Chapter 10

The Finite Volume Method for Convection-Diffusion Problems

Outline

10-1 Introduction

- Convection important where fluid flow is important. Steady convection equation

$$\nabla \cdot (\varrho \mathbf{u} \phi) = \nabla \cdot (\Gamma \nabla \phi) + S_\phi$$

- After formal integration over a CV

$$\int_{\partial CV} \mathbf{n} \cdot (\varrho \mathbf{u} \phi)\, dA = \int_{\partial CV} \mathbf{n} \cdot (\Gamma \nabla \phi)\, dA + \int_{CV} S_\phi dV \quad (15)$$

- Eq. (15) represents the flux balance in a CV

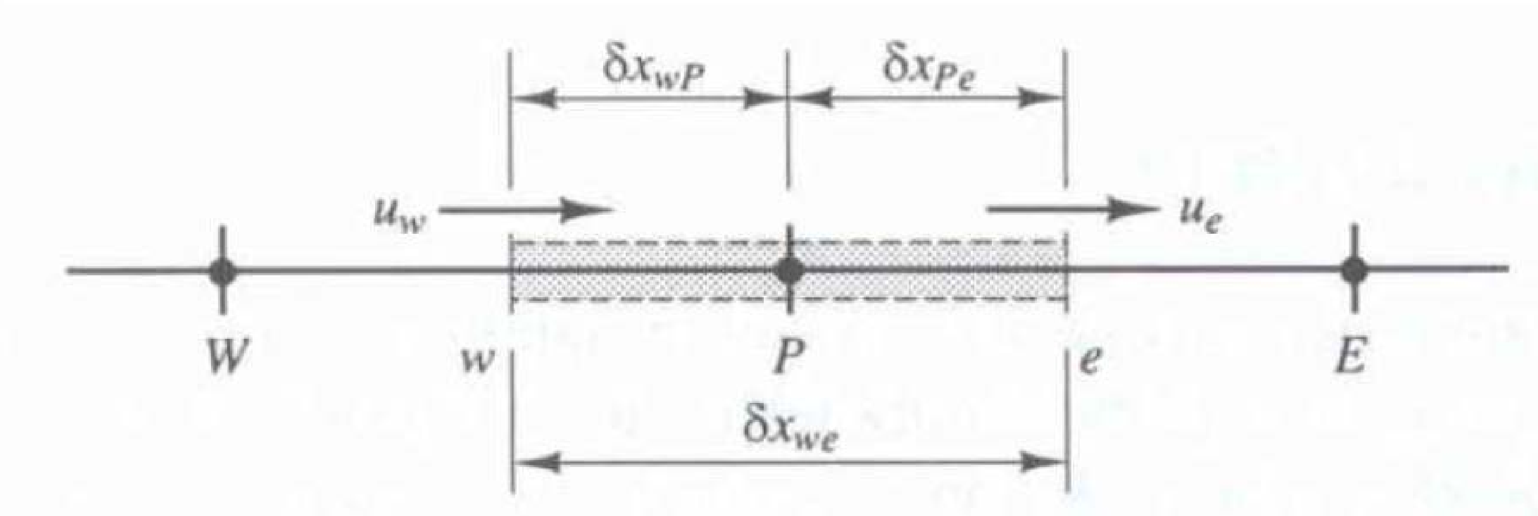
- **Main Problem**: calculation of the values of ϕ in the CV faces. Diffusion implies gradients of ϕ in all directions whilst Convection spreads influence only in the flow direction $\rightarrow$ upper limit to the grid size.

Steady 1-D convection and diffusion

- In absence of sources the flow equations for ϕ are:

$$\frac{d}{dx}(\varrho \mathbf{u} \phi) = \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right); \qquad \frac{d(\varrho \mathbf{u})}{dx} = 0$$

- Definition of the grid



Steady 1-D convection and diffusion

- Integration of transport equation leads to

$$(\varrho u A \phi)_e - (\varrho u A \phi)_w = \left(\Gamma A \frac{d\phi}{dx}\right)_e - \left(\Gamma A \frac{d\phi}{dx}\right)_w$$

$$(\varrho u A)_e - (\varrho u A)_w = 0$$

- Definition of

$$F = \varrho u; \quad D = \frac{\Gamma}{\delta x}$$

representing the convective mass flux per unit area and diffusion flux conductance at cell faces respectively.

Steady 1-D convection and diffusion

$$\left(\Gamma A \frac{d\phi}{dx}\right)_e = \Gamma_e A_e \left(\frac{\phi_E - \phi_P}{\delta x_{PE}}\right)$$

$$\left(\Gamma A \frac{d\phi}{dx}\right)_w = \Gamma_w A_w \left(\frac{\phi_P - \phi_W}{\delta x_{WP}}\right)$$

- If $A_w = A_e = A$ and central differencing approach is used to represent the contribution of diffusion terms

$$F_e\phi_e - F_w\phi_w = D_e(\phi_E - \phi_P) - D_w(\phi_P - \phi_W) \qquad (16)$$

$$F_e - F_w = 0 \qquad (17)$$

- The velocity field is 'somehow known' (i.e. F_w and F_e). Need to estimate ϕ at e and w faces. $\rightarrow$ Just for now!

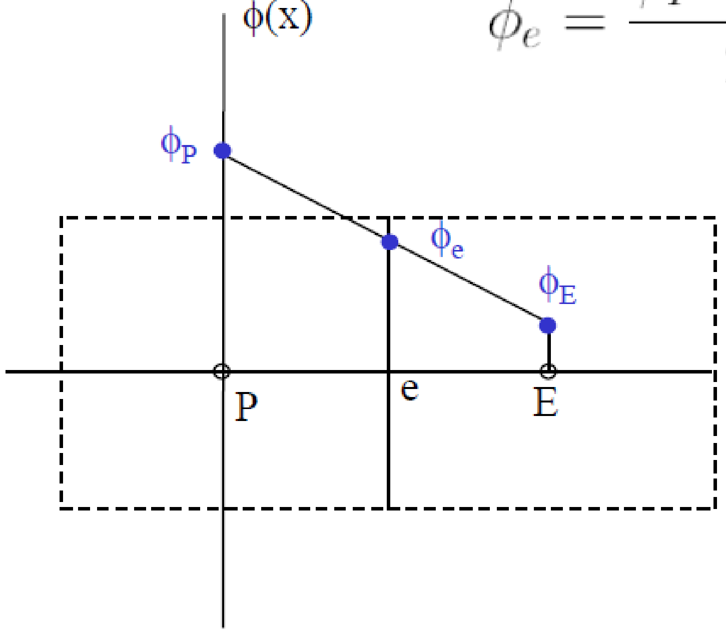
10-3 Central Differencing Scheme (1)

Steady 1-D convection and diffusion

The central differencing scheme

- The same method used in the diffusion terms

$$\phi_e = \frac{\phi_P + \phi_E}{2}; \qquad \phi_w = \frac{\phi_P + \phi_W}{2}$$



- We determine the value of ϕ at the face by linear interpolation between the cell centered values.

