

流體力學 HW#1 Solution

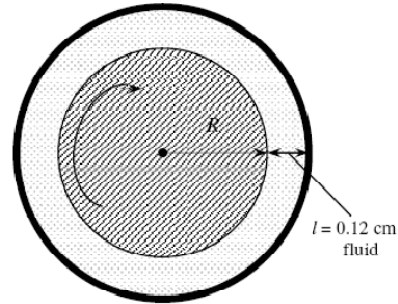
2-51 The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in oil. **2** The viscous effects on the two ends of the inner cylinder are negligible. **3** The fluid is Newtonian.

Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n}L} = \frac{(0.8 \text{ N} \cdot \text{m})(0.012 \text{ m})}{4\pi^2 (0.075 \text{ m})^3 (200/60 \text{ s}^{-1})(0.75 \text{ m})} = 0.0231 \text{ N} \cdot \text{s/m}^2$$

Discussion This is the viscosity value at the temperature that existed during the experiment. Viscosity is a strong function of temperature, and the values can be significantly different at different temperatures.



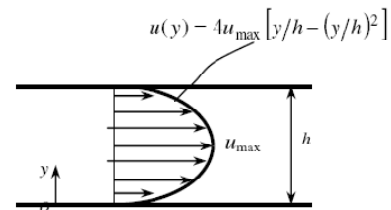
2-73 The velocity profile for laminar one-dimensional flow between two parallel plates is given. A relation for friction drag force exerted on the plates per unit area of the plates is to be obtained.

Assumptions **1** The flow between the plates is one-dimensional. **2** The fluid is Newtonian.

Analysis The velocity profile is given by $u(y) = Au_{\max} [y/h - (y/h)^2]$

where h is the distance between the two plates, y is the vertical distance from the bottom plate, and $u_{\max}$ is the maximum flow velocity that occurs at midplane. The shear stress at the bottom surface can be expressed as

$$\tau_w = \mu \left. \frac{du}{dy} \right|_{y=0} = 4\mu u_{\max} \frac{d}{dy} \left(\frac{y}{h} - \frac{y^2}{h^2} \right) \bigg|_{y=0} = 4\mu u_{\max} \left(\frac{1}{h} - \frac{2y}{h^2} \right) \bigg|_{y=0} = \frac{4\mu u_{\max}}{h}$$



Because of symmetry, the wall shear stress is identical at both bottom and top plates. Then the friction drag force exerted by the fluid on the inner surface of the plates becomes

$$F_D = 2\tau_w A_{\text{plate}} = \frac{8\mu u_{\max}}{h} A_{\text{plate}}$$

Therefore, the friction drag per unit plate area is

$$F_D / A_{\text{plate}} = \frac{8\mu u_{\max}}{h}$$

Discussion Note that the friction drag force acting on the plates is inversely proportional to the distance between plates.

3-36 Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

Assumptions 1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible.

Properties The densities of seawater and mercury are given to be $\rho_{\text{sea}} = 1035 \text{ kg/m}^3$ and $\rho_{\text{Hg}} = 13,600 \text{ kg/m}^3$. We take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$.

Analysis Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the sea water pipe (point 2), and setting the result equal to P_2 gives

$$P_1 + \rho_w gh_w - \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{air}} gh_{\text{air}} + \rho_{\text{sea}} gh_{\text{sea}} = P_2$$

Rearranging and neglecting the effect of air column on pressure,

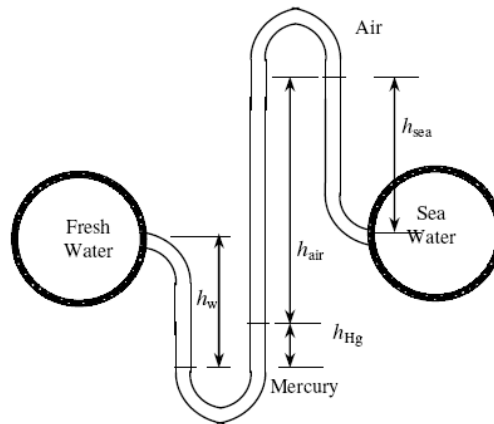
$$P_1 - P_2 = -\rho_w gh_w + \rho_{\text{Hg}} gh_{\text{Hg}} - \rho_{\text{sea}} gh_{\text{sea}} = g(\rho_{\text{Hg}} h_{\text{Hg}} - \rho_w h_w - \rho_{\text{sea}} h_{\text{sea}})$$

