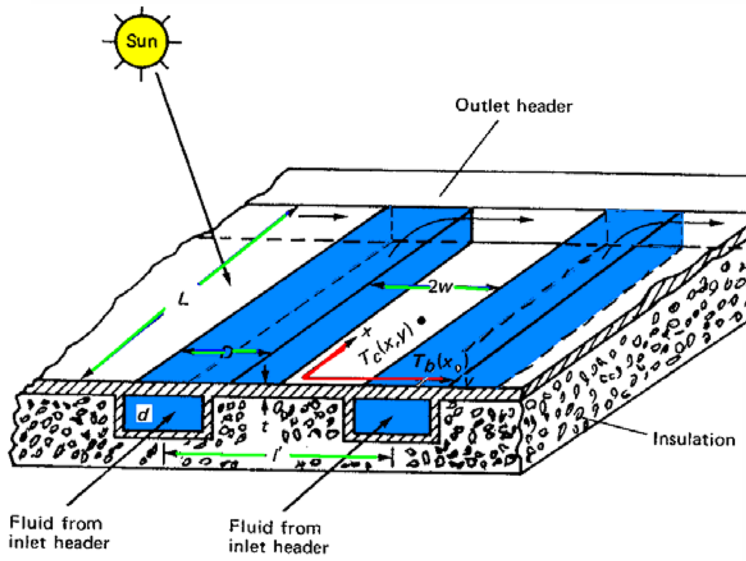
The rate of heat loss from a given segment of the collector plate at x,y is

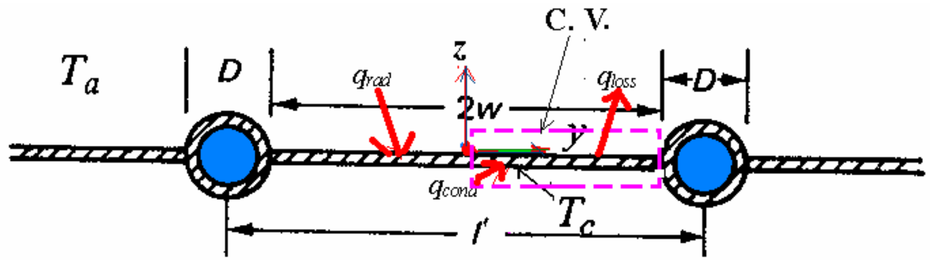
$$\boxed{q(x,y) = U_c\,[T_c(x,y) - T_a]\; dx\; dy} \qquad (4.13)$$

where T_c = local collector-plate temperature ($T_c > T_a$)
T_a = ambient air temperature
U_c = overall unit conductance between the plate and the ambient air

4.1.2 The thermal analysis of a flat-plate collector

To derive a relation to evaluate this efficiency for a flat-plate solar collector





$dq_{rad} - d\,q_{loss} - d\,q_{cond} = 0$

$$\alpha_s I_s\,dy - U_c(T_c - T_a)\,dy + \left(-kt \left.\frac{dT_c}{dy}\right|_{y,x_0}\right) - \left(-kt \left.\frac{dT_c}{dy}\right|_{y+dy,x_0}\right) = 0 \quad (4.14)$$

$$\left.\frac{dT_c}{dy}\right|_{y+dy,x_0} = \left.\frac{dT_c}{dy}\right|_{y,x_0} + \left(\frac{d^2T_c}{dy^2}\right)_{y,x_0} dy$$

$$\alpha_s I_s\,dy - U_c(T_c - T_a)\,dy + kt\,\frac{d^2T_c}{dy^2}\,dy = 0 \quad (4.15)$$

$$\frac{d^2T_c}{dy^2} = \frac{U_c}{kt}\left[T_c - \left(T_a + \frac{\alpha_s I_s}{U_c}\right)\right]$$

The boundary conditions are

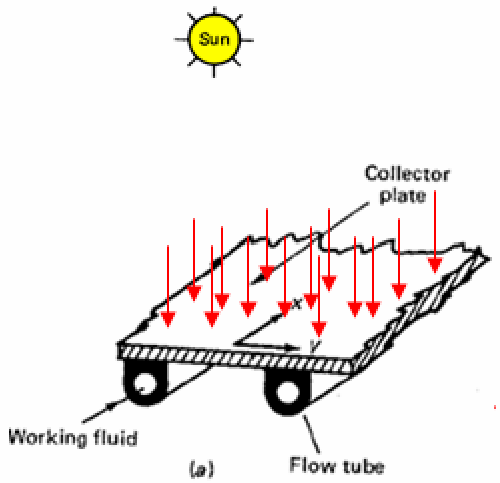
1. $\boxed{\text{at } y = 0,\ dT_c/dy = 0.}$
2. at $y = w = (l' - D)/2,\ T_c = T_b(x_0)$ (4.16)

let $m^2 = U_c/kt$ and $\phi = T_c - (T_a + \alpha_s I_s/U_c)$

$$\frac{d^2\phi}{dy^2} = m^2\phi$$

The general solution of Eq. (4.16) is

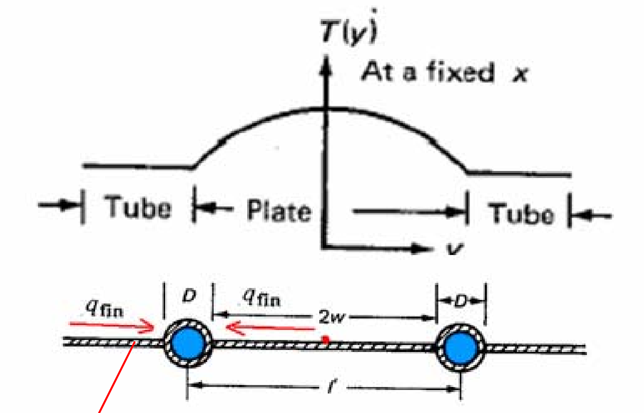
$$\phi = C_1 \sinh my + C_2 \cosh my \quad (4.17)$$

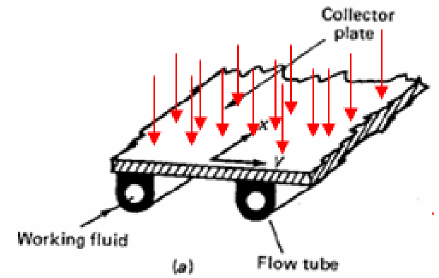


The constants C_1 and C_2 can be determined by substituting the two boundary conditions and solving the two resulting equations for C_1 and C_2. This gives

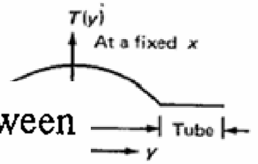
溫度分布

$$\boxed{\frac{T_c-(T_a+\alpha_s I_s/U_c)}{T_b(x_0)-(T_a+\alpha_s I_s/U_c)}=\frac{\cosh my}{\cosh mw}} \tag{4.18}$$





solar absober plate



- the rate of heat transfer to the conduit from the portion of the plate between two conduits

$$\boxed{q_{\text{fin}}=-kt\left.\frac{dT_c}{dy}\right|_{y=w}=\frac{1}{m}\{\alpha_s I_s-U_c[T_b(x_0)-T_a]\}\tanh mw} \tag{4.19}$$

- Since the conduit is connected to fins on both sides, the total rate of heat transfer is

$$\boxed{q_{\text{total}}(x_0)=2w\{\alpha_s I_s-U_c[T_b(x_0)-T_a]\}\frac{\tanh mw}{mw}} \tag{4.20}$$

- If the entire fin were at the temperature $T_b(x_0)$

The fin efficiency $\eta_f \equiv \tanh mw/mw$.

$$\boxed{q_{\text{total}}(x_0)=q_{\text{total, max}}=2w\eta_f\{\alpha_s I_s-U_c[T_b(x_0)-T_a]\}} \tag{4.21}$$

