

微積分應用

Matlab 在微積分可求極限、導函數及積分等運算。

例一：求極限

syms x y a t h; %syms 定義符號變數

- `limit(sin(x)/x)`
`ans = 1`
- `limit((x-2)/(x^2-4),2)`
`ans = 1/4`
- `limit((1+2*t/x)^(3*x),x,inf)`
`ans = exp(6*t)`
- `limit(1/x,x,0,'right')`
`ans = inf`
- `limit(1/x,x,0,'left')`
`ans = -inf`
- `limit((sin(x+h)-sin(x))/h,h,0)`
`ans = cos(x)`
- `v = [(1 + a/x)^x, exp(-x)];`
`limit(v,x,inf,'left')`
`ans = [exp(a), 0]`

例二：求導函數

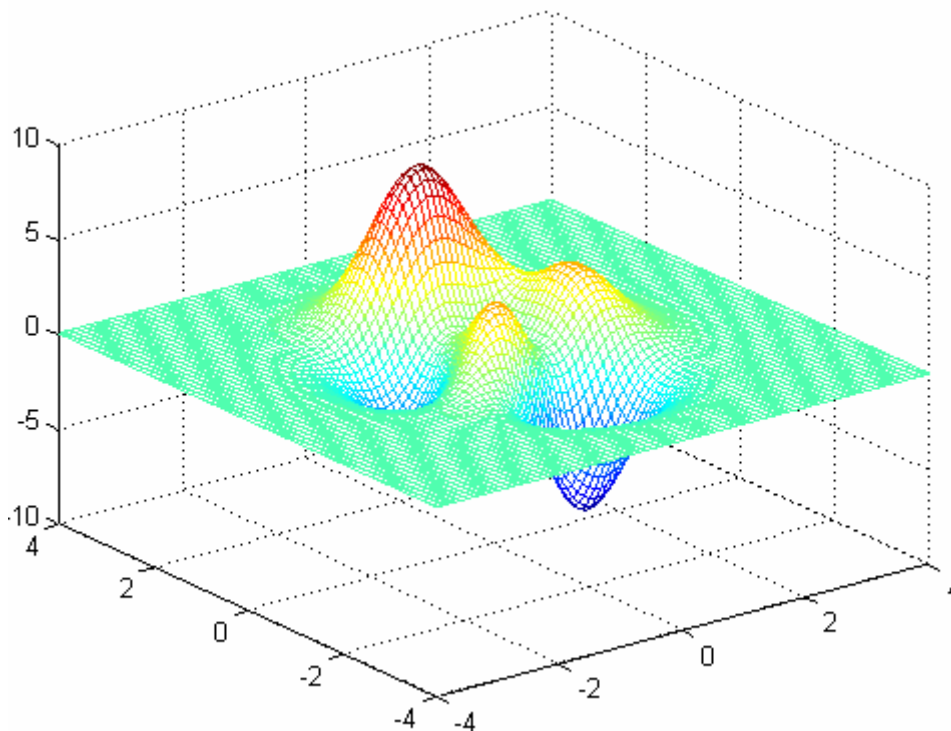
syms x y; %syms 定義符號變數

- %求 $y=3x^4+2x^3-x+7$ 之導函數
`diff('3*x^4+2*x^3-x+7')`
`ans=12*x^3+6*x^2-1`
- %求 $y=(x^3+x^2+2)(2x^3-x^2+3)$ 之導函數
`diff('(x^3+x^2+2)*(2*x^3-x^2+3)')`
`ans=(3*x^2+2*x)*(2*x^3-x^2+3)+(x^3+x^2+2)*(6*x^2-2*x)`
- %求 $y= x / x^{1/2}$ 之導函數
`diff('x/sqrt(x)')`
`ans=1/2/x^(1/2)`
- %求 $y= \sec^4(x)$ 之導函數
`diff('sec(x)^4')`
`ans=4*sec(x)^4*tan(x)`

- 求 $z=3*(1-x)^2*\exp(-(x^2)-(y+1)^2)-10*(x/5-x^3-y^5)*\exp(-x^2-y^2)-1/3*\exp(-(x+1)^2-y^2)$ 之一階和二階偏導函數

