Test Sample-3

1. As shown in Fig. 1, a tornado may be simulated as a Rankine vortex which has a two-port circulating flow in plane polar coordinates, with $u_\theta = U$ at $r = R$ and $u_r = 0$. For a Rankine vortex, the velocity distribution is as follows:

$$u_\theta = \frac{U}{R} r \quad , \quad u_r = 0 \quad \text{if} \quad r \le R$$

$$u_\theta = \frac{C}{r} \quad , \quad u_r = 0 \quad \text{if} \quad r > R$$

where C is a constant. Please determine

(a) constant C.

(b) an expression of stream function φ for a Rankine vortex.

(c) the vorticity in each part of the Rankine vortex.

The relation between velocity components and stream function is expressed by $u_\theta = -\frac{\partial \varphi}{\partial r}$, $u_r = \frac{1}{r}\frac{\partial \varphi}{\partial \theta}$. The vorticity component about the z-axis is $\omega_z = \frac{1}{r}\frac{\partial}{\partial r}\left(r u_\theta\right) - \frac{1}{r}\frac{\partial u_r}{\partial u_\theta}$

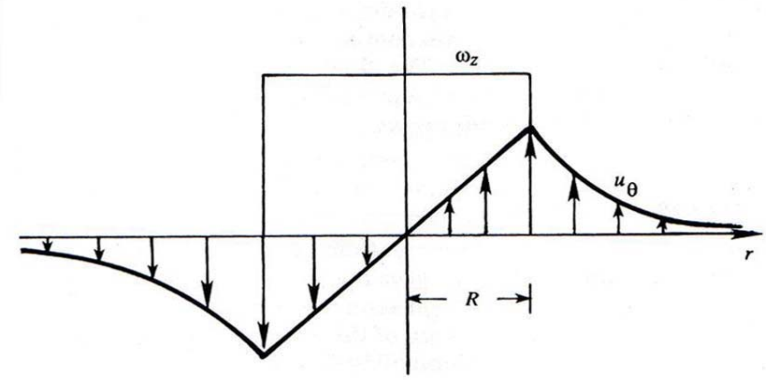

Fig. 1

2. Consider the laminar flow of a fluid layer falling down a plane inclined at an angle α with the horizontal, as depicted in Fig. 2. Assume that air resistance is negligible at the free surface. If h is the thickness of the layer in the fully developed stage, please

(a) determine the velocity distribution.

(b) find the volume flow rate per unit width.

(c) find he frictional stress on the wall.

(d) plot the distribution of shear stress between the bottom wall and free surface.

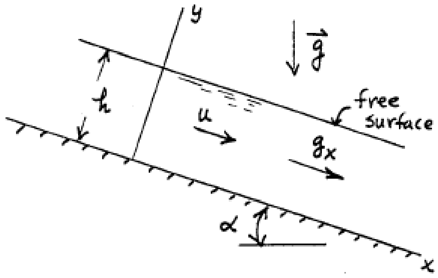

Fig. 2

3. As shown in Fig. 3, a liquid flows from a large tank (d=0.4 m) into a small tube (d=1.2 mm) in the center of the bottom of the large tank. There is 0.4 m of liquid in the large tank and the length of the small capillary tube is 0.5 m. Assume the flow is laminar. Both the inlet of the large tank and the discharge of small tube are open to the atmosphere. The flow is supported by gravity alone, and the level of liquid in the large tank is maintained constant. You may neglect the friction effect of the large tank, but the friction effect of the small capillary tube cannot be neglected. The loss coefficient, k_m accounting for the total minor losses of the entrance and the exit of the small capillary tube is 1.41. Calculate the kinematic viscosity of the liquid.