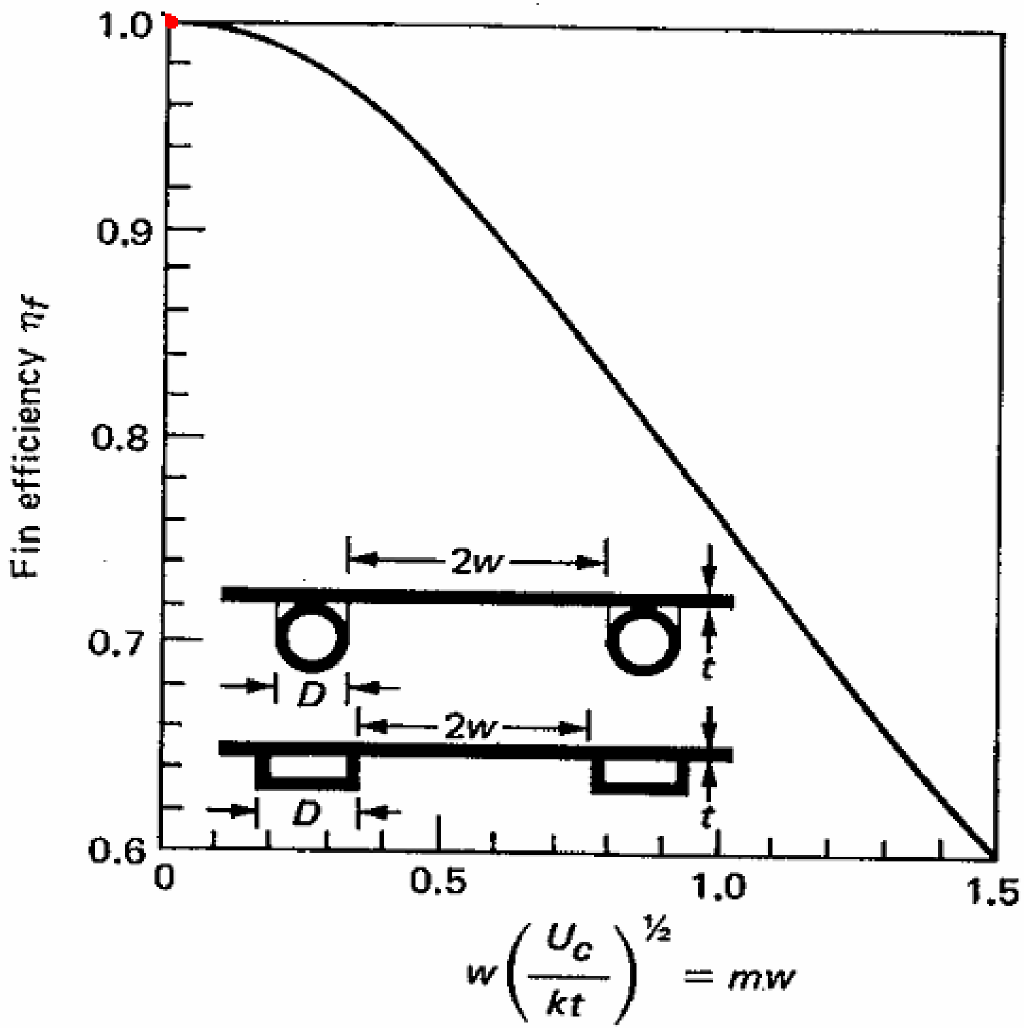
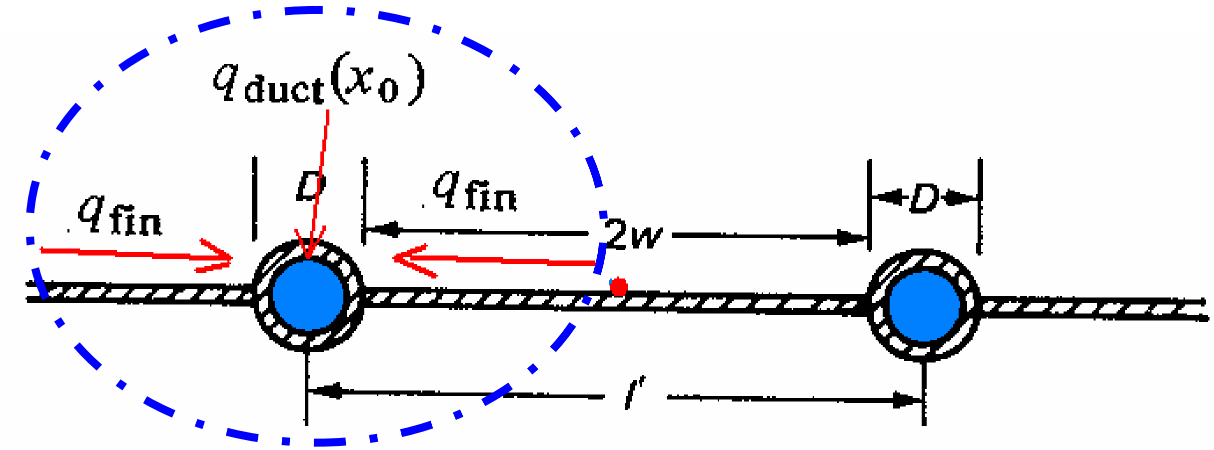


FIGURE 4.5 Fin efficiency for tube and sheet flat-plate solar collectors.

$$U_c = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4 + R_5}$$ 並聯電路

$\eta_f \equiv 1$

☛ When the fin efficiency approaches unity, the maximum portion of the radiant energy impinging on the fin becomes available for heating the fluid.



- In addition, the energy impinging on the portion of the plate above the flow passage provides useful energy to heat the working fluid.

$$\boxed{q_{duct}(x_0) = D\{\alpha_s I_s - U_c[T_b(x_0) - T_a]\}} \tag{4.22}$$

- Thus, the total useful energy per unit length in the flow direction becomes

$$q_u(x_0) = 2 \cdot q_{fin} + q_{duct}(x_0)$$

$$\boxed{q_u(x_0) = (D + 2w\eta)\{\alpha_s I_s - U_c[T_b(x_0) - T_a]\}} \tag{4.23}$$

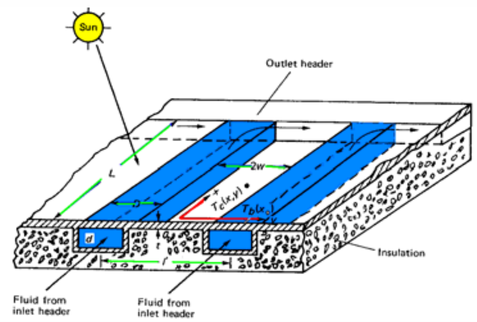
- Thus, the useful energy per unit length transferred as heat to the working fluid becomes

$$\boxed{q_u(x_0) = (D + 2w\eta)\{\alpha_s I_s - U_c[T_b(x_0) - T_a]\}}$$

or

$$\boxed{q_u(x_0) = 2(D + d)\bar{h}_{c,i}[T_b(x_0) - T_f(x_0)]}$$

(矩形管)



$$\Rightarrow \quad T_b(x_0) = \frac{q_u(x_0)}{2(D + d)\bar{h}_{c,i}} + T_f(x_0)$$

$$q_u(x_0) = (D + 2w\eta)\{\alpha_s I_s - U_c[T_b(x_0) - T_a]\}$$

$$= (D + 2w\eta)\{\alpha_s I_s - U_c[\frac{q_u(x_0)}{2(D + d)\bar{h}_{c,i}} + T_f(x_0) - T_a]\}$$

$$\frac{q_u(x_0)}{(D + 2w\eta)} = -U_c[\frac{q_u(x_0)}{2(D + d)\bar{h}_{c,i}}] + \{\alpha_s I_s + T_f(x_0) - T_a]\}$$

$$\Rightarrow \boxed{q_u(x_0) = \frac{\{\alpha_s I_s + T_f(x_0) - T_a]\}}{1/U_c(D + 2w\eta_f) + 1/\bar{h}_{c,i}(2D + 2d)} = l'F'\{\alpha_s I_s - U_c[T_f(x_0) - T_a]\}}$$

Collector Efficiency Factor

$$\boxed{F' = \frac{1/U_c l'}{1/U_c(D + 2w\eta_f) + 1/\bar{h}_{c,i}(2D + 2d)}}$$

is called the collector efficiency factor

- The collector-plate efficiency factor F' depends on U_c, $\bar{h}_{c,i}$, and η_f. only slightly dependent on temperature

- Typical values for the factors determining the value of F' are given in Table 4.1.