10-3 Central Differencing Scheme (2)

Steady 1-D convection and diffusion

The central differencing scheme

- Substituting in the discretised Eq. (16)

$$\frac{F_e}{2}(\phi_P+\phi_E)-\frac{F_w}{2}(\phi_W+\phi_P)=D_e(\phi_E-\phi_P)-D_w(\phi_P-\phi_W)$$

$$\Downarrow$$

$$\left[\left(D_w+\frac{F_w}{2}\right)+\left(D_e-\frac{F_e}{2}\right)+(F_e-F_w)\right]\phi_P=$$

$$\left(D_w+\frac{F_w}{2}\right)\phi_W+\left(D_e-\frac{F_e}{2}\right)\phi_E$$

10-3 Central Differencing Scheme (3)

Steady 1-D convection and diffusion

The central differencing scheme

- Indentifying coefficients the **Central Differencing** expressions are obtained

$$a_P\phi_P = a_W\phi_W + a_E\phi_E \tag{18}$$

a_W	a_E	a_P
$D_w + \frac{F_w}{2}$	$D_e - \frac{F_e}{2}$	$a_W + a_E + (F_e - F_w)$

- Eq. (18) is written for each node $\rightarrow$ set of algebraic equations solved using linear algebra. Boundary conditions are introduced in the limiting CV's.

10-4 Properties of Discretization Schemes (1)

Properties of discretisation schemes

- Theoretically, the numerical results and the 'exact' solution may be indistinguishable when the number of computational cells in infinitely large irrespective of the differencing method used.
- In practice, only finite (and frequently coarse) grids can be used. Therefore, the numerical results will only be realistic when the discretisation scheme has certain fundamental properties as: *Conservativeness*, *Boundedness* and *Transportiveness*.

- Other properties are also considered: Consistency, stability, convergence and accuracy

10-4 Properties of Discretization Schemes (2)

Properties of discretisation schemes

Consistency

The discretization should become exact as the grid spacing tends to zero. The difference between the discretized equation and the exact one is called the *truncation error*. It is usually estimated by replacing all the nodal values in the discrete approximation by a Taylor series expansion about a single point. As a result one recovers the original differential equation plus a remainder, which represents the truncation error. For a method to be *consistent*, the truncation error must become zero when the mesh spacing $\Delta t \to 0$ and/or $\Delta x_i \to 0$.

10-4 Properties of Discretization Schemes (3)

Properties of discretisation schemes

Consistency

$$\boxed{\mathcal{L}\ u = f}$$

differential equation

$$\boxed{\hat{\mathcal{L}}\ \underline{\hat{u}} = \underline{\hat{f}}}$$

Discrete approximation

The discrete approximation is consistent with the differential equation iff

For **all** smooth functions v

$$(\hat{\mathcal{L}}\underline{v} - \underline{\hat{f}})_j - (\mathcal{L}v - f)_j \ \to 0, \quad \text{for} \ \ j = 1, \ldots, n$$

when $\Delta x \to 0$.

Properties of discretisation schemes

Stability

A numerical solution method is said to be stable if it does not magnify the errors that appear in the course of numerical solution process. For temporal problems, stability guarantees that the method produces a bounded solution whenever the solution of the exact equation is bounded. For iterative methods, a stable method is one that does not diverge.

The difference scheme $\underline{\hat{u}}^{n+1} = \hat{S}\underline{\hat{u}}^n$ is **stable** if:

> there exists C_T such that
>
> $$\|\underline{v}^n\| = \|\hat{S}^n \, \underline{v}^0\| \leq C_T \|\underline{v}^0\|$$
>
> for all $\underline{v}^0$; and n, Δt such that $0 \leq n\Delta t \leq T$

10-4 Properties of Discretization Schemes (5)

Properties of discretisation schemes

Convergence

A numerical method is said to be convergent if the solution of the discretized equations tends to the exact solution of the differential equation as the grid spacing tends to zero.

Lax Equivalence Theorem

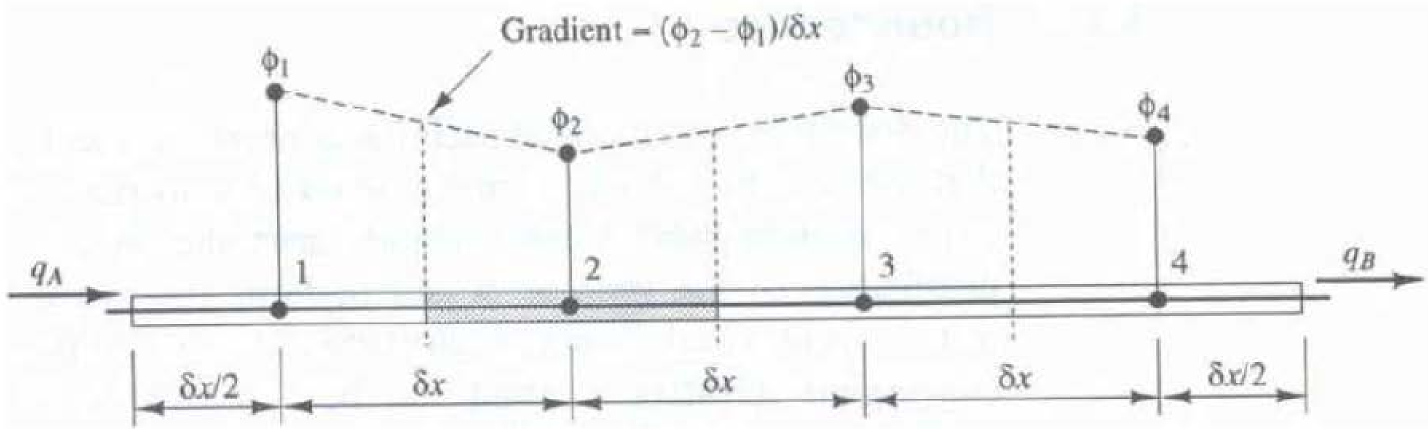
> A **consistent** finite difference scheme for a partial differential equation for which the initial value problem is well-posed is **convergent** if and only if it is **stable**.

For non-linear problems which are strongly influenced by boundary conditions, the stability and convergence of a method are difficult to demonstrate. Therefore convergence is usually checked using numerical experiments, i.e. repeating the calculation on a series of successively refined grids. If the method is stable and if all approximations used in the discretization process are consistent, we usually find that the solution does converge to a *grid-independent solution*.

