Solution of HW#1

7.

(a) $\cancelto{0}{\rho c \frac{\partial T}{\partial t}} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \underbrace{\cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}}_{1-D} + \cancel{\dot{q}}\,\dot{s}$

$\Rightarrow \frac{d}{dx}\left(k\frac{dT}{dx}\right) + \overset{\dot{s}}{\cancel{\dot{q}}} = 0$

(b) $\dot{E}_{in} + \dot{E}_g = \dot{E}_{out} + \cancelto{0}{\dot{E}_{st}}$

$\Rightarrow q_1'' + \dot{s}L = q_2''$

$q_2'' = h_B(T_2 - T_B) = 125 \times (100-20) = 10000 \ W/m^2$

$q_1'' = q_2'' - \dot{s}L = 10000 - 5\times10^3 \times 0.8 = 6000 \ W/m^2 = h_A(T_A - T_1)$

$= h_A \cdot (300-100) \Rightarrow h_A = 60 \ W/m^2\cdot K$

(c) $k\frac{dT}{dx} = -\dot{s}x + C$

$q_1'' = -k\frac{dT}{dx}\Big|_{x=0} = \dot{s}(0) - C = 6000 \Rightarrow C = -6000$

$\rightarrow k\frac{dT}{dx} = -5\times10^3 x - 6000 \Rightarrow k \cdot \frac{100-200}{0.8} = -5000x - 6000 \Rightarrow k = 40x + 48$ #

9.

$$q'' = \frac{T_{s,i} - T_{s,o}}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{(600-20)^\circ C}{\frac{0.3 \text{ m}}{20 \text{ W/m}\cdot\text{K}} + \frac{0.15 \text{ m}}{k_B} + \frac{0.15 \text{ m}}{50 \text{ W/m}\cdot\text{K}}}$$

$$q'' = \frac{580}{0.018 + 0.15/k_B} \text{ W/m}^2. \qquad (1)$$

The heat flux may be obtained from

$$q'' = h\left(T_\infty - T_{s,i}\right) = 25 \text{ W/m}^2\cdot\text{K}\,(800\text{-}600)^\circ C \qquad (2)$$

$$q'' = 5000 \text{ W/m}^2.$$

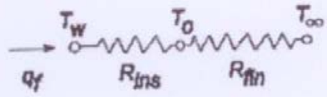
Substituting for the heat flux from Eq. (2) into Eq. (1), find

$$\frac{0.15}{k_B} = \frac{580}{q''} - 0.018 = \frac{580}{5000} - 0.018 = 0.098$$

$$k_B = 1.53 \text{ W/m}\cdot\text{K}. \qquad <$$

12.

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.76,

$$R_{fin} = \theta_b/q_f = \frac{1}{(hPkA_c)^{1/2}\tanh(mL_o)} \qquad (2)$$

$$m = (hP/kA_c)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}(T_w - T_\infty) \qquad (6) <$$

(b) Substituting numerical values into Eqs. (1) - (6) with $L_o = 200$ mm,

$$T_o = 25^\circ C + \frac{6.298}{6.790 + 6.298}(200 - 25)^\circ C = 109^\circ C \qquad <$$

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909 \times 10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi(0.025\,m)^2/4 = 4.909 \times 10^{-4}\,m^2$$

$$R_{fin} = 1/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh(6.324 \times 0.200) = 6.298\,K/W$$

$$(hPkA_c) = \left(15\,W/m^2\cdot K \times \pi(0.025\,m) \times 60\,W/m\cdot K \times 4.909 \times 10^{-4}\,m^2\right) = 0.0347\,W^2/K^2$$

Continued...

PROBLEM 3.111 (Cont.)

$$m = (hP/kA_c)^{1/2} = \left(15\,W/m^2\cdot K \times \pi(0.025\,m)/60\,W/m\cdot K \times 4.909 \times 10^{-4}\,m^2\right)^{1/2} = 6.324\,m^{-1}$$

13.

For 1 meter axial length; L; $A_b = \pi\cdot d - 12\cdot t = \pi\cdot 0.028 - 12\cdot 0.002 = 0.064\ m^2$

$$R_p = \frac{\ln(r_{out}/r_{in})}{2\cdot\pi\cdot k_{bronze}\cdot L} = \frac{\ln(2.8/2.0)}{2\cdot\pi\cdot 52\cdot 1} = 1.03 \times 10^{-3}\left[\frac{^\circ C}{W}\right] \qquad R_b = \frac{1}{h_{out}\cdot A_b} = \frac{1}{5\cdot 0.064} = 3.125\left[\frac{^\circ C}{W}\right]$$

$$A_c = L \cdot \text{thickness} = 1\cdot 0.002 = 0.002\,[m^2] \qquad P = 2\,(L) = 2.0\ [m]$$

$$L_C = L + A_C/P = 0.01 + \frac{0.002}{2.0}[m] = 0.011\ [m] \qquad m = \sqrt{\frac{h\cdot P}{k\cdot A_C}} = \sqrt{\frac{5\cdot 2.0}{52\cdot 0.002}} = 9.806$$

$$R_f = \frac{1}{N\sqrt{h\cdot P\cdot k\cdot A_C}\,\tanh m\cdot L_C} = \frac{1}{12\sqrt{5\cdot 2.0\cdot 52\cdot 0.002}\,\tanh(9.8\cdot 0.011)} = 0.760$$