TABLE 4.1 Typical Values for the Parameters That Determine the Collector Efficiency Factor F' for a Flat-Plate Collector in Eq. (4.26)

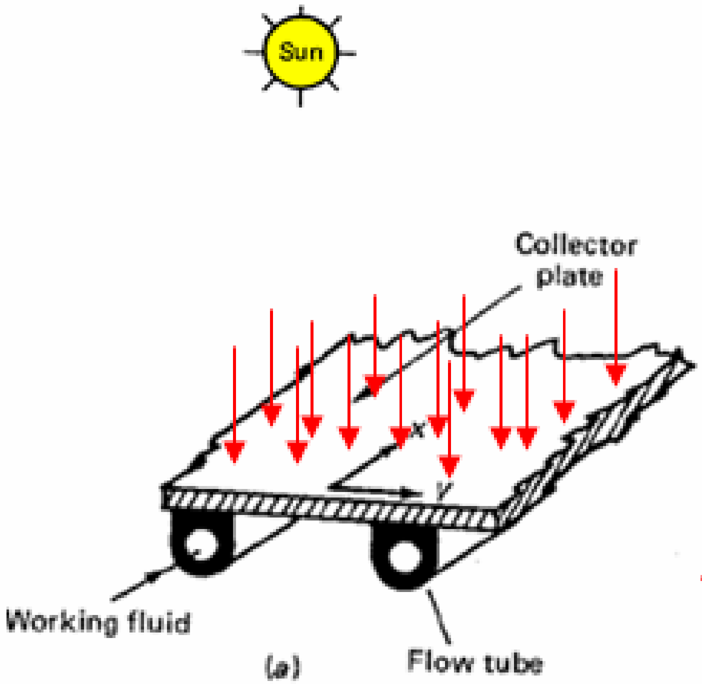
	U_c	
2 glass covers	4 W/m² · K	0.685 Btu/hr · ft² · °F
1 glass cover	8 W/m² · K	1.37 Btu/hr · ft² · °F
	kt	
copper plate, 1 mm thick	0.4 W/K	41.5 Btu/hr · °F
steel plate, 1 mm thick	0.005 W/K	0.52 Btu/hr · °F
	$\bar{h}_{c,i}$	
water in laminar flow forced convection	300 W/m² · K	52 Btu/hr · ft² · °F
water in turbulent flow forced convection	1500 W/m² · K	254 Btu/hr · ft² · °F
air in turbulent forced convection	100 W/m² · K	17.6 Btu/hr · ft² · °F

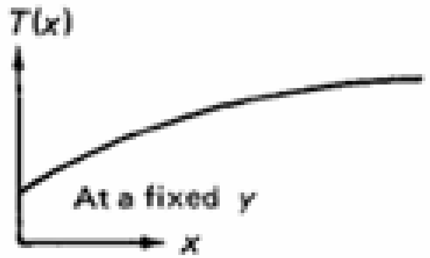
- $F' \propto kt \quad \bar{h}_{c,i} \quad 1/l'$

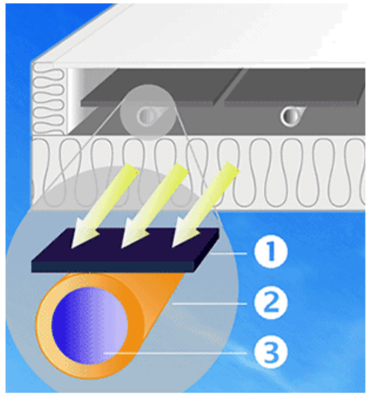
• *Collector Heat-Removal Factor*

However, in a real collector the fluid temperature increases in the direction of flow as heat is transferred to it.

An energy balance for a section of flow duct dx can be written in the form

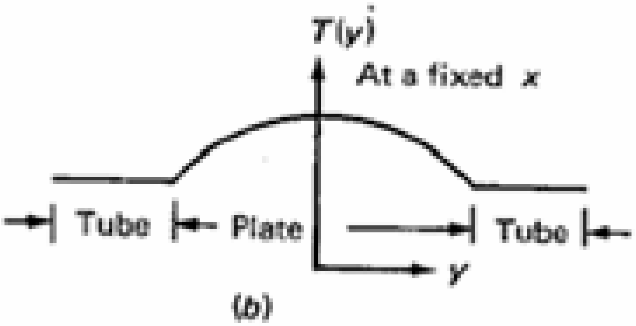




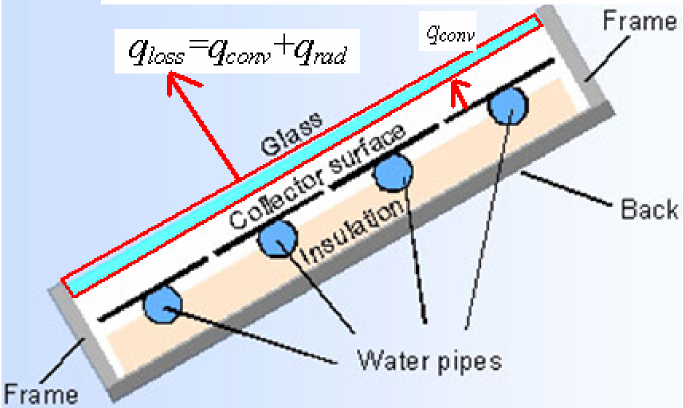


temperature profile in working fluid

(a)



- Similarly, a heat balance on the collector cover gives



$$q_{conv}-q_{top\ loss}=dq/dt=(mc)_g d\bar{T}_g(t)/dt$$

$$q_{conv}=A_cU_p[\bar{T}_p(t)-\bar{T}_g(t)]$$

$$q_{top\ loss}=q_{conv}+q_{rad}=A_c\bar{h}_{c2}(\bar{T}_g(t)-T_a)+\frac{\sigma A_c(\bar{T}_g^4(t)-T_a^{\ 4})}{1/\epsilon_{p,i}+1/\epsilon_{g2,i}-1} \quad (4.7)$$

$$=(\bar{h}_{c2}+h_{r2})A_c(\bar{T}_g(t)-T_a)=A_cU_\infty[\bar{T}_g(t)-T_a]$$

$$h_{r2}=\frac{\sigma(\bar{T}_g(t)-T_a)(\bar{T}_g^2(t)-T_a^2)}{(1/\epsilon_{p,i}+1/\epsilon_{g2,i}-1)} \qquad U_\infty=(\bar{h}_{c2}+h_{r2})$$

- $$\boxed{(mc)_g\frac{d\bar{T}_g(t)}{dt}=A_cU_p[\bar{T}_p(t)-\bar{T}_g(t)]-A_cU_\infty[\bar{T}_g(t)-T_a]} \quad (4.35)$$

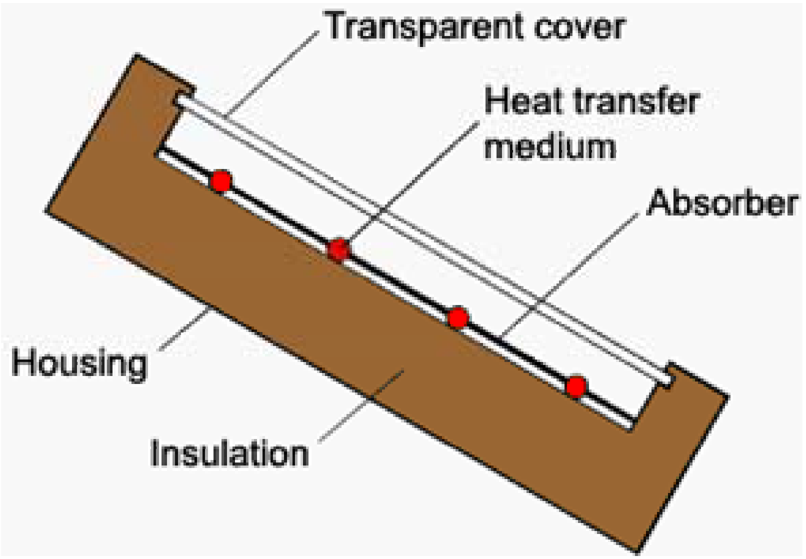
$U_\infty=(h_{c,\infty}+h_{r,\infty})$

$(mc)_g$ = thermal capacity of the cover plate

EXAMPLE 4.2

Calculate the temperature rise between 8 and 10 a.m. of a 1 m $\times$ 2 m single-glazed water collector with a 0.3-cm-thick glass cover if the heat capacities of the plate, water, and back insulation, are 5, 3, and 2 kJ/K, respectively. Assume that the unit surface conductance from the cover to ambient air is 18 W/m^2 · K and the unit surface conductance between the collector and the ambient air is $U_c = 6$ W/m^2 · K. Assume that the collector is initially at the ambient temperature. The absorbed insolation $\alpha_s I_s$ during the first hour averages 90 W/m^2 and between 9 and 10 a.m., 180 W/m^2. The air temperature between 8 and 9 a.m. is 273 K and that between 9 and 10 a.m., 278 K.