Properties of discretisation schemes

Conservativeness

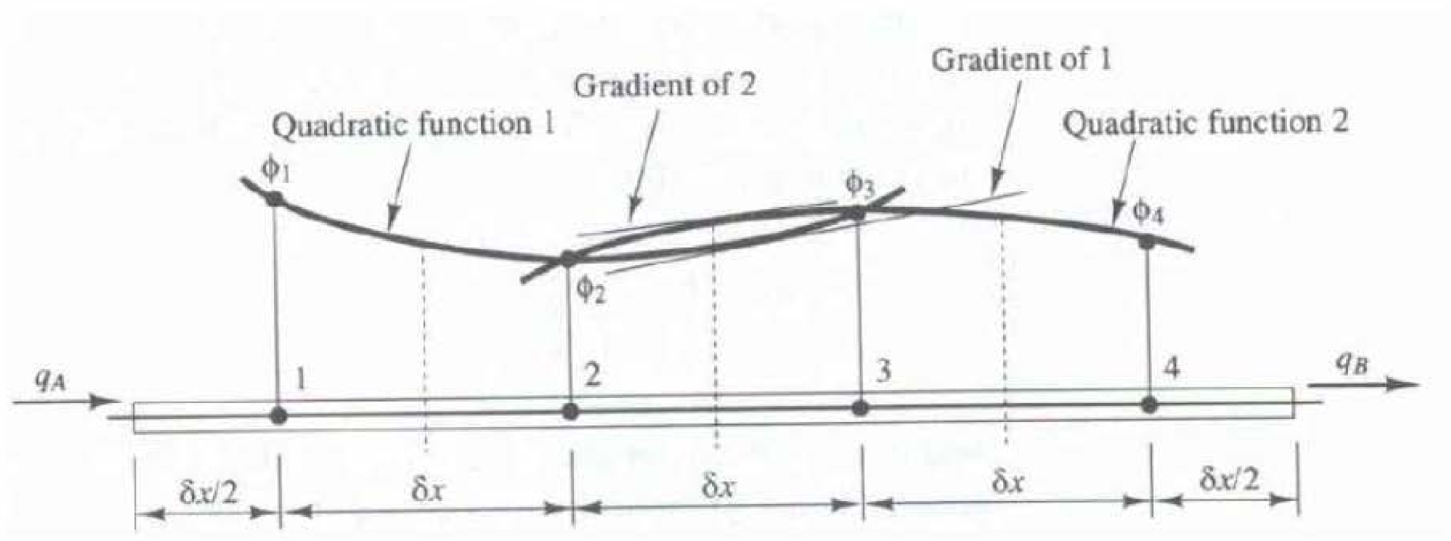
- The discretised convection-diffusion eq. implies fluxes of ϕ through CV faces. To ensure conservation of ϕ, the flux leaving a CV across a face must be equal to the flux entering the adjacent CV through the same face.
- Consistent specification of diffusive fluxes (central differencing)



Properties of discretisation schemes

Conservativeness

- Example of inconsistent specification of diffusive fluxes



- Inconsistent flux interpolation generates unsuitable schemes that do not satisfy overall conservation (e.g. quadratic interpolation formula).

Properties of discretisation schemes

Boundedness

Numerical solutions should lie within proper bounds. Physically non-negative quantities (like density, kinetic energy of turbulence) must always be positive; other quantities, such as concentration, must lie between 0% and 100%. In the absence of sources, some equations (e.g. the heat equation for the temperature when no heat sources are present) require that the minimum and maximum values of the variable be found on the boundaries of the domain.

- Discretised eq's at each node represent a set of algebraic equations that is solved by iterative numerical techniques. Initially a guessed distribution of ϕ is considered and the iterations provide succesive updates until a converged solution is obtained.

Properties of discretisation schemes

Boundedness

- **Scagborough criterium**: Sufficient condition for a convergent iterative method

$$\frac{\Sigma\,|a_{nb}|}{|a_P - S_p|}\begin{cases} \leq 1 & \text{at all nodes} \\ < 1 & \text{at one node at least} \end{cases} \qquad (19)$$

- If this criterium is satisfied, the resulting matrix of coefficients is **diagonally dominant** $\Rightarrow S_p$ must be always negative.
- **Boundedness criterium**: in absence of sources the values of ϕ must be bounded by its boundary values.
- **Essential requirement** for boundedness: all coefficients of the discretised equations should have the same sign (usually positive).

10-4 Properties of Discretization Schemes (10)

Properties of discretisation schemes

Accuracy

Numerical solutions of fluid flow and heat transfer problems are only *approximate solutions.*

kinds of systematic errors

- *Modeling errors*, which are defined as the difference between the actual flow and the exact solution of the mathematical model;
- *Discretization errors*, defined as the difference between the exact solution of the conservation equations and the exact solution of the algebraic system of equations obtained by discretizing these equations, and
- *Iteration errors*, defined as the difference between the iterative and exact solutions of the algebraic equations systems.

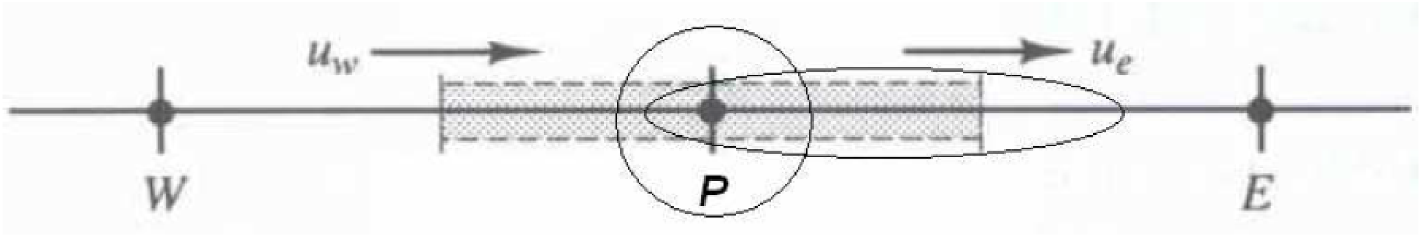
Properties of discretisation schemes

Transportiveness → Property of fluid flow

- Definition of Peclet number (measure of the relative strengths of convection and diffusion)

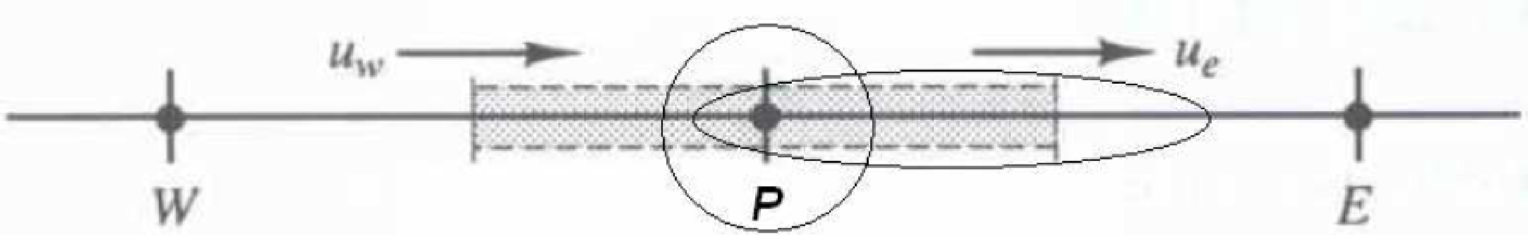
$$Pe = \frac{F}{D} = \frac{\rho u}{\Gamma/\delta x}$$

- Case of pure diffusion ($Pe = 0$): fluid stagnant and contours of constant ϕ are concentric circles centered in node P (the diffusion tends to spread ϕ in all directions)



