熱傳 HW#1 Solution

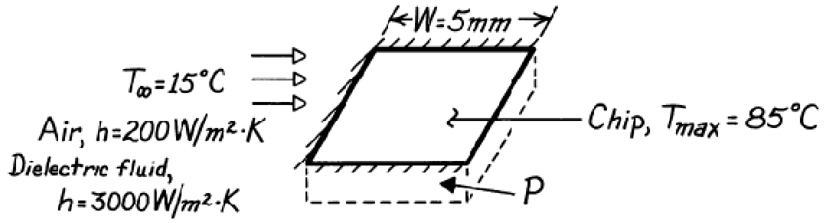
1.

PROBLEM 1.18

KNOWN: Chip width and maximum allowable temperature. Coolant conditions.

FIND: Maximum allowable chip power for air and liquid coolants.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

ANALYSIS: All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_\infty) = h\,W^2(T - T_\infty).$$

In *air*,

$$P_{max} = 200\ \text{W/m}^2\cdot\text{K}(0.005\ \text{m})^2(85 - 15)\ ^\circ\text{C} = 0.35\ \text{W}. \qquad <$$

In the *dielectric liquid*

$$P_{max} = 3000\ \text{W/m}^2\cdot\text{K}(0.005\ \text{m})^2(85\text{-}15)\ ^\circ\text{C} = 5.25\ \text{W}. \qquad <$$

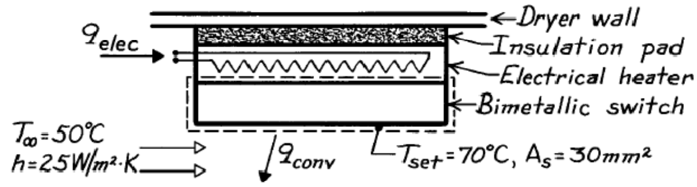
2.

PROBLEM 1.21

KNOWN: Upper temperature set point, T_{set}, of a bimetallic switch and convection heat transfer coefficient between clothes dryer air and exposed surface of switch.

FIND: Electrical power for heater to maintain T_{set} when air temperature is $T_\infty = 50°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Electrical heater is perfectly insulated from dryer wall, (3) Heater and switch are isothermal at T_{set}, (4) Negligible heat transfer from sides of heater or switch, (5) Switch surface, A_s, loses heat only by convection.

ANALYSIS: Define a control volume around the bimetallic switch which experiences heat input from the heater and convection heat transfer to the dryer air. That is,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$
$$q_{elec} - hA_s(T_{set} - T_\infty) = 0.$$

The electrical power required is,

$$q_{elec} = hA_s(T_{set} - T_\infty)$$

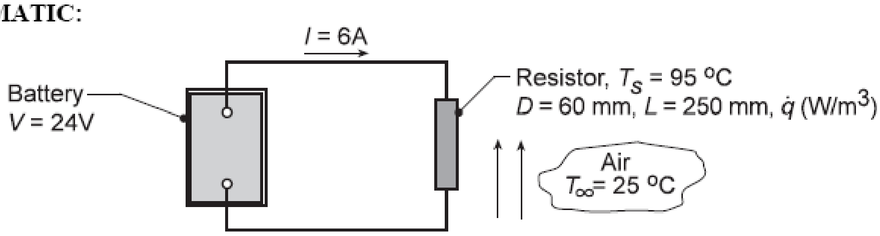
$$q_{elec} = 25\ W/m^2 \cdot K \times 30 \times 10^{-6}\ m^2 (70 - 50) K = 15\ mW. \qquad <$$

3.

KNOWN: Resistor connected to a battery operating at a prescribed temperature in air.