SOLUTION



The combined collector, water, and insulation thermal capacity is equal to

$$(\overline{mc})_p + \frac{U_c}{U_\infty}(mc)_g = 5 + 3 + 2 + \frac{6}{18}\,15 = 15\ \mathrm{kJ/K}$$

the temperature rise of the collector between 8 and 9 a.m. is

$$\bar{T}_p(t) - T_a = \frac{\alpha_s I_s}{U_c} - \left[\frac{\alpha_s I_s}{U_c} - (T_{p,0} - T_a)\right] \exp - \left[\frac{U_c A_c t}{(\overline{mc})_p + (U_c/U_\infty)_c (mc)_g}\right]$$

$$= \frac{\alpha_s I_s}{U_c} \left\{1 - \exp - \left[\frac{U_c A_c t}{(\overline{mc})_p + (U_c/U_\infty)(mc)_g}\right]\right\} \qquad \left(\because \overset{\text{at the initial}}{T_{p,0}} = T_a\right)$$

$$= \frac{90}{6}\left[1 - \exp - \left(\frac{2 \times 6 \times 3600}{15{,}000}\right)\right] = 15 \times 0.944 = 14.2\ \mathrm{K}$$

Thus, at 9 a.m. the collector temperature will be

$$\bar{T}_p = 14.2\ \mathrm{K} + T_a = 14.2\ \mathrm{K} + 273\ \mathrm{K} = 287.2\ \mathrm{K}$$

Between 9 and 10 a.m. the collector temperature will rise as

$$T_p = T_a + \frac{\alpha_s I_s}{U_c} - \left[\frac{\alpha_s I_s}{U_c} - (T_{p,0} - T_a)\right] \exp - \left[\frac{U_c A_c t}{(\overline{mc})_p + (U_c/U_\infty)(mc)_g}\right]$$

$$= 278 + \frac{180}{6} - (30 - 9.2)0.056 = 306.8\ \mathrm{K}\ (91^\circ\mathrm{F})$$

The basic air-cooled flat-plate collector

(i) Since poor heat-transfer properties of air (the convection heat-transfer coefficient $h_{water} >> 50\ h_{air}$), it is essential to provide the largest heat-transfer area possible to remove heat from the absorber surface of an air-heating collector. The heat-transfer

cess needs an unsymmetrically heated duct of large aspect ratio

(typically 20–40). 高長寬比矩形管道

(ii) Malik and Buelow (38, 39) a suitable expression for the Nusselt number for a smooth air-heating collector is

$$\mathrm{Nu}_{sm} = \frac{0.0192\ \mathrm{Re}^{3/4}\ \mathrm{Pr}}{1 + 1.22\ \mathrm{Re}^{-1/8}\ (\mathrm{Pr} - 2)} \qquad (4.45)$$

For rough-surface air-heating collector., they recommended multiplying a friction factor f_{sm} (Blasius equation)

$$f_{sm} = 0.079\ \mathrm{Re}^{-1/4} \qquad (4.46)$$

$$\mathrm{Nu}_{sm} = 0.079\ \mathrm{Re}^{-1/4}\ \frac{0.0192\ \mathrm{Re}^{3/4}\ \mathrm{Pr}}{1 + 1.22\ \mathrm{Re}^{-1/8}\ (\mathrm{Pr} - 2)}$$

the hydraulic diameter D_H in the Nusselt and Reynolds numbers is based on the *entire duct perimeter.* $D_h \equiv \frac{4A_c}{P}$

Air-Collector Efficiency Factor

$$F' = \frac{\text{energy collection}}{\text{the collection rate if the absorber plate were at the local fluid temperature.}}$$

$$F' = \frac{\bar{h}_c}{\bar{h}_c + U_c} \qquad (4.47)$$

where

$$\mathrm{Nu}_{sm} = \frac{0.0192\ \mathrm{Re}^{3/4}\ \mathrm{Pr}}{1 + 1.22\ \mathrm{Re}^{-1/8}\ (\mathrm{Pr} - 2)} = \frac{\bar{h}_c D_H}{k} \qquad (4.45)$$

$$q_{\text{top loss}} = \frac{(T_c - T_a)A_c}{N/\{(C/T_c)[(T_c - T_a)/(N + f)]^{0.33}\} + 1/h_{c,\infty}}$$

$$+ \frac{\sigma(T_c{}^4 - T_a{}^4)A_c}{1/[\epsilon_{p,i} + 0.05N(1 + \epsilon_{p,i})] + (2N + f - 1)/\epsilon_{g,i} - N} \qquad (4.12)$$

$$= U_c A_c(T_c - T_a)$$

• Air-Collector Heat-Removal Factor F_R

$$F_R = \frac{\text{the actual rate of heat transfer to the working fluid}}{\text{the rate of heat transfer at the minimum temperature difference between the absorber and the environment}}$$

$$F_R = \frac{Gc_p(T_{f,\,\text{out}} - T_{f,i})}{\alpha_s I_s - U_c(T_{f,i} - T_a)} \tag{4.31}$$

$$F_R = \frac{\dot{m}_a c_{p,a}}{U_c A_c}\left[1 - \exp\left(-\frac{F' U_c A_c}{\dot{m}_a c_{p,a}}\right)\right] \tag{4.48}$$

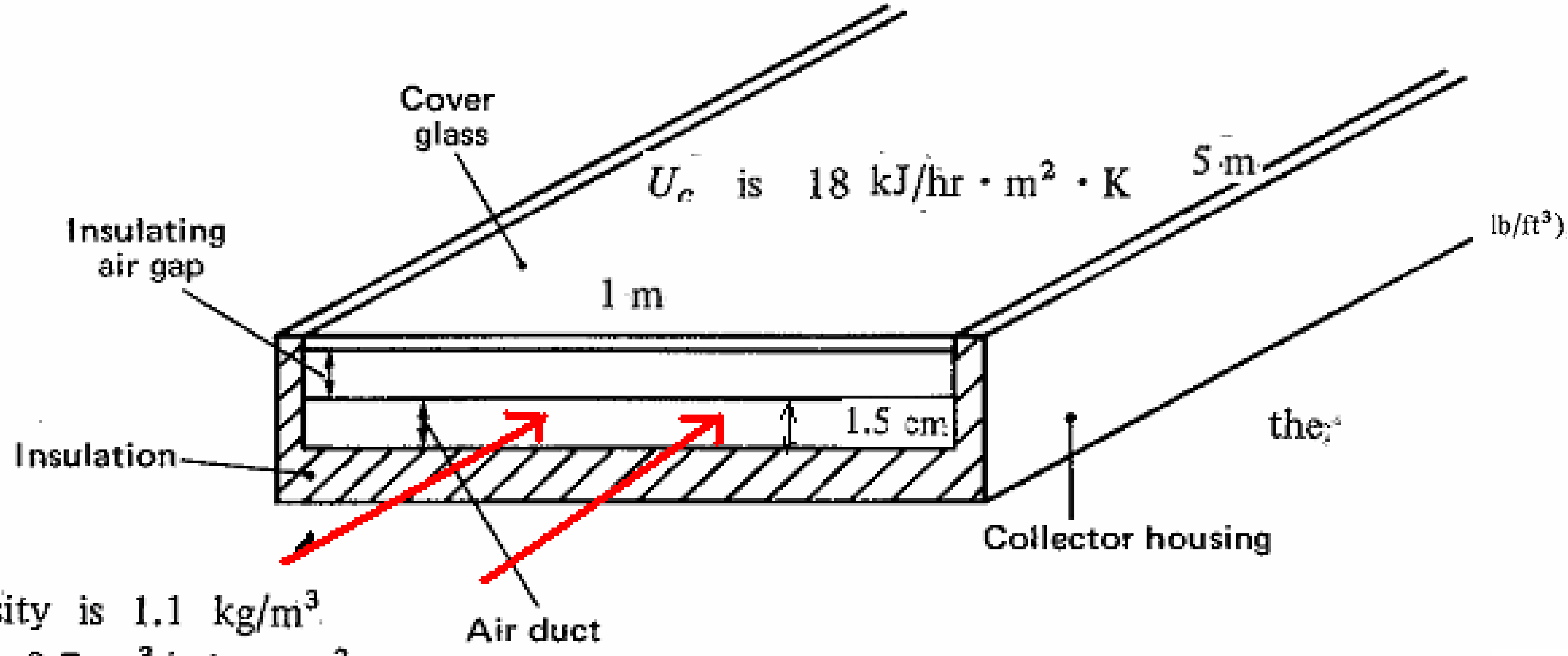
$\dot{m}_a$ is the air flow rate (kg/sec)

$c_{p,a}$ is its specific heat

The heat-removal factor for air collectors is usually significantly less than that for liquid collectors because h_c for air is much smaller than for a liquid such as water.

