

流體力學 HW#3 Solution

6-25 A reducing elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The anchoring force needed to hold the elbow in place is to be determined. ✓

Assumptions 1 The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). 2 The weight of the elbow and the water in it is considered. 3 The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. 4 The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be 1000 kg/m^3 .

Analysis The weight of the elbow and the water in it is

$$W = mg = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N} = 0.4905 \text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z . The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30 \text{ kg/s}$. Noting that $\dot{m} = \rho AV$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.0150 \text{ m}^2)} = 2.0 \text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.0025 \text{ m}^2)} = 12 \text{ m/s}$$

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow P_1 - P_2 = \rho g \left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1 \right) \rightarrow P_{1, \text{gage}} = \rho g \left(\frac{V_2^2 - V_1^2}{2g} + z_2 \right)$$

Substituting,

$$P_{1, \text{gage}} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \left(\frac{(12 \text{ m/s})^2 - (2 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0.4 \right) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 73.9 \text{ kN/m}^2 = 73.9 \text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$. We let the x - and

z - components of the anchoring force of the elbow be F_{Rx} and F_{Rz} , and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and z axes become

$$F_{Rx} + P_{1, \text{gage}} A_1 = \beta \dot{m} V_2 \cos \theta - \beta \dot{m} V_1 \quad \text{and} \quad F_{Rz} - W = \beta \dot{m} V_2 \sin \theta$$

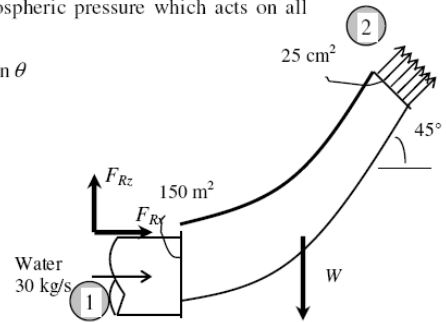
Solving for F_{Rx} and F_{Rz} , and substituting the given values,

$$\begin{aligned} F_{Rx} &= \beta \dot{m} (V_2 \cos \theta - V_1) - P_{1, \text{gage}} A_1 \\ &= 1.03(30 \text{ kg/s})[(12 \cos 45^\circ - 2) \text{ m/s}] \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &\quad - (73.9 \text{ kN/m}^2)(0.0150 \text{ m}^2) \\ &= -0.908 \text{ kN} \end{aligned}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin \theta + W = 1.03(30 \text{ kg/s})(12 \sin 45^\circ \text{ m/s}) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) + 0.4905 \text{ kN} = 0.753 \text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-0.908)^2 + (0.753)^2} = \mathbf{1.18 \text{ kN}}, \quad \theta = \tan^{-1} \frac{F_{Rz}}{F_{Rx}} = \tan^{-1} \frac{0.753}{-0.908} = \mathbf{-39.7^\circ}$$

Discussion Note that the magnitude of the anchoring force is 0.936 kN , and its line of action makes -39.7° from $+x$ direction. Negative value for F_{Rx} indicates the assumed direction is wrong.



6-27 Water accelerated by a nozzle strikes the back surface of a cart moving horizontally at a constant velocity. The braking force and the power wasted by the brakes are to be determined. .

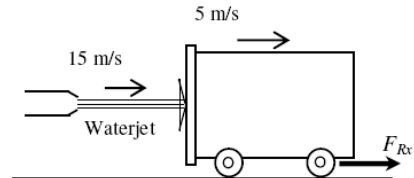
Assumptions **1** The flow is steady and incompressible. **2** The water splatters off the sides of the plate in all directions in the plane of the back surface. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** Friction during motion is negligible. **5** There is no acceleration of the cart. **7** The motions of the water jet and the cart are horizontal. **6** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Analysis We take the cart as the control volume, and the direction of flow as the positive direction of x axis. The relative velocity between the cart and the jet is

$$V_r = V_{\text{jet}} - V_{\text{cart}} = 15 - 10 = 10 \text{ m/s}$$

Therefore, we can assume the cart to be stationary and the jet to move with a velocity of 10 m/s. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{\text{brake}} = -\dot{m} V_r$$