Properties of discretisation schemes

Transportiveness

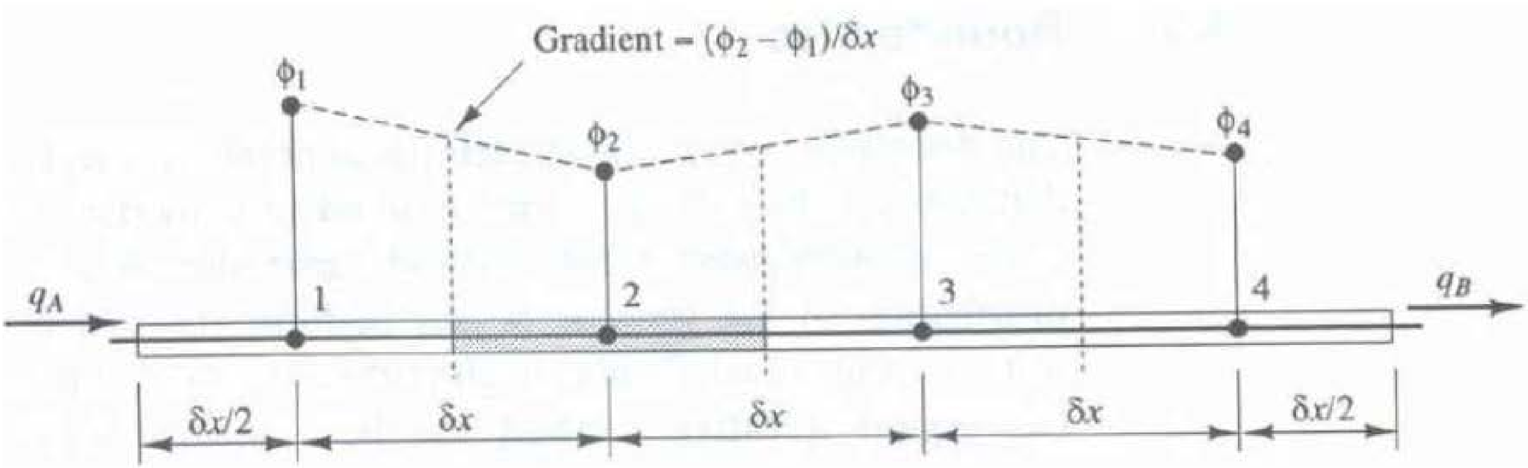


- As Pe ↑, the contours become more and more elliptic. Influencing is increasingly biased towards upstream for large values of Pe.
- In case of pure convection ($Pe \rightarrow \infty$) all of ϕ emanating from node P is immediately transported downstream.
- It is essential that this relationship between the value of Pe and directionality of influencing (transportiveness) is borne out in the discretisation scheme.

10-4 Properties of Discretization Schemes (13)

Assesment of central differencing scheme for convection-diffusion problems

Conservativeness



$\Gamma_{e\,i} = \Gamma_{w\,i-1}$ Guarantees that diffusive fluxes are equal from both sides of the CV

10-4 Properties of Discretization Schemes (14)

Assesment of central differencing scheme for convection-diffusion problems

Boundedness

$$a_P \phi_P = a_W \phi_W + a_E \phi_E$$

$$F = \rho u$$

$$D = \frac{\Gamma}{\delta x}$$

a_W	a_E	a_P
$D_w + \frac{F_w}{2}$	$D_e - \frac{F_e}{2}$	$a_W + a_E + (F_e - F_w)$

It is satisfied: $a_P = a_W + a_E + F_e - F_w$

but the continuity equation establishes that $F_e - F_w = 0$.
Therefore, the criterim of Scarborough is satisfied.

10-4 Properties of Discretization Schemes (15)

Assesment of central differencing scheme for convection-diffusion problems

Boundedness

a_W	a_E	a_P
$D_w + \frac{F_w}{2}$	$D_e - \frac{F_e}{2}$	$a_W + a_E + (F_e - F_w)$

$a_E = D_e - F_e/2$. If convection dominates a_E can be negative. The condition for $a_E > 0$ (with F_e and $F_w > 0$) is:

$$\frac{F_e}{D_e} = Pe < 2$$

So, if $Pe > 2$, $a_E < 0$ and the essential requirement for boundedness is violated.

10-4 Properties of Discretization Schemes (16)

Assesment of central differencing scheme for convection-diffusion problems

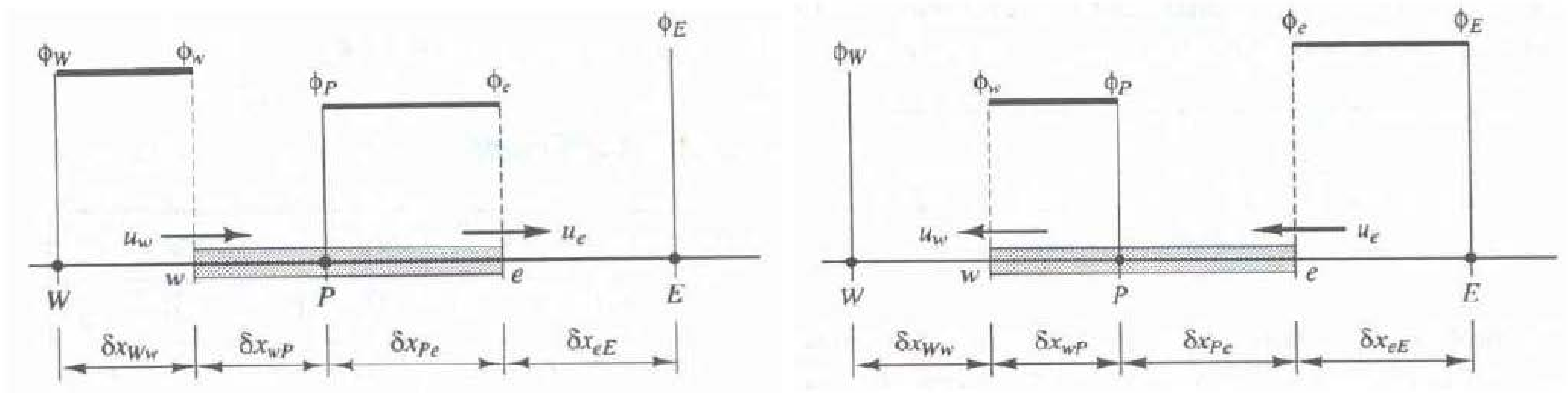
• *Transportiveness*: Central differencing introuduces influence at node P from all the directions. Thus, it does no recognise the direction of the flow. It does not fullfil transportiveness at high Pe.

• *Precision*: The Taylor series trucation error of central differencing is second order. The scheme will be stable and accurate if $Pe < 2 \Rightarrow$ central differencing is not a suitable discretisation practice for general purpose flow calculations.

10-5 Upwind Differencing Scheme (1)

The upwind differencing scheme

- It takes into account the flow direction: the convected value of ϕ at a cell face is equal to the value at upstream node.



The upwind differencing scheme

- If the flow is in the positive direction $F_w > 0;\ \ F_e > 0$

$$\phi_w = \phi_W; \quad \phi_e = \phi_P$$

$$\Downarrow$$

$$F_e\phi_P - F_w\phi_W = D_e(\phi_E - \phi_P) - D_w(\phi_P - \phi_W)$$

$$\Downarrow$$

$$[(D_w + F_w) + D_e + (F_e - F_w)]\phi_P = (D_w + F_w)\phi_W + D_e\phi_E$$

- If the flow is in the negative direction $F_w < 0;\ \ F_e < 0$

$$\phi_w = \phi_P; \quad \phi_e = \phi_E$$

$$\Downarrow$$

$$[D_w + (D_e - F_e) + (F_e - F_w)]\phi_P = D_w\phi_W + (D_e + F_e)\phi_E$$

10-5 Upwind Differencing Scheme (3)

The upwind differencing scheme

- Coefficients for the Upwind difference scheme

$$a_P\phi_P = a_E\phi_E + a_W\phi_W \tag{20}$$

a_W	a_E	a_P
$D_w + max(F_w, 0)$	$D_e - max(0, -F_e)$	$a_W + a_E + (F_e - F_w)$

10-5 Upwind Differencing Scheme (4)

Assesment of the upwind scheme

Conservativeness: the calculation of fluxes through the cell faces is consistent, so the scheme is conservative

Boundedness: the coefficients are always positive by construction and satisfy the requirements for boundedness. Also the matrix of coefficients is diagonally dominant.