EXAMPLE 4.3

Calculate the collector-plate efficiency factor F' and heat-removal factor F_R for a smooth, 1-m-wide, 5-m-long air collector with the following design.



air density is 1.1 kg/m³

flow rate is 0.7 $m^3/min \cdot m_c^2$

specific heat is 1 kJ/kg · K

viscosity is 1.79×10^{-5} kg/m · sec

SOLUTION

The first step is to determine the duct heat-transfer coefficient h_c from Eq. (4.45). The Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho \bar{V} D_H}{\mu}$$

in which the average velocity $\bar{V}$ is the volume flow rate divided by the flow area.

$$\bar{V} = \frac{0.7 \times 1 \times 5}{1 \times 0.015} = 233 \text{ m/min} = 3.89 \text{ m/sec}$$

and the hydraulic diameter D_H is

$$D_H = \frac{4(0.015 \times 1)}{1 + 1 + 0.015 + 0.015} = 0.0296 \text{ m}$$

$$\mathrm{Re} = \frac{(1.1)(3.89)(0.0296)}{1.79 \times 10^{-5}} = 7066$$

From Eq. (4.45) the Nusselt number is

$$\mathrm{Nu}_{sm} = \frac{0.0192\, \mathrm{Re}^{3/4}\, \mathrm{Pr}}{1 + 1.22\, \mathrm{Re}^{-1/8}\, (\mathrm{Pr} - 2)} = \frac{k\, h_c}{D_H}$$

$$= \frac{0.0192(7066)^{3/4}(0.72)}{1 + 1.22(7066)^{-1/8}(0.72 - 2.0)} = 22.0$$

The heat-transfer coefficient is

$$h_c = \mathrm{Nu}\frac{k}{D_H} = \frac{\mathrm{Nu}\, c_p \mu}{\mathrm{Pr}\, D_H}$$

The Prandtl number for air is ~ 0.72. Therefore,

$$h_c = \frac{(22.0)(1.0)(1.79 \times 10^{-5})}{(0.72)(0.0296)} = 0.0185 \text{ kJ/m}^2 \cdot \text{sec} \cdot \text{K}$$

$$= 66.5 \text{ kJ/m}^2 \cdot \text{hr} \cdot {}^\circ\text{C} \ (3.26 \text{ Btu/hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F})$$

The plate efficiency is then, from Eq. (4.47),

$$F' = \frac{\bar{h}_c}{\bar{h}_c + U_c} = \frac{66.5}{66.5 + 18} = 0.787$$

The mass flow rate per unit area is

$$\frac{\dot{m}}{A_c} = \frac{\rho q}{A_c} = \frac{1.1 \times 0.7}{60} = 0.0128 \text{ kg/sec} \cdot \text{m}_c^2$$

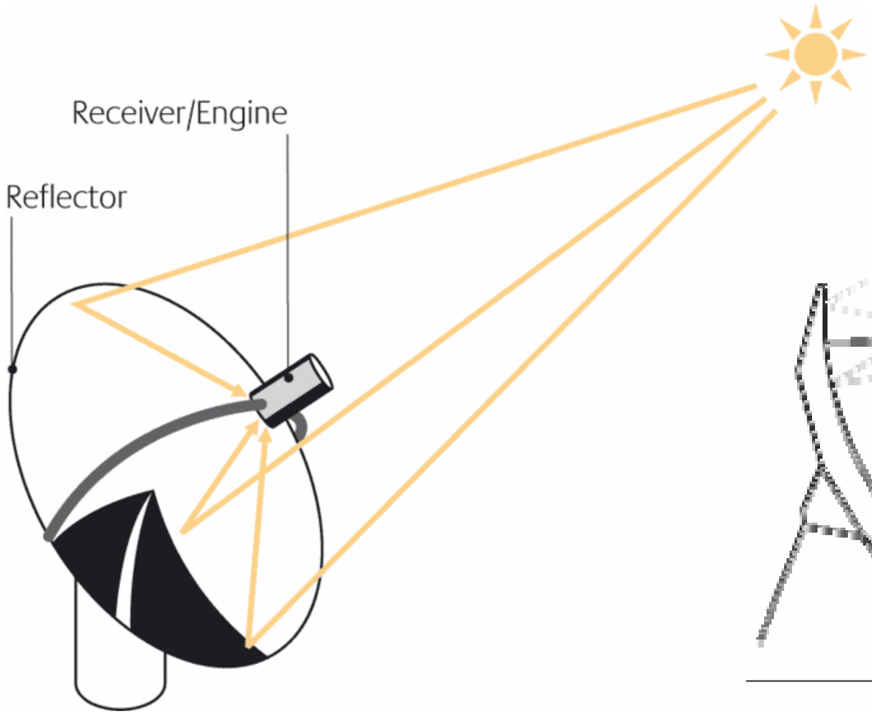
the heat-removal factor F_R can be calculated from Eq. (4.48).

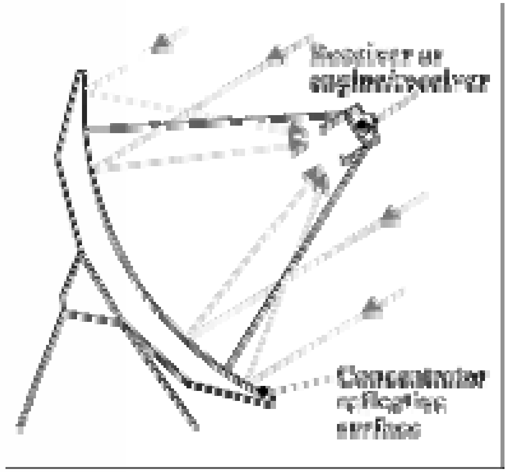
$$F_R = \frac{\dot{m}_a c_{p,a}}{U_c A_c}\left[1 - \exp\left(-\frac{F' U_c A_c}{\dot{m}_a c_{p,a}}\right)\right]$$

$$= \frac{(0.0128)(1)}{(18/3600)}\left\{1 - \exp - \left[\frac{(0.787)(18/3600)}{(0.0128)(1)}\right]\right\} = 0.677$$