We note that the brake force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x -direction. Substituting the given values,

$$F_{\text{brake}} = -\dot{m} V_r = -(25 \text{ kg/s})(+10 \text{ m/s}) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = -250 \text{ N}$$

The negative sign indicates that the braking force acts in the opposite direction to motion, as expected. Noting that work is force times distance and the distance traveled by the cart per unit time is the cart velocity, the power wasted by the brakes is

$$\dot{W} = F_{\text{brake}} V_{\text{cart}} = (250 \text{ N})(5 \text{ m/s}) \left(\frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}} \right) = 1.25 \text{ kW}$$

6-42 The flow rate in a channel is controlled by a sluice gate by raising or lowering a vertical plate. A relation for the force acting on a sluice gate of width w for steady and uniform flow is to be developed.

Assumptions 1 The flow is steady, incompressible, frictionless, and uniform (and thus the Bernoulli equation is applicable.) 2 Wall shear forces at surfaces are negligible. 3 The channel is exposed to the atmosphere, and thus the pressure at free surfaces is the atmospheric pressure. 4 The flow is horizontal. 5 Water flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Analysis We take point 1 at the free surface of the upstream flow before the gate and point 2 at the free surface of the downstream flow after the gate. We also take the bottom surface of the channel as the reference level so that the elevations of points 1 and 2 are y_1 and y_2 , respectively. The application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + y_2 \rightarrow V_2^2 - V_1^2 = 2g(y_1 - y_2) \quad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1 = \dot{V}_2 = \dot{V} \rightarrow A_1 V_1 = A_2 V_2 = \dot{V} \rightarrow V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{wy_1} \quad \text{and} \quad V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{wy_2} \quad (2)$$

Substituting into Eq. (1),

$$\left(\frac{\dot{V}}{wy_2} \right)^2 - \left(\frac{\dot{V}}{wy_1} \right)^2 = 2g(y_1 - y_2) \rightarrow \dot{V} = w \sqrt{\frac{2g(y_1 - y_2)}{1/y_2^2 - 1/y_1^2}} \rightarrow \dot{V} = wy_2 \sqrt{\frac{2g(y_1 - y_2)}{1 - y_2^2/y_1^2}} \quad (3)$$

Substituting Eq. (3) into Eqs. (2) gives the following relations for velocities,

$$V_1 = \frac{y_2}{y_1} \sqrt{\frac{2g(y_1 - y_2)}{1 - y_2^2/y_1^2}} \quad \text{and} \quad V_2 = \sqrt{\frac{2g(y_1 - y_2)}{1 - y_2^2/y_1^2}} \quad (4)$$

We choose the control volume as the water body surrounded by the vertical cross-sections of the upstream and downstream flows, free surfaces of water, the inner surface of the sluice gate, and the bottom surface of the channel. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$. The

force acting on the sluice gate F_{Rx} is horizontal since the wall shear at the surfaces is negligible, and it is equal and opposite to the force applied on water by the sluice gate. Noting that the pressure force acting on a vertical surface is equal to the product of the pressure at the centroid of the surface and the surface area, the momentum equation along the x direction gives

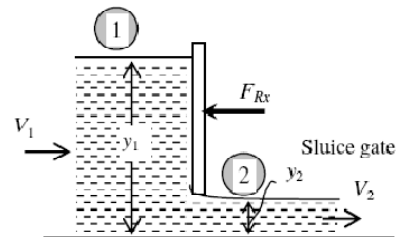
$$-F_{Rx} + P_1 A_1 - P_2 A_2 = \dot{m} V_2 - \dot{m} V_1 \rightarrow -F_{Rx} + \left(\rho g \frac{y_1}{2} \right) (wy_1) - \left(\rho g \frac{y_2}{2} \right) (wy_2) = \dot{m} (V_2 - V_1)$$

Rearranging, the force acting on the sluice gate is determined to be

$$F_{Rx} = \dot{m} (V_1 - V_2) + \frac{w}{2} \rho g (y_1^2 - y_2^2) \quad (5)$$

where V_1 and V_2 are given in Eq. (4).