Transportiveness: the scheme accounts for the direction of the flow and transportiveness is built into the formulation.

Accuracy: Formally the accuracy is only first order.

10-6 Exponential Scheme (1)

- The governing transport equation:

$$\frac{d}{dx}(\rho u\phi) = \frac{d}{dx}(\Gamma\frac{d\phi}{dx})$$

- If = constant, the equation can be solved exactly

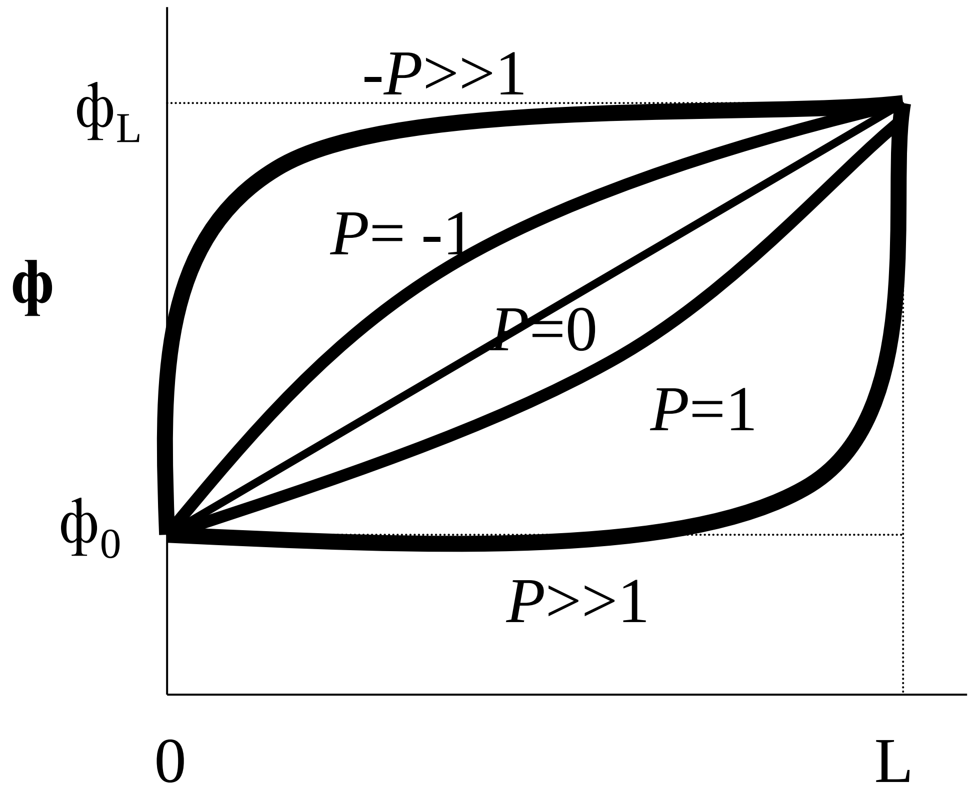
- Boundary conditions: $at\ x = 0 \quad \phi = \phi_0 \;,\quad at\ x = L \quad \phi = \phi_L$

- Solution: $$\frac{\phi - \phi_0}{\phi_L - \phi_0} = (e^{\frac{Px}{L}} - 1)/(e^P - 1)$$

where, $P = \rho u L/\Gamma$

10-6 Exponential Scheme (2)

The nature of the exact solution:



Exact solution for the one-dimensional convection-diffusion problem

10-6 Exponential Scheme (3)

- Define $$J = \rho u \phi - \Gamma \frac{d\phi}{dx}$$
- Our transport equation becomes, $$\frac{dJ}{dx} = 0$$
- Integrating over CV, $J_e - J_w = 0$
- The exact solution derived above can be used as profile assumption with $\phi_0 \rightarrow \phi_{P,}\ \phi_L \rightarrow \phi_{E,}\ L \rightarrow \delta x_e$
- Substitution gives $$J_e = F_e\left(\phi_P + \frac{\phi_P - \phi_E}{e^{P_e} - 1}\right)$$

where $$P_e = \frac{(\rho u)_e \partial x_e}{\Gamma_e} = \frac{F_e}{D_e}$$

10-6 Exponential Scheme (4)

- After substitution of similar expression for J_w , equation in our standard form can be written as:

$$a_P\phi_P = a_W\phi_W + a_E\phi_E \quad , \quad a_P = a_W + a_E + (F_e - F_w)$$

a_W	a_E
$F_w e^{F_w/D_w}/(e^{F_w/D_w} - 1)$	$F_e/(e^{F_e/D_e} - 1)$

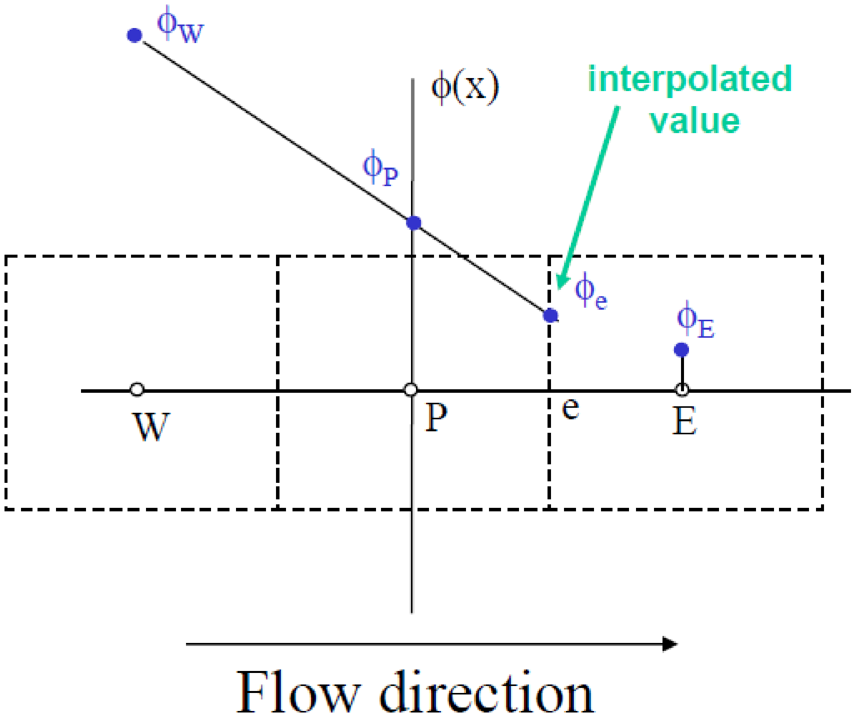
- Merit: Guaranteed to produce exact solution for any Peclet number for 1-D steady convection-diffusion