FIND: (a) Considering the resistor as the system, determine corresponding values for $\dot{E}_{in}(W)$, $\dot{E}_g(W)$, $\dot{E}_{out}(W)$ and $\dot{E}_{st}(W)$. If a control surface is placed about the entire system, determine the values for $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$. (b) Determine the volumetric heat generation rate within the resistor, $\dot{q}$ (W/m^3), (c) Neglecting radiation from the resistor, determine the convection coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Electrical power is dissipated uniformly within the resistor, (2) Temperature of the resistor is uniform, (3) Negligible electrical power dissipated in the lead wires, (4) Negligible radiation exchange between the resistor and the surroundings, (5) No heat transfer occurs from the battery, (5) Steady-state conditions.

ANALYSIS: (a) Referring to Section 1.3.1, the conservation of energy requirement for a control volume at an instant of time, Eq 1.11a, is

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

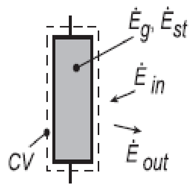
where $\dot{E}_{in}$, $\dot{E}_{out}$ correspond to *surface* inflow and outflow processes, respectively. The energy generation term $\dot{E}_g$ is associated with conversion of some other energy form (chemical, electrical, electromagnetic or nuclear) to thermal energy. The energy storage term $\dot{E}_{st}$ is associated with changes in the internal, kinetic and/or potential energies of the matter in the control volume. $\dot{E}_g$, $\dot{E}_{st}$ are *volumetric* phenomena. The electrical power delivered by the battery is P = VI = 24V×6A = 144 W.

Control volume: Resistor.

$\dot{E}_{in} = 0$ $\quad\quad$ $\dot{E}_{out} = 144\,\text{W}$

<

$\dot{E}_g = 144\,\text{W}$ $\quad\quad$ $\dot{E}_{st} = 0$



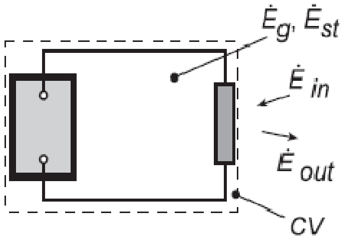
The $\dot{E}_g$ term is due to conversion of electrical energy to thermal energy. The term $\dot{E}_{out}$ is due to convection from the resistor surface to the air.

Continued...

Control volume: Battery-Resistor System.

$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,\text{W} \qquad <$$

$$\dot{E}_g = 0 \qquad \dot{E}_{st} = -144\,\text{W}$$



The $\dot{E}_{st}$ term represents the decrease in the chemical energy within the battery. The conversion of chemical energy to electrical energy and its subsequent conversion to thermal energy are processes internal to the system which are not associated with $\dot{E}_{st}$ or $\dot{E}_g$. The $\dot{E}_{out}$ term is due to convection from the resistor surface to the air.

(b) From the energy balance on the resistor with volume, $\forall = (\pi D^2/4)L$,

$$\dot{E}_g = \dot{q}\forall \qquad 144\,\text{W} = \dot{q}\left(\pi\,(0.06\,\text{m})^2/4\right)\times 0.25\,\text{m} \qquad \dot{q} = 2.04\times10^5\ \text{W/m}^3 \qquad <$$

(c) From the energy balance on the resistor and Newton's law of cooling with $A_s = \pi DL + 2(\pi D^2/4)$,

$$\dot{E}_{out} = q_{cv} = hA_s\,(T_s - T_\infty)$$

$$144\,\text{W} = h\left[\pi\times0.06\,\text{m}\times0.25\,\text{m} + 2\left(\pi\times0.06^2\,\text{m}^2/4\right)\right](95-25)^\circ\text{C}$$

$$144\,\text{W} = h\,[0.0471 + 0.0057]\,\text{m}^2\,(95-25)^\circ\text{C}$$

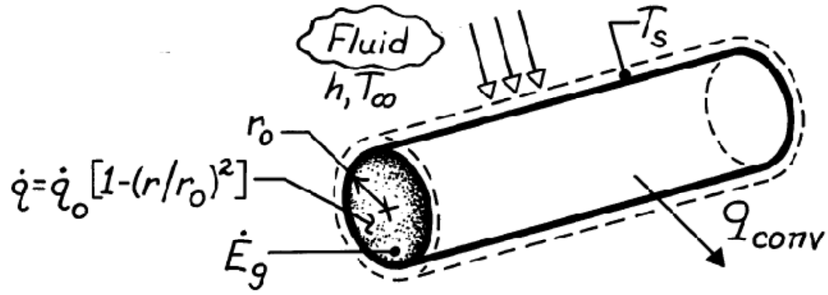
$$h = 39.0\,\text{W/m}^2\!\cdot\!\text{K} \qquad <$$

4.