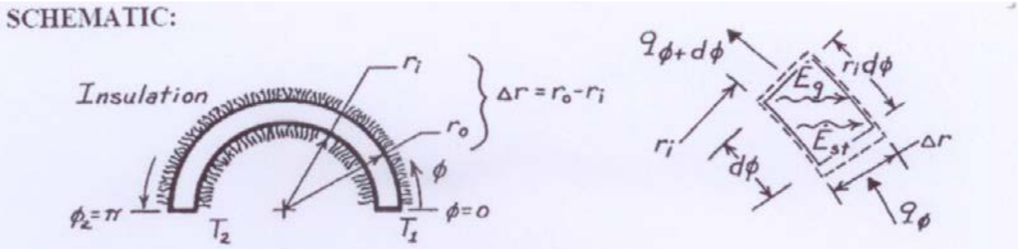
$$R_{eq} = \frac{R_b\cdot R_f}{R_b + R_f} = \frac{3.125\cdot 0.760}{3.125 + 0.760} = 0.611\ [^\circ C/W]$$

$$\sum R = \frac{1}{h_{in}\cdot A_{in}} + R_p + R_{eq} = \frac{1}{1200\cdot\pi\cdot 0.02} + 0.00103 + 0.611 = 0.625\ [^\circ C/W]$$

$$\therefore \frac{\dot q}{L} = \frac{T_{water} - T_{air}}{\sum R} = \frac{(98 - 15)[^\circ C]}{0.625\,[^\circ C/W]} = 132.8\left[\frac{W}{m}\right] \qquad \Leftarrow\Leftarrow\Leftarrow\Leftarrow$$

16.

SCHEMATIC:



ASSUMPTIONS: (1) T is independent of r,z, (2) $\Delta r = (r_o - r_i) << r_i$.

ANALYSIS: (a) Define the control volume as $V = r_i d\phi \cdot \Delta r \cdot L$ where L is length normal to page. Apply the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} \qquad q_\phi - q_{\phi+d\phi} + \dot{q}V = \rho V c \frac{\partial T}{\partial t} \tag{1,2}$$

where
$$q_\phi = -k(\Delta r \cdot L)\frac{\partial T}{r_i \partial \phi} \qquad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)d\phi. \tag{3,4}$$

Eqs. (3) and (4) follow from Fourier's law, Eq. 2.1, and from Eq. 2.7, respectively. Combining Eqs. (3) and (4) with Eq. (2) and canceling like terms, find

$$\frac{1}{r_i^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c \frac{\partial T}{\partial t}. \tag{5} <$$

Since temperature is independent of r and z, this form agrees with Eq. 2.20.

(b) For steady-state conditions with $\dot{q} = 0$, the heat equation, (5), becomes

$$\frac{d}{d\phi}\left[k\frac{dT}{d\phi}\right] = 0. \tag{6}$$

With constant properties, it follows that $dT/d\phi$ is constant which implies $T(\phi)$ is linear in ϕ. That is,

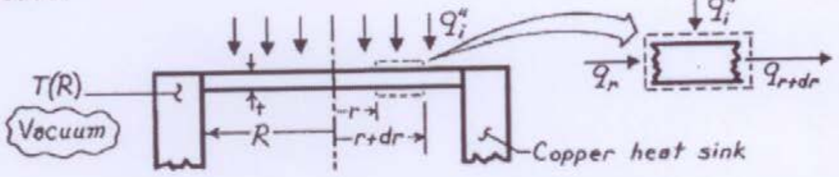
$$\frac{dT}{d\phi} = \frac{T_2 - T_1}{\phi_2 - \phi_1} = +\frac{1}{\pi}(T_2 - T_1) \quad \text{or} \quad T(\phi) = T_1 + \frac{1}{\pi}(T_2 - T_1)\phi. \tag{7,8} <$$

(c) The heat rate for the conditions of Part (b) follows from Fourier's law, Eq. (3), using the temperature gradient of Eq. (7). That is,

$$q_\phi = -k(\Delta r \cdot L)\frac{1}{r_i}\left[+\frac{1}{\pi}(T_2 - T_1)\right] = -k\left[\frac{r_o - r_i}{\pi r_i}\right]L(T_2 - T_1). \tag{9} <$$

17.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r (negligible temperature drop across foil thickness), (3) Constant properties, (4) Uniform incident flux, (5) Negligible heat loss from foil due to radiation exchange with enclosure wall, (6) Negligible contact resistance between foil and heat sink.