- Demerits: 1. exponentials expensive to compute

  2. not exact for 2-D, 3-D

Second-order upwind scheme

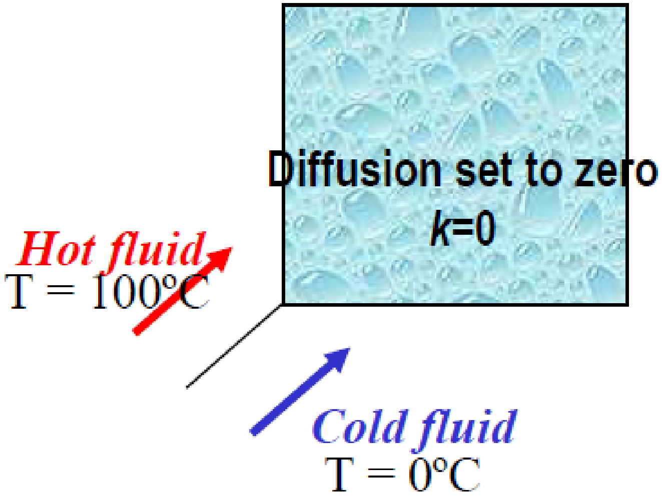
- We determine the value of ϕ from the cell values in the two cells upstream of the face.
- This is more accurate than the first order upwind scheme, but in regions with strong gradients it can result in face values that are outside of the range of cell values. It is then necessary to apply limiters to the predicted face values.



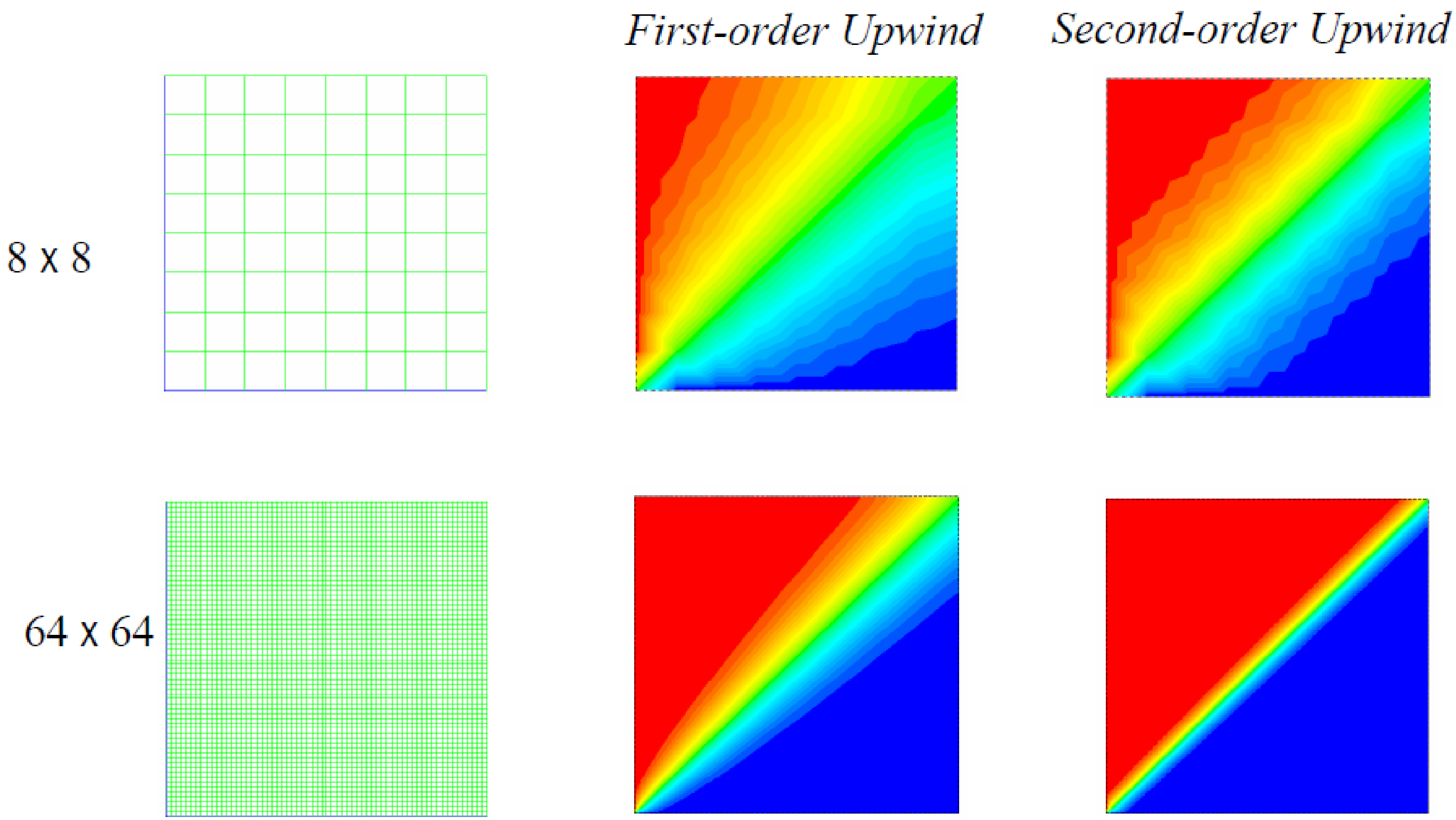
10-7 Second-order Upwind Scheme (2)

- Because its simplicity, the upwind scheme is applied widely in CFD calculations. But it has a drawback that is known as **false diffusion**.

When the flow is not aligned with the grid lines, the upwind scheme produces distributions of transported properties which are artificially smeared out.



10-7 Second-order Upwind Scheme (3)



Grid refinement coupled with a higher-order interpolation scheme will minimize the false diffusion

10-8 Hybrid Differencing Scheme (1)

The hybrid differencing scheme

- Combination of central and upwind schemes. Central differences are used for $Pe < 2$ (second order accuracy) and upwind for $Pe > 2$ (first order accuracy)
- Use of piecewise formulae based on local Pe in the face of CV

$$Pe_w = \frac{F_w}{D_w} = \frac{(\varrho u)_w}{\Gamma_w/\delta x_{WP}}$$

- Net flux per unit area through the west face

$$q_w = F_w \left[\frac{1}{2}\left(1 + \frac{2}{Pe_w}\right)\phi_W + \frac{1}{2}\left(1 - \frac{2}{Pe_w}\right)\phi_P\right] \quad -2 < Pe_w < 2$$

$$q_w = F_w \phi_W \quad Pe_w \geq 2$$

$$q_w = F_w \phi_P \quad Pe_w \leq -2$$

10-8 Hybrid Differencing Scheme (2)

The hybrid differencing scheme

- General form of the discretised equation

$$a_P\phi_P = a_E\phi_E + a_W\phi_W$$

a_W	a_E	a_P
$max\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right]$	$max\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right]$	$a_W + a_E + (F_e - F_w)$

- The scheme is fully conservative and incondicionally bounded. Also satisfies the transportivity requirements because for high Pe is the upwind scheme. Hybrid scheme is widely used in CFD and has proved very useful for predicting practical flows.