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Total energy generation rate and surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature drop across thin container wall.

ANALYSIS: The rate of energy generation is

$$\dot{E}_g = \int \dot{q}dV = \dot{q}_o \int_0^{r_o} \left[1-(r/r_o)^2\right] 2\pi r L dr$$

$$\dot{E}_g = 2\pi L \dot{q}_o \left(r_o^2/2 - r_o^2/4\right)$$

or per unit length,

$$\dot{E}'_g = \frac{\pi \dot{q}_o r_o^2}{2}.$$ <

Performing an energy balance for a control surface about the container yields, at an instant,

$$\dot{E}'_g - \dot{E}'_{out} = 0$$

and substituting for the convection heat rate per unit length,

$$\frac{\pi \dot{q}_o r_o^2}{2} = h(2\pi r_o)(T_s - T_\infty)$$

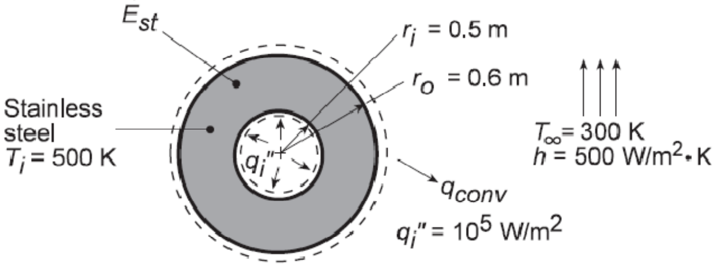
$$T_s = T_\infty + \frac{\dot{q}_o r_o}{4h}.$$ <

5.

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time. Initial rate of temperature change, (b) Steady-state wall temperature, (c) Effect of convection coefficient on canister temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

PROPERTIES: *Table A.1*, Stainless Steel, AISI 302: $\rho = 8055\ \mathrm{kg/m^3}$, $c_p = 510\ \mathrm{J/kg{\cdot}K}$.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$. Identifying relevant processes and solving for dT/dt,

$$q_i''\left(4\pi r_i^2\right) - h\left(4\pi r_o^2\right)\left(T - T_\infty\right) = \rho\frac{4}{3}\pi\left(r_o^3 - r_i^3\right)c_p\frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p\left(r_o^3 - r_i^3\right)}\left[q_i'' r_i^2 - h r_o^2\left(T - T_\infty\right)\right].$$

Substituting numerical values for the initial condition, find

$$\left.\frac{dT}{dt}\right)_i = \frac{3\left[10^5\,\dfrac{W}{m^2}(0.5m)^2 - 500\dfrac{W}{m^2\cdot K}(0.6m)^2(500-300)K\right]}{8055\dfrac{kg}{m^3}\,510\dfrac{J}{kg\cdot K}\left[(0.6)^3 - (0.5)^3\right]m^3}$$

$$\left.\frac{dT}{dt}\right)_i = -0.089\ \mathrm{K/s}.$$ <

(b) Under steady-state conditions with $\dot{E}_{st} = 0$, it follows that

$$q_i''\left(4\pi r_i^2\right) = h\left(4\pi r_o^2\right)\left(T - T_\infty\right)$$

$$T = T_\infty + \frac{q_i''}{h}\left(\frac{r_i}{r_o}\right)^2 = 300K + \frac{10^5\,W/m^2}{500W/m^2\cdot K}\left(\frac{0.5m}{0.6m}\right)^2 = 439K$$