4. 3 Introduction to Concentrating Solar Energy Collectors

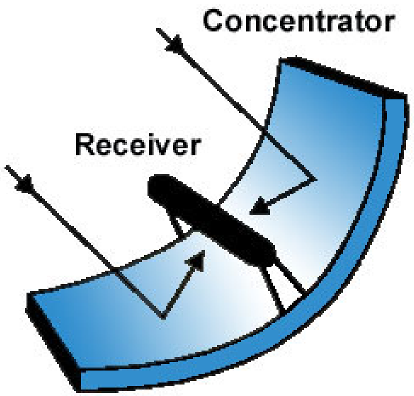
太陽能集中收集器

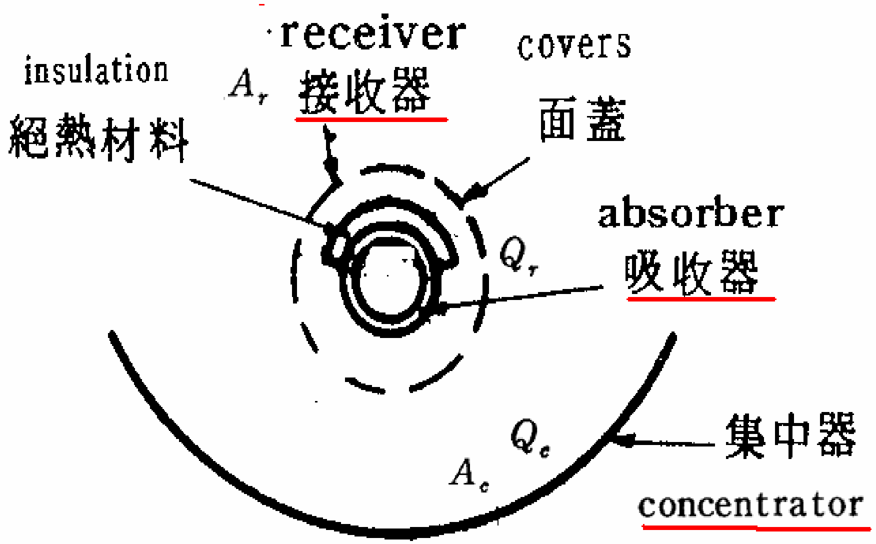




Solar collection devices that increase the receiver surface flux intensity over that incident on the collector aperture are called concentrators.

集中比由1.5或2至10000倍，亦即能使溫度提高





太陽能集中收集器

(1) 接收器（ receiver ），它包括吸收太陽輻射轉換成另外能量形式的吸收器（ absorber ），以及面蓋、絕熱材料等。

(2) 集中器（ concentrator ），將直達輻射，經反射或折射，使之聚集到吸收器上者。

(A) 反射集中器

(a) 圓錐反射面鏡，圓柱形吸收器，最常見用于太陽能烹調器，或利用爲蒸汽發生器。

(b) 與(a)類似，但經多次反射集中，集中比可由多外接反射面鏡面增大。

(c) 拋物線面（柱形）或拋物線體（球形）之集中器，爲最常見之集中器，集中比範圍大

(d) 與(c)同，只是吸收器形狀不同之拋物線集中器。

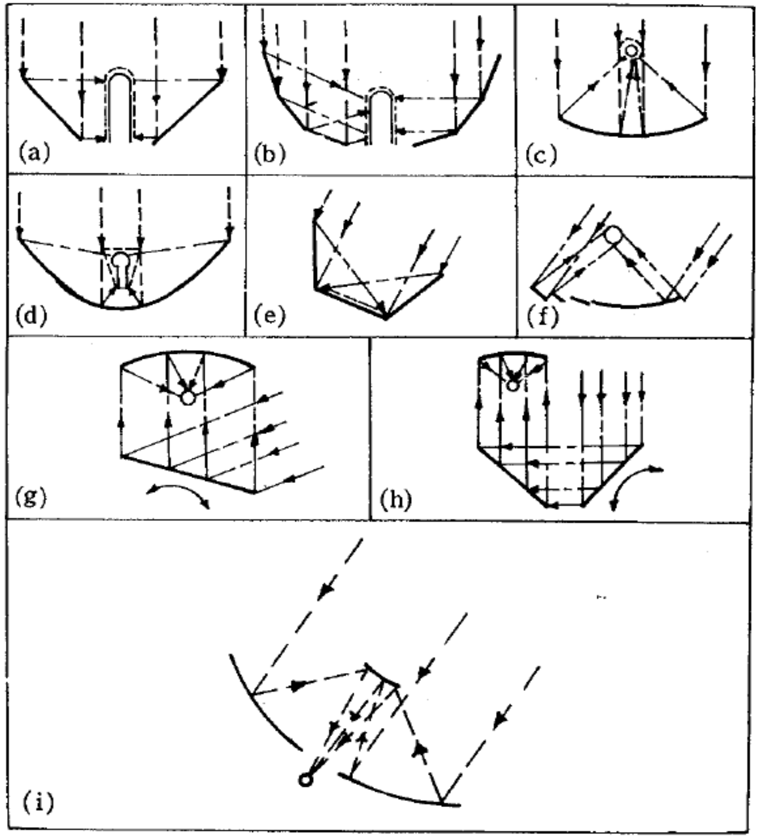
(e) 平面反射、平面吸收之集中器，此種 $C \leq 4$，不過因平面反射，部分漫射也會爲吸收器吸收，反射面鏡增大，可增大集中比。

(f) 分段鏡面反射集中，分段鏡面（mirror segments）可成拋物面或平面，以集中入射輻射，優點爲不必太精確對準太陽，缺點爲損失部分可用之反射面積。Fresnel concentrator