10-8 Hybrid Differencing Scheme (3)

The hybrid differencing scheme multi-dimensional

	One-dimensional flow	Two-dimensional flow	Three-dimensional flow
a_W	$\max\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right]$	$\max\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right]$	$\max\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right]$
a_E	$\max\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right]$	$\max\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right]$	$\max\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right]$
a_S	–	$\max\left[F_s, \left(D_s + \frac{F_s}{2}\right), 0\right]$	$\max\left[F_s, \left(D_s + \frac{F_s}{2}\right), 0\right]$
a_N	–	$\max\left[-F_n, \left(D_n - \frac{F_n}{2}\right), 0\right]$	$\max\left[-F_n, \left(D_n - \frac{F_n}{2}\right), 0\right]$
a_B	–	–	$\max\left[F_b, \left(D_b + \frac{F_b}{2}\right), 0\right]$
a_T	–	–	$\max\left[-F_t, \left(D_t - \frac{F_t}{2}\right), 0\right]$
ΔF	$F_e - F_w$	$F_e - F_w + F_n - F_s$	$F_e - F_w + F_n - F_s + F_t - F_b$

Face	w	e	s	n	b	t
F	$(\rho u)_w A_w$	$(\rho u)_e A_e$	$(\rho v)_s A_s$	$(\rho v)_n A_n$	$(\rho w)_b A_b$	$(\rho w)_t A_t$
D	$\frac{\Gamma_w}{\delta x_{WP}} A_w$	$\frac{\Gamma_e}{\delta x_{PE}} A_e$	$\frac{\Gamma_s}{\delta y_{SP}} A_s$	$\frac{\Gamma_n}{\delta y_{PN}} A_n$	$\frac{\Gamma_b}{\delta z_{PN}} A_b$	$\frac{\Gamma_t}{\delta z_{PT}} A_t$

10-9 Power-Law Scheme (1)

The power-law scheme

- More accurate approximation to the 1-D exact solution and produces better results that hybrid scheme.
- In this scheme, diffusion is set to zero when $Pe > 10$. Otherwise, the flux is evaluated using a polynomial expresion; for example, for the west CV face

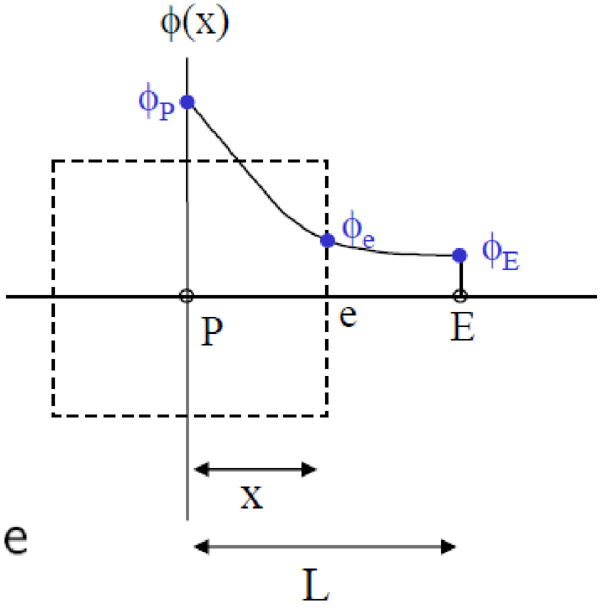
$$q_w = F_w[\phi_W - \beta_w(\phi_P - \phi_W)];\ \beta_w = \frac{(1-0.1Pe_w)^5}{Pe_w};\ 0 < Pe < 10$$

$$q_w = F_w\phi_W;\ \ Pe > 10$$

10-9 Power-Law Scheme (2)

The power-law scheme

This is based on the analytical solution of the one-dimensional convection-diffusion equation.



- Discretised equation for power-law scheme

a_W	a_E	a_P
$D_w max\left[0, (1-0.1\lvert Pe_w\rvert)^5\right]$ $+max[F_w, 0]$	$D_e max\left[0, (1-0.1\lvert Pe_e\rvert)^5\right]$ $+max[-F_e, 0]$	$a_W + a_E +$ $(F_e - F_w)$

10-9 Power-Law Scheme (3)