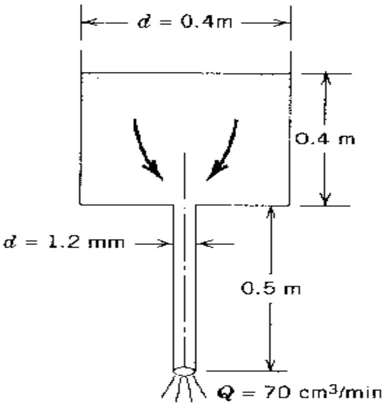

Fig. 3

4. By consideration of the cylindrical elemental control volume shown in Fig. 4, use the conservation of mass to derive the continuity equation in cylindrical coordinates, that is,

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r \mathrm{v}_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho \mathrm{v}_\theta)+\frac{\partial}{\partial z}(\rho \mathrm{v}_z)=0$$

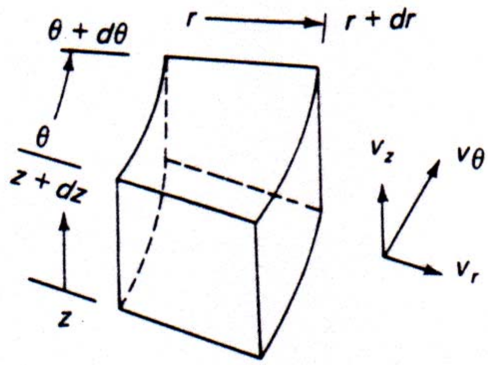

Fig. 4

5. Fig. 5 displays a 2-D circular Couette flow. Let the radius and angular velocity of the inner cylinder be r_1 and ω_1, and those for the outer cylinder be r_2 and ω_2.

(a) Please find the velocity and pressure distributions.

(b)From the result of (a), please find velocity distribution if $r_2 \to \infty$ and $\omega_2 = 0$. Other operating conditions are kept the same as (a).

(c)From the result of (a), please find velocity distribution if $r_1 = 0$ and $\omega_1 = 0$. Other operating conditions are kept the same as (a).

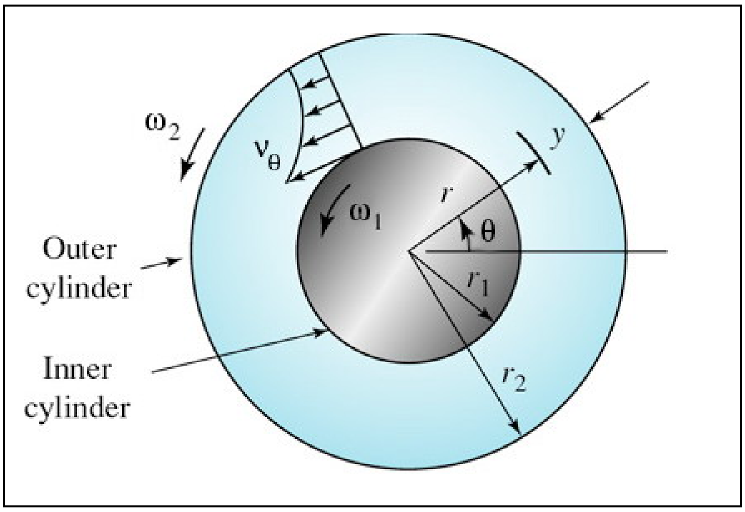

Fig. 5

6. Consider the steady laminar flow through the annular space formed by two coaxial tubes, as shown in Fig 6. The flow is along the axis of the tubes and is maintained by a pressure gradient *dp/dx*, where the *x* direction is taken along the axis of the tubes. Please find
   (a) the velocity distribution at any radius *r*.
   (b) the radius at which the maximum velocity is reached.
   (c) the stress distribution.

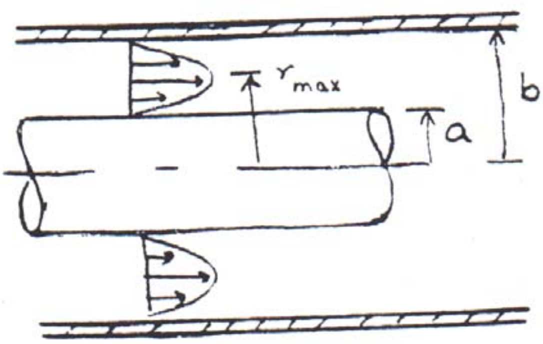

Fig. 6