(g) (h)補助反射平面鏡之集中器（heliostat），此種型式，可將拋物面集中器之光軸水平或垂直放置，補助之平面反射鏡可以一面或數面裝置。

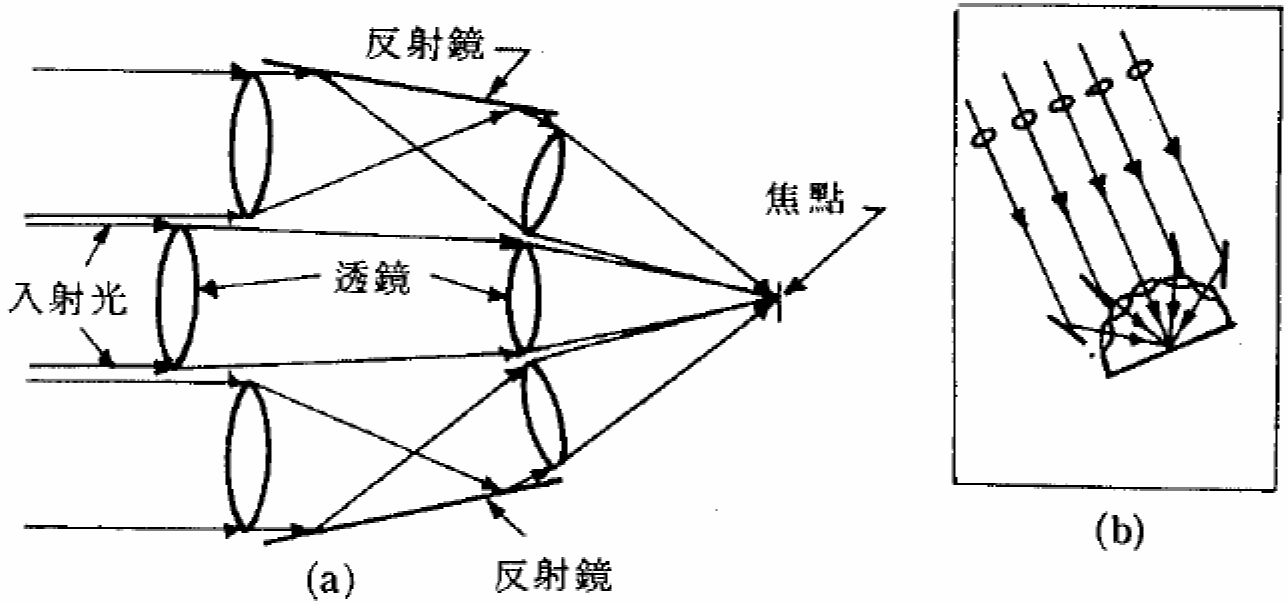
(i) 拋物線體之二次反射集中器，此種優點爲可將聚焦位置改變，方便放置吸收器。

(j) 其他之不必追踪太陽之SRTA，半圓柱形，CPC，等反射集中器，

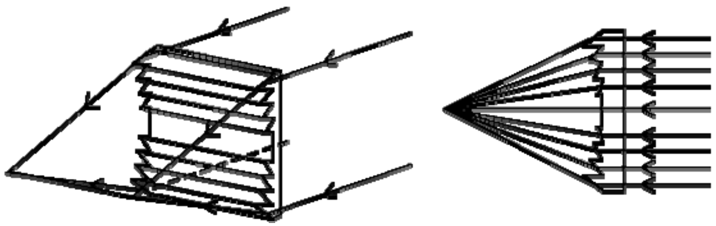


(B) 折射集中

(a) 透鏡集中器，或透鏡組輔以反射鏡面之集中器 (a)，為美國加州理工學院（ CIT ）之設計，或 (b) 所示者。



透鏡集中器



集中器優點爲：

(1) 反射集中器之反射面雖然製造上較精密，但所須材料少，結構單純，因此以單位太陽能收集面積之成本計算，較平板收集器便宜。

(2) 所得能量密度較平板型爲高。

(3) 因吸收器面積較入射到集中器之面積小，散熱損失少，且日射集中，因此作用流體溫度在相同 A_c 下，集中器可得溫度較平板型爲高。

(4) 由于可得較高溫度，可用于較高溫度系統，並可得較高系統效率。

(5) 因較高溫，單位體積之熱儲存量較大，因此熱儲存成本較低。

集中器之缺點：

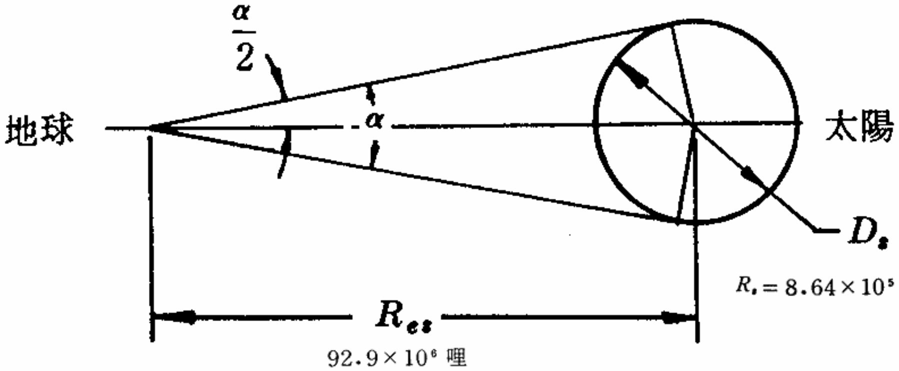
(1) 比平板收集器收集較少之漫射輻射（一般直達日射與漫射日射之比約爲3：5）。

(2) 有些集中器需追踪並對準太陽。

(3) 反射面之反射率會隨使用時間而變化，且需定期擦光或磨光。

(4) 集中器之操作經驗仍不足，需科技上繼續努力。但是，集中器之最大特點在得到高溫，高系統熱效率，這是平板收集器絕對做不到的優點，因此它的使用于某種系統有時是不可或缺的。

● solar image(太陽影像)



地球－太陽之幾何關係

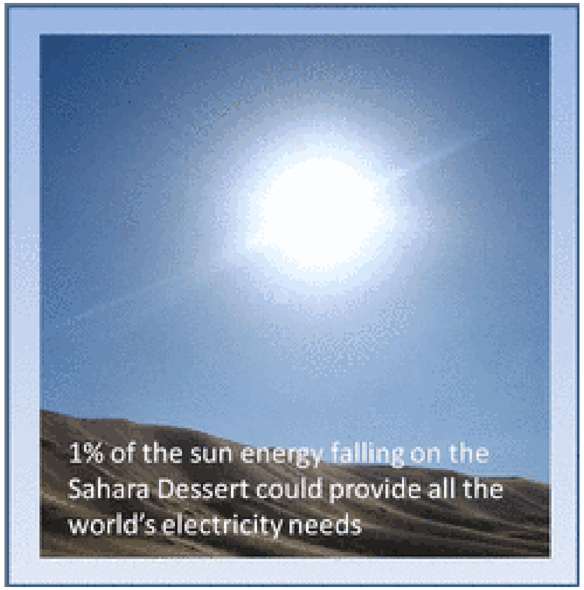
solar intercept angle（太陽截角）α

$$\sin\frac{\alpha}{2} = \frac{R_s}{R_{es}} = \frac{\text{太陽平均直徑}}{\text{地球與太陽平均距離}}$$

$$= \frac{8.64 \times 10^5}{92.9 \times 10^6 \times 2} = 0.00467$$

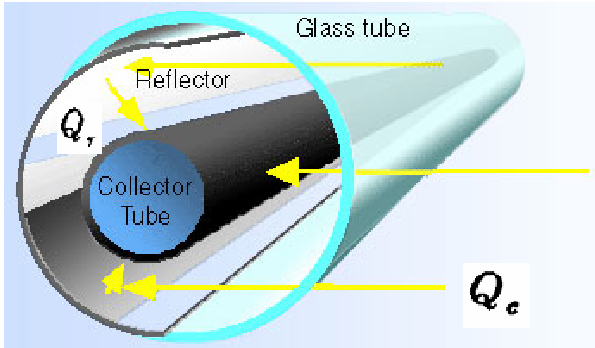
$$\therefore \alpha = 32'$$

● 32′夾角對平面之反射不重要，但是對于曲面反射聚焦時相當重要，它是集中比（concentration ratio）之極限原因，也就是不可能反射到焦點後，將輻射聚成一點的原因，因爲“點”的面積爲零，如能聚焦成點，則焦點上之輻射強度爲相當大，溫度亦將無限高，當然實際上這是不可能的。



concentration ratio

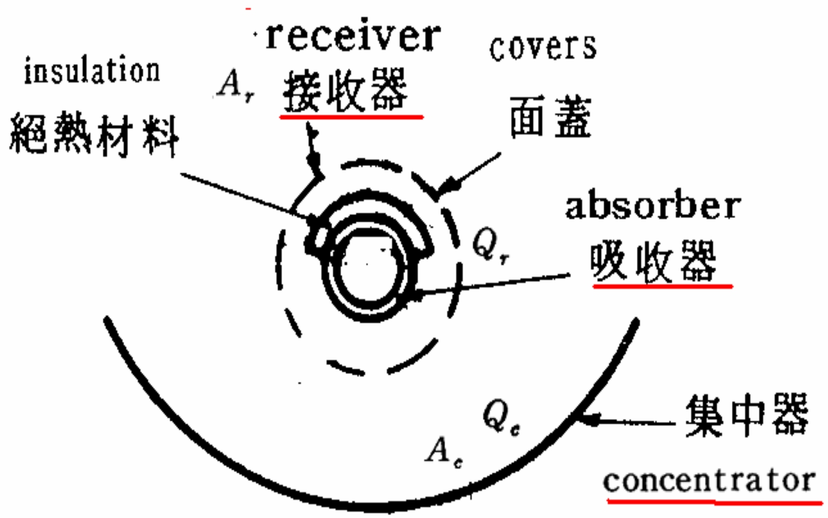
CR 集中比



假設一具理想反射集中器

入射到反射鏡之能量爲Q_c = 由此鏡面反射到接收器之能量爲Q_r

$$Q_c = Q_r$$



太陽能集中收集器

整理 $$Q_c \frac{A_c}{A_c} = Q_r \frac{A_r}{A_r}$$

$$\therefore \quad \frac{Q_r / A_r}{Q_c / A_c} = \frac{A_c}{A_r} = CR$$

定義 $$CR = \frac{A_c}{A_r} = \frac{Q_r / A_r}{Q_c / A_c}$$

CR: 集中比

A_c : 集中器之有效投影面積

A_r : 接收器之表面積

concentration ratio CR

集中比有下列意義：

(1) 它是A_c/A_r的幾何面積比，可做爲成本計算（ 因成本與面積成正比），集中比越高則成本亦高。

(2) 集中比是吸收器之能量密度與入射到集中器之能量密度之比，即（Q_r/A_r）/（Q_c/A_c），這種說法可知道經過集中器後之能量供應量，或可到達之溫度。

平板收集器因$A_c/A_r=1$，即集中比爲1，無法增大能量密度，但是集中器就可把能量密度增大，且因散熱面積（即吸收器面積）較入射面積小，因此散熱損失小，不過因A_c大，反射損失、形狀因素損失及無法收集之散射光損失相對增高，所以C值增大，截取因素會下降，因此設計時要考慮各種因素，以便決定所要達到之溫度。