From eq(14), we deduce that

$$\frac{a_E}{D_e} = \frac{p_e}{\exp(p_e) - 1}$$

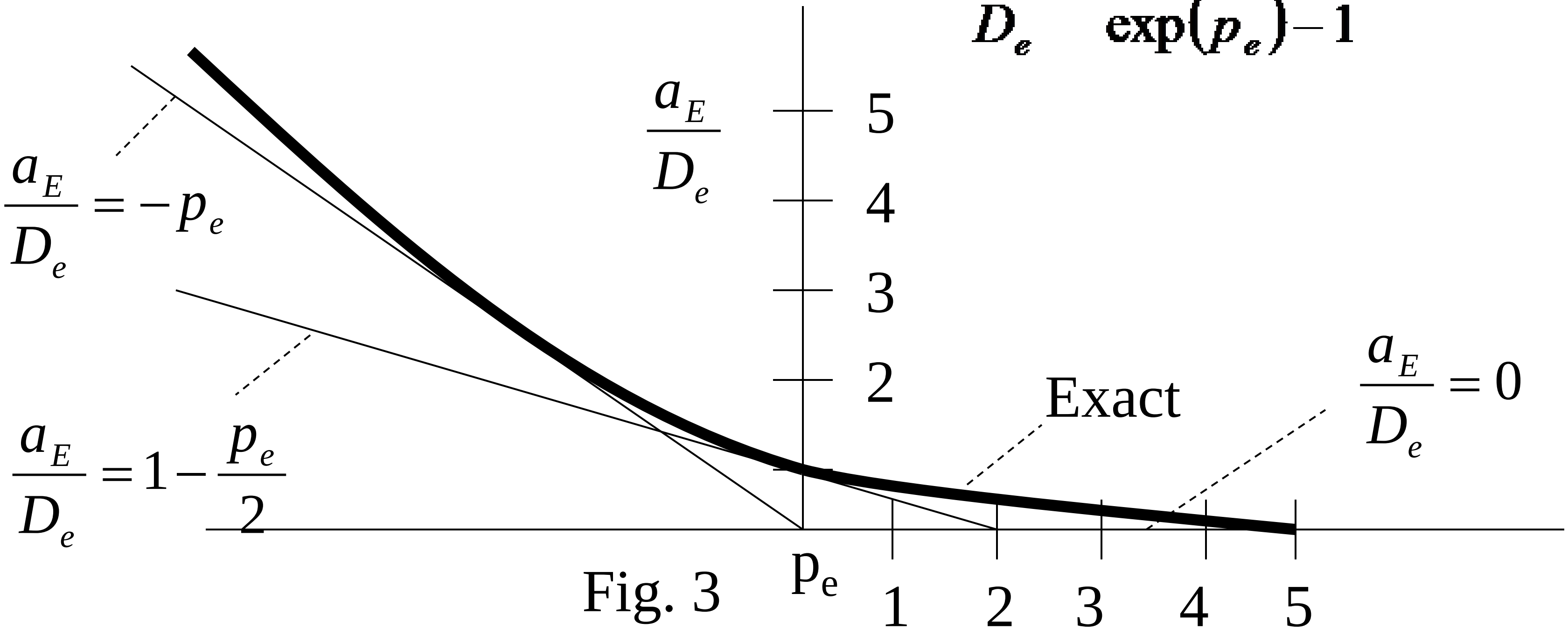


Fig. 3

10-10 QUICK Scheme

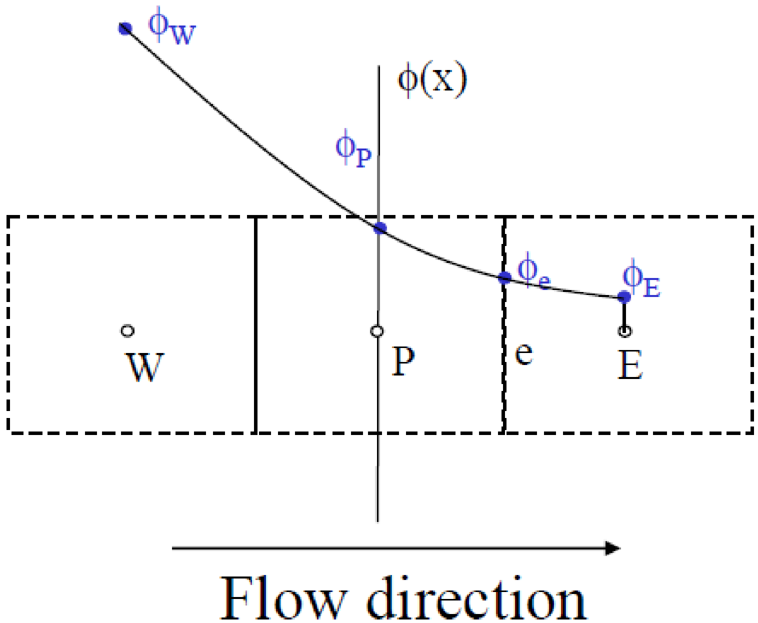
Quadratic Upwind Interpolation (QUICK)

- A quadratic curve is fitted through two upstream nodes and one downstream node.

- This is a very accurate scheme, but in regions with strong gradients, overshoots and undershoots can occur. This can lead to stability problems in the calculation.

$u_x > 0$

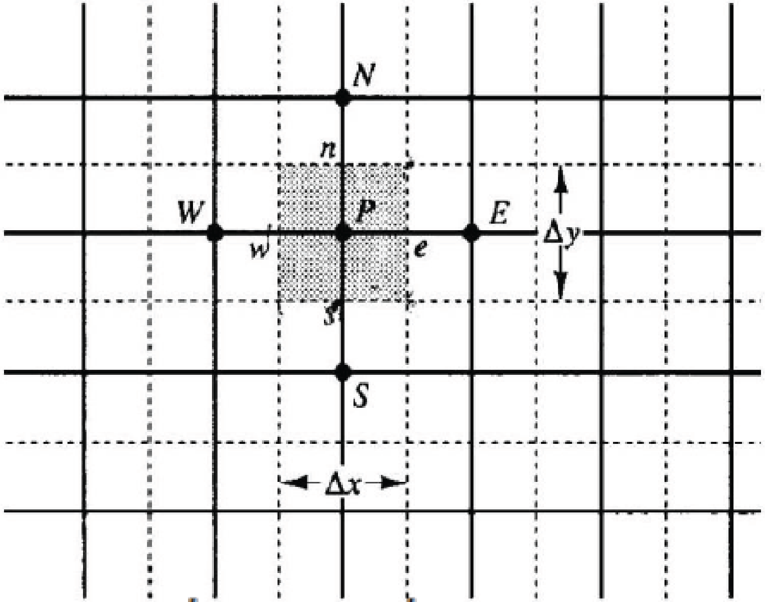
$$\phi_e = \frac{6}{8}\phi_P + \frac{3}{8}\phi_E - \frac{1}{8}\phi_W$$



conservative with 3^{rd} order accuracy

violates the tranportivity and boundedness.

10-11 Discretization for 2D, 3D (1)



Discretization Equation for 2-D

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + a_S\phi_S + a_N\phi_N$$

$$a_P = a_W + a_E + a_S + a_N + \Delta F$$

Discretization Equation for 3-D

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + a_S\phi_S + a_N\phi_N + a_B\phi_B + a_T\phi_T$$

$$a_P = a_W + a_E + a_S + a_N + a_B + a_T + \Delta F$$

The coefficients for 2-D, 3-D for hybrid differencing scheme are shown on next page

10-11 Discretization for 2D, 3D (2)

	One Dimensional Flow	Two Dimensional Flow	Three Dimensional Flow
a_W	$\left[\!\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right]\!\right]$	$\left[\!\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right]\!\right]$	$\left[\!\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right]\!\right]$
a_E	$\left[\!\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right]\!\right]$	$\left[\!\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right]\!\right]$	$\left[\!\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right]\!\right]$
a_S	-	$\left[\!\left[F_s, \left(D_s + \frac{F_s}{2}\right), 0\right]\!\right]$	$\left[\!\left[F_s, \left(D_s + \frac{F_s}{2}\right), 0\right]\!\right]$
a_N	-	$\left[\!\left[-F_n, \left(D_n - \frac{F_n}{2}\right), 0\right]\!\right]$	$\left[\!\left[-F_n, \left(D_n - \frac{F_n}{2}\right), 0\right]\!\right]$
a_B	-		$\left[\!\left[F_b, \left(D_b + \frac{F_b}{2}\right), 0\right]\!\right]$
a_T	-		$\left[\!\left[-F_t, \left(D_t - \frac{F_t}{2}\right), 0\right]\!\right]$
ΔF	$F_e - F_w$	$F_e - F_w + F_n - F_s$	$F_e - F_w + F_n - F_s + F_t - F_b$

10-12 Summary of Numerical Schemes (1)

Note on accuracy of numerical schemes

- Each of the previously discussed numerical schemes assumes some shape of the function ϕ. These functions can be approximated by Taylor series polynomials:

$$\phi(x_e) = \phi(x_P) + \frac{\phi'(x_P)}{1!}(x_e - x_P) + \frac{\phi''(x_P)}{2!}(x_e - x_P)^2 + \ldots + \frac{\phi^n(x_P)}{n!}(x_e - x_P)^n + \ldots$$

- The first order upwind scheme only uses the constant and ignores the first derivative and consecutive terms. This scheme is therefore considered first order accurate.
- For high Peclet numbers the power law scheme reduces to the first order upwind scheme, so it is also considered first order accurate.
- The central differencing scheme and second order upwind scheme do include the first order derivative, but ignore the second order derivative. These schemes are therefore considered second order accurate. QUICK does take the second order derivative into account, but ignores the third order derivative. This is then considered third order accurate.

10-12 Summary of Numerical Schemes (2)

Summary

Scheme	Conser-vative	Bounded	Accuracy	Transp-ortive	Remarks
CDS	Yes	Conditionally bounded	Second order	No	Unrealistic solutions at large Peclet number
UDS, HDS, PLS	Yes	Unconditionally bounded	First Order	Yes	Induce false diffusion if the velocity vector is not parallel to one of the coordinate directions
QUICK	Yes	Unconditionally bounded	Third Order	Yes	Less computationally stable. Can give small undershoots and overshoots

10-13 Handling the Source Term

- For 1-D, Discretization equation simply becomes,

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + b$$

- If the source term is a constant $S = S_C$, then all other coefficients remain same and,

$$b = S_C\Delta x$$

- If source term is dependent on $\boxed{A}$, linearization is done as:

$$S = S_C + S_P\phi_P$$

In this case, b and a_P become,

$$b = S_C\Delta x, \qquad a_P' = a_P - S_P\Delta x$$

All other coefficients remain same

In a similar way, Source term can be incorporated in 2-D, 3-D