Substituting,

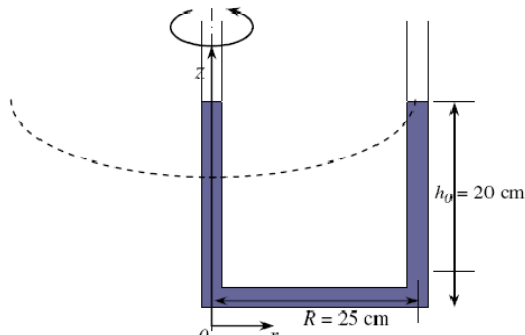
$$\begin{aligned} P_1 - P_2 &= (9.81 \text{ m/s}^2)[(13600 \text{ kg/m}^3)(0.1 \text{ m}) \\ &\quad - (1000 \text{ kg/m}^3)(0.6 \text{ m}) - (1035 \text{ kg/m}^3)(0.4 \text{ m})] \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) \\ &= 3.39 \text{ kN/m}^2 = \mathbf{3.39 \text{ kPa}} \end{aligned}$$

Therefore, the pressure in the fresh water pipe is 3.39 kPa higher than the pressure in the sea water pipe.

Discussion A 0.70-m high air column with a density of 1.2 kg/m^3 corresponds to a pressure difference of 0.008 kPa. Therefore, its effect on the pressure difference between the two pipes is negligible.



3-104 A vertical U-tube partially filled with alcohol is rotated at a specified rate about one of its arms. The elevation difference between the fluid levels in the two arms is to be determined.



Assumptions 1 Alcohol is an incompressible fluid.

Analysis Taking the base of the left arm of the U-tube as the origin ($r = 0$, $z = 0$), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where $h_0 = 0.20$ m is the original height of the liquid before rotation, and $\omega = 4.2$ rad/s. The fluid rise at the right arm relative to the fluid level in the left arm (the center of rotation) is

$$\Delta h = z_s(R) - z_s(0) = \left(h_0 + \frac{\omega^2 R^2}{4g}\right) - \left(h_0 - \frac{\omega^2 R^2}{4g}\right) = \frac{\omega^2 R^2}{2g} = \frac{(4.2 \text{ rad/s})^2 (0.25 \text{ m})^2}{2(9.81 \text{ m/s}^2)} = \mathbf{0.056 \text{ m}}$$

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property.

3-111 A helium balloon tied to the ground carries 2 people. The acceleration of the balloon when it is first released is to be determined.

Assumptions The weight of the cage and the ropes of the balloon is negligible.

Properties The density of air is given to be $\rho = 1.16 \text{ kg/m}^3$. The density of helium gas is 1/7th of this.

Analysis The buoyancy force acting on the balloon is

$$V_{\text{balloon}} = 4\pi r^3 / 3 = 4\pi (5 \text{ m})^3 / 3 = 523.6 \text{ m}^3$$

$$F_B = \rho_{\text{air}} g V_{\text{balloon}} = (1.16 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(523.6 \text{ m}^3) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 5956.5 \text{ N}$$

The total mass is

$$m_{\text{He}} = \rho_{\text{He}} V = \left(\frac{1.16}{7} \text{ kg/m}^3 \right) (523.6 \text{ m}^3) = 86.8 \text{ kg}$$

$$m_{\text{total}} = m_{\text{He}} + m_{\text{people}} = 86.8 + 2 \times 70 = 226.8 \text{ kg}$$

The total weight is

$$W = m_{\text{total}} g = (226.8 \text{ kg})(9.807 \text{ m/s}^2) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 2224.2 \text{ N}$$

Thus the net force acting on the balloon is

$$F_{\text{net}} = F_B - W = 5956.5 - 2224.2 = 3732.3 \text{ N}$$

Then the acceleration becomes

$$a = \frac{F_{\text{net}}}{m_{\text{total}}} = \frac{3732.3 \text{ N}}{226.8 \text{ kg}} \left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = \mathbf{16.5 \text{ m/s}^2}$$