拋物線集中收集器之集中比

(1) 拋物線面（ parabolic cylinder ），吸收面為平板，平板寬度恰為影像直徑，如圖 5-7

$C = A_c / A_r = (a \times L)/(d \times L) = a/d$

a：反射鏡之開口大小（ aperture ）。

L：長度

d：吸收器寬度（即太陽影像直徑）。

n

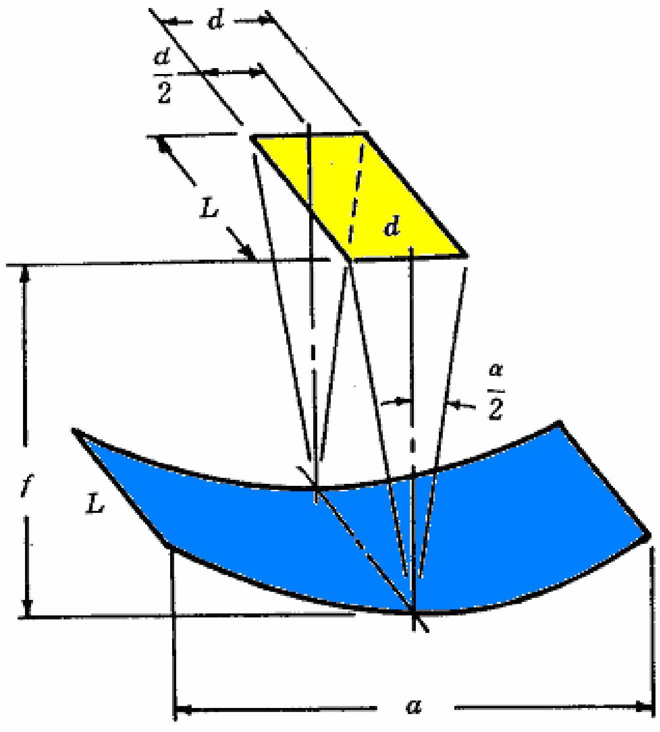
$$\tan\frac{\alpha}{2} = \frac{d/2}{f} = \frac{d}{2f} \qquad (\alpha = 32' = 0.5339^\circ)$$

$$d = f\left(2\tan\frac{\alpha}{2}\right)$$

$$C = a/d = \frac{nf}{f\left(2\tan\frac{\alpha}{2}\right)} = \frac{n}{2\tan\frac{\alpha}{2}}$$

$$\doteqdot 107.3n = 107.3\,a/f \ (n \le 0.5) \quad a/f = 0.5$$

可見此種反射集中器之理論集中比大小。



拋物線面反射鏡——影像直徑寬度之吸收面

(6)　抛物線體集中器，圓盤平板吸收器

$$C=\left(\frac{\pi}{4}a^2\right)/\left(\frac{\pi}{4}d_a{}^2\right)=a^2/d_a{}^2$$

$$d_a\approx\frac{f\left(1+\frac{n^2}{16}\right)^2\left(2\tan\frac{\alpha}{2}\right)}{\left(1-\frac{n^2}{16}\right)}$$

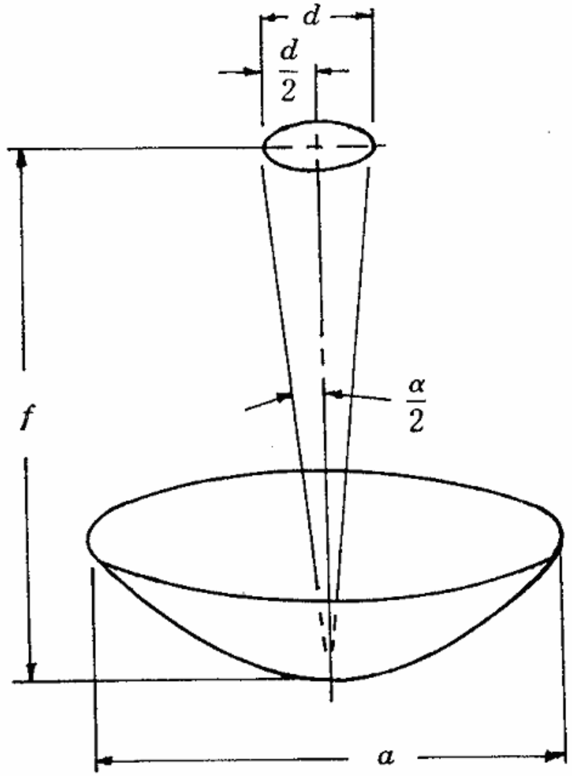
$$C=\frac{(nf)^2}{\dfrac{\left[f\left(1+\frac{n^2}{16}\right)^2\left(2\tan\frac{\alpha}{2}\right)\right]^2}{\left(1-\frac{n^2}{16}\right)^2}}$$

$$\approx\frac{11{,}550\ n^2\left(1-\frac{n^2}{16}\right)^2}{\left(1+\frac{n^2}{16}\right)^4}$$

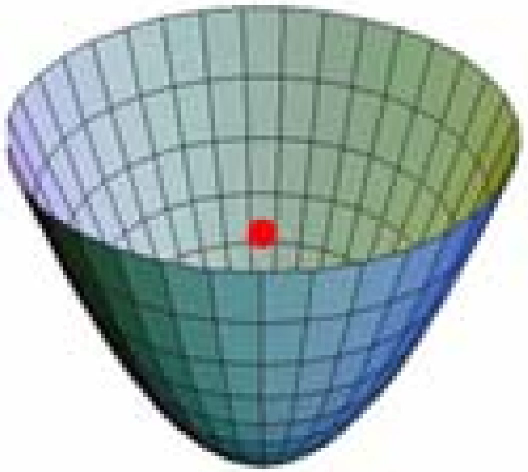
最大集中比 $C_{\max}$ ， $\dfrac{dC}{dn}=0$

$$n^4-96n^2+256=0\qquad n=1.65$$

$$C_{\max}=11.550$$



抛物線體集中器 —— 圓板吸收器



- ## Reasons for Using Concentrating Collectors

Three reasons are commonly cited for using concentrators:

1. To increase energy delivery temperatures in order to achieve a thermodynamic match between temperature level and task. The task may be to operate thermionic, magnetohydrodynamic, thermodynamic, or other higher temperature devices.
2. To improve thermal efficiency by reducing the heat-loss area relative to the receiver area. There would also be a reduction in transient effects, since the thermal mass is usually much smaller than for a flat-plate collector.
3. To reduce cost by replacing an expensive receiver by a less expensive reflecting or refracting area.

Flat-plat solar collector   T=384°k η_{carnot}=20%

Solar concentrator   T=580°k η_{carnot}=40%

Intermittently Turned Concentrators-The Compound parabolic Concentrator (CPC) Family

- The compound parabolic concentrator (CPC) family developed by Winston (60-62) and Rabl (41-44), and described by Baranov and Melnikov (2), will be described in detail.
- The CPC family can achieve in practice the upper bound of concentration ratio as set by the laws of physics. Most other concentrators fall short of the ideal limit by a factor of 2 or more.
- These CPC collectors are of either single or compound curvature and are characterized by the highest concentration permitted for a given acceptance angle.
- Since the compound-curvature (dishlike) CPCs require almost continuous tracking at useful values of the concentration ratio, the single- curvature devices requiring only periodical movement are described herein.

TABLE 4.5 Optical and Geometric Properties of Full and Truncated Compound Parabolic Concentrator Solar Collectors[a]

Acceptance half-angle of CPC (degrees)	Concentration ratio CR	Height/aperture ratio	Reflector/aperture ratio	Average number of reflections $\bar{n}$
36	1.40	0.38	0.80	0.25
	1.50 (10%)↓	0.50 (50%)↓	1.06	0.34 (20%)↑
	1.60	0.65	1.36	0.42
	1.70[b]	1.09	2.24	0.61
11.5	3.65	1.04	2.22	0.60
	4.51	1.62	3.38	0.79
	4.90	2.25	4.62	0.91
	5.0[b]	2.94	6.00	0.99
5.7	7.28	1.86	3.86	0.87
	9.08	3.03	6.17	1.06
	9.80	4.17	8.44	1.17
	10.0[b]	5.47	11.05	1.25

[a] truncated. From (41). (50%)↓

1. a 50 percent truncation (50%)↓ , that is, 50 percent reduction in height/aperture ratio (50%)↓ , causes a *CR* reduction of only about 10 percent (10%)↓ while reducing the average number of reflections by 20 percent or more.(20%)↑

2. The average number of reflections $\bar{n}$ increases with *CR* roughly as $\ln\sqrt{CR}$.

$$\bar{n} \propto \ln\sqrt{CR}.$$