Discussion Note that for $y_1 \gg y_2$, Eq. (3) simplifies to $\dot{V} = y_2 w \sqrt{2gy_1}$ or $V_2 = \sqrt{2gy_1}$ which is the Toricelli equation for frictionless flow from a tank through a hole a distance y_1 below the free surface.



6-58 Water is flowing into and discharging from a pipe U-section with a secondary discharge section normal to return flow. Net x - and z - forces at the two flanges that connect the pipes are to be determined.

Assumptions 1 The flow is steady and incompressible. 2 The weight of the U-turn and the water in it is negligible. 4 The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

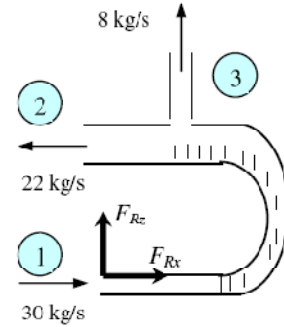
Properties We take the density of water to be 1000 kg/m^3 .

Analysis The flow velocities of the 3 streams are

$$V_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{\dot{m}_1}{\rho(\pi D_1^2 / 4)} = \frac{30 \text{ kg/s}}{(1000 \text{ kg/m}^3)[\pi(0.05 \text{ m})^2 / 4]} = 0.51 \text{ m/s}$$

$$V_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{\dot{m}_2}{\rho(\pi D_2^2 / 4)} = \frac{22 \text{ kg/s}}{(1000 \text{ kg/m}^3)[\pi(0.10 \text{ m})^2 / 4]} = 2.80 \text{ m/s}$$

$$V_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{\dot{m}_3}{\rho(\pi D_3^2 / 4)} = \frac{8 \text{ kg/s}}{(1000 \text{ kg/m}^3)[\pi(0.03 \text{ m})^2 / 4]} = 11.3 \text{ m/s}$$



We take the entire U-section as the control volume. We designate the horizontal coordinate by x with the direction of incoming flow as being the positive direction and the vertical coordinate by z . The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$. We let the x - and z - components

of the anchoring force of the cone be F_{Rx} and F_{Rz} , and assume them to be in the positive directions. Then the momentum equations along the x and z axes become

$$\begin{aligned} F_{Rx} + P_1 A_1 + P_2 A_2 &= \beta \dot{m}_2 (-V_2) - \beta \dot{m}_1 V_1 \quad \rightarrow \quad F_{Rx} = -P_1 A_1 - P_2 A_2 - \beta(\dot{m}_2 V_2 + \dot{m}_1 V_1) \\ F_{Rz} + 0 - \dot{m}_3 V_3 &= 0 \quad \rightarrow \quad F_{Rz} = \beta \dot{m}_3 V_3 \end{aligned}$$

Substituting the given values,

$$\begin{aligned} F_{Rx} &= -[(200 - 100) \text{ kN/m}^2] \frac{\pi(0.05 \text{ m})^2}{4} - [(150 - 100) \text{ kN/m}^2] \frac{\pi(0.10 \text{ m})^2}{4} \\ &\quad - 1.03 \left[(22 \text{ kg/s})(2.80 \text{ m/s}) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) + (30 \text{ kg/s})(0.51 \text{ m/s}) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \right] \\ &= -0.668 \text{ kN} = \mathbf{-668 \text{ N}} \\ F_{Rz} &= 1.03(8 \text{ kg/s})(11.3 \text{ m/s}) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = \mathbf{93.1 \text{ N}} \end{aligned}$$

The negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed. Therefore, a force of 668 N acts on the flanges in the opposite direction. A vertical force of 93.1 N acts on the flange in the vertical direction.