ANALYSIS: Applying energy conservation to a circular ring extending from r to r + dr,

$$q_r + q_i''(2\pi\ rdr) = q_{r+dr}, \qquad q_r = -k(2\pi\ rt)\frac{dT}{dr}, \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr.$$

Rearranging, find that

$$q_i''(2\pi\ rdr) = \frac{d}{dr}\left[(-k2\pi\ rt)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_i''}{kt}r.$$

Integrating,

$$r\frac{dT}{dr} = -\frac{q_i''r^2}{2kt} + C_1 \qquad \text{and} \qquad T(r) = -\frac{q_i''r^2}{4kt} + C_1\ln r + C_2.$$

With $dT/dr|_{r=0} = 0$, $C_1 = 0$ and with $T(r = R) = T(R)$,

$$T(R) = -\frac{q_i''R^2}{4kt} + C_2 \qquad \text{or} \qquad C_2 = T(R) + \frac{q_i''R^2}{4kt}.$$

Hence, the temperature distribution is

$$T(r) = \frac{q_i''}{4kt}\left(R^2 - r^2\right) + T(R).$$

Applying this result at r = 0, it follows that

$$q_i'' = \frac{4kt}{R^2}\left[T(0) - T(R)\right] = \frac{4kt}{R^2}\Delta T. \qquad <$$

18.

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot q}{k} = -\frac{q_0}{k}\left[1-\left(\frac{r}{r_0}\right)^2\right]$$

$$\Rightarrow \frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{q_0}{k}\left(r^2-\frac{r^4}{r_0^2}\right)$$

$$\Rightarrow r^2\frac{dT}{dr} = -\frac{q_0}{k}\left(\frac{1}{3}r^3-\frac{r^5}{5r_0^2}\right)+C_1$$

$$\Rightarrow \frac{dT}{dr} = -\frac{q_0}{k}\left(\frac{1}{3}r-\frac{r^3}{5r_0^2}\right)+\frac{C_1}{r^2}$$

$$\Rightarrow T = -\frac{q_0}{k}\left(\frac{1}{6}r^2-\frac{r^4}{20r_0^2}\right)-\frac{C_1}{r}+C_2$$

B.Cs: $\left.\frac{dT}{dr}\right|_{r=0} = 0$ or $r\to 0: T\to$ finite

$$\Rightarrow C_1 = 0$$

@ $r=r_0$: $\cancel{k(T-T_\infty)}$ $-k\left.\frac{dT}{dr}\right|_{r=r_0} = h(T-T_\infty)$

$$-k\left[-\frac{q_0}{k}\left(\frac{1}{3}r_0-\frac{r_0^3}{5r_0^2}\right)\right] = h\left[-\frac{q_0}{k}\left(\frac{1}{6}r_0^2-\frac{1}{20}r_0^2\right)+C_2-T_\infty\right]$$

$$\frac{2q_0}{15}r_0 = h\left(-\frac{7}{60}\frac{q_0}{k}r_0^2+C_2-T_\infty\right)$$

$$\Rightarrow C_2 = T_\infty+\frac{2q_0}{15h}r_0+\frac{7}{60}\frac{q_0}{k}r_0^2$$

$$\therefore T(r) = T_\infty-\frac{q_0}{k}\left(\frac{1}{6}r^2-\frac{r^4}{20r_0^2}\right)+\frac{2q_0}{15h}r_0+\frac{7}{60}\frac{q_0}{k}r_0^2$$

$$T(r_0) = T_\infty-\frac{q_0}{k}\underbrace{\left(\frac{1}{6}r_0^2-\frac{1}{20}r_0^2\right)}_{\frac{7}{60}r_0^2}+\frac{2}{15}\frac{q_0 r_0}{h}\cancel{+\frac{7}{60}\frac{q_0}{k}r_0^2}$$

$$= T_\infty+\frac{2}{15}\frac{q_0 r_0}{h}$$

0.

$$\cancel{\dot{E}_{in}} + \dot{E}_g = \dot{E}_{out} + \cancel{\dot{E}_{st}}$$

$$\Rightarrow \dot{E}_g = \dot{E}_{out} \Rightarrow \int \dot{q}_g \, dv = hA(T_s - T_\infty)$$

$$\Rightarrow \int_0^{r_0} \dot{q}_0 \left[1 - \left(\frac{r}{r_0}\right)^2\right] 4\pi r^2 dr = h \overbrace{}^{4\pi r_0^2} (T_s - T_\infty)$$

$$\Rightarrow \int_0^{r_0} 4\pi \dot{q}_0 \left(r^2 - \frac{1}{r_0^2} r^4\right) dr$$

$$= 4\pi \dot{q}_0 \left(\frac{1}{3} r^3 - \frac{1}{5} \frac{1}{r_0^2} r^5\right)\Big|_0^{r_0}$$

$$= \frac{8}{15} \pi \dot{q}_0 r_0^3 = 4\pi r_0^2 h (T_s - T_\infty)$$

$$\therefore T_s = T_\infty + \frac{2 \dot{q}_0 r_0}{15 h}$$