圖形

```
>> [x,y]=meshgrid(-4:0.1:4);
>> z=3*(1-x).^2.*exp(-(x.^2)-(y+1).^2)-10.*(x./5-x.^3-y.^5).*exp(-x.^2-y.^2)
-1/3.*exp(-(x+1).^2-y.^2);
>> mesh(x,y,z)
```



```
>> syms x y
>> z=3*(1-x).^2.*exp(-(x.^2)-(y+1).^2)-10.*(x./5-x.^3-y.^5).*exp(-x.^2-y.^2)
-1/3.*exp(-(x+1).^2-y.^2);
>> diff(z,x) (求  $\frac{\partial z}{\partial x}$ )
ans = -6*(1-x)*exp(-x^2-(y+1)^2)-6*(1-x)^2*x*exp(-x^2-(y+1)^2)
-(2-30*x^2)*exp(-x^2-y^2)+2*(2*x-10*x^3-10*y^5)*x*exp(-x^2-y^2)
-1/3*(-2*x-2)*exp(-(x+1)^2-y^2)
>> diff(z,y) (求  $\frac{\partial z}{\partial y}$ )
ans = 3*(1-x)^2*(-2*y-2)*exp(-x^2-(y+1)^2)+50*y^4*exp(-x^2-y^2)
+2*(2*x-10*x^3-10*y^5)*y*exp(-x^2-y^2)+2/3*y*exp(-(x+1)^2-y^2)
>> diff(diff(z,x),x) (求  $\frac{\partial^2 z}{\partial x^2}$ )
ans = 6*exp(-x^2-(y+1)^2)+24*(1-x)*x*exp(-x^2-(y+1)^2)
-6*(1-x)^2*exp(-x^2-(y+1)^2)+12*(1-x)^2*x^2*exp(-x^2-(y+1)^2)
+60*x*exp(-x^2-y^2)+4*(2-30*x^2)*x*exp(-x^2-y^2)+2*(2*x-10*x^3-10*y^5)*exp(-x^2-y^2)-4*(2*x-10*x^3-10*y^5)*x^2*exp(-x^2-y^2)
```

```

+2/3*exp(-(x+1)^2-y^2)-1/3*(-2*x-2)^2*exp(-(x+1)^2-y^2)
>> diff(diff(z,x),y) (求  $\frac{\partial^2 z}{\partial x \partial y}$ )
ans = -6*(1-x)*(-2*y-2)*exp(-x^2-(y+1)^2)-6*(1-x)^2*x*(-2*y-2)
      *exp(-x^2-(y+1)^2)+2*(2-30*x^2)*y*exp(-x^2-y^2)-100*y^4*x
      *exp(-x^2-y^2)-4*(2*x-10*x^3-10*y^5)*x*y*exp(-x^2-y^2)
      +2/3*(-2*x-2)*y*exp(-(x+1)^2-y^2)
>> diff(diff(z,y),x) (求  $\frac{\partial^2 z}{\partial y \partial x}$ )
ans = -6*(1-x)*(-2*y-2)*exp(-x^2-(y+1)^2)-6*(1-x)^2*x*(-2*y-2)
      *exp(-x^2-(y+1)^2)+2*(2-30*x^2)*y*exp(-x^2-y^2)-100*y^4*x
      *exp(-x^2-y^2)-4*(2*x-10*x^3-10*y^5)*x*y*exp(-x^2-y^2)
      +2/3*(-2*x-2)*y*exp(-(x+1)^2-y^2)
>> diff(diff(z,y),y) (求  $\frac{\partial^2 z}{\partial y^2}$ )
ans = -6*(1-x)^2*exp(-x^2-(y+1)^2)
      +3*(1-x)^2*(-2*y-2)^2*exp(-x^2-(y+1)^2)
      +200*y^3*exp(-x^2-y^2)-200*y^5*exp(-x^2-y^2)
      +2*(2*x-10*x^3-10*y^5)*exp(-x^2-y^2)
      -4*(2*x-10*x^3-10*y^5)*y^2*exp(-x^2-y^2)
      +2/3*exp(-(x+1)^2-y^2)-4/3*y^2*exp(-(x+1)^2-y^2)

```

Matlab 中以 int 指令作積分運算

例三：積分運算（定積分）

```

syms x y; %syms 定義符號變數
int('x^2+1','x',0,1)
ans = 4/3

```

```

int('1/sqrt(4-x^2)','x',0,1)
ans = 1/12*4^(1/2)*pi

```

例四：積分運算（不定積分）

```

syms x y; %syms 定義符號變數
int(x)
ans = 1/2*x^2

```

```

int(sin(x))
ans = -cos(x)

```

例五：泰勒級數

```

syms x y; %syms 定義符號變數
taylor(1/(1-x))
ans = 1+x+x^2+x^3+x^4+x^5

```

```

taylor(exp(x))
ans = 1+x+1/2*x^2+1/6*x^3+1/24*x^4+1/120*x^5

```

```
taylor(sin(x))  
ans= x-1/6*x^3+1/120*x^5
```

```
taylor(cos(x))  
ans= 1-1/2*x^2+1/24*x^4
```

```
taylor(cos(x),10)  
ans= 1-1/2*x^2+1/24*x^4-1/720*x^6+1/40320*x^8
```

```
taylor(log(1+x))  
ans= x-1/2*x^2+1/3*x^3-1/4*x^4+1/5*x^5
```

```
taylor(exp(-x))  
ans=1-x+1/2*x^2-1/6*x^3+1/24*x^4-1/120*x^5
```

```
taylor(log(x),6,1)  
ans=x-1-1/2*(x-1)^2+1/3*(x-1)^3-1/4*(x-1)^4+1/5*(x-1)^5
```

```
taylor(sin(x),pi/2,6)  
ans=1-1/2*(x-1/2*pi)^2+1/24*(x-1/2*pi)^4
```

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