Mid-term Exam-1 (2010/04/06)
Advanced Heat Transfer

1. (a) Please explain the physical meaning of Biot number. What is the criterion for the Biot number when the lumped capacitance method can be used? (3%)

   (b) Consider a 1-D space bounded by two rigid boundaries at $y= 0$ and $y= h$. Initially, the whole space, including both boundaries, is held at a uniform temperature, T_0. Now, the lower plate (y=0) is suddenly kept at a constant temperature T_1 ($\neq T_0$). Does this problem exist similarity solution? Why? (3%)

   (c) If the similarity solution can be applied to the problem of part (b), when does the similarity solution fail? (2%)

   (d) If two plates (one is made by wood and the other is made by copper) with the same size are subjected to the same thermal boundary conditions, are their temperature distributions at steady state the same? Please give your reason. Assume that both materials have constant thermal conductivities. Also, there are no heat generations within both plates. (2%)

   (e) As shown in Fig. 1, heat is supplied to one end of A, and the whole system is insulated from heat loss. At steady state, the temperature profiles within A and within B are also plotted in Fig. 1. If no heat sources exist within materials A and B, please find the relationship between thermal conductivities, k_A and k_B. Note that the temperature drop (T_{1a}-T_{1b}) is caused by the thermal contact resistance between the surfaces of rod A and rod B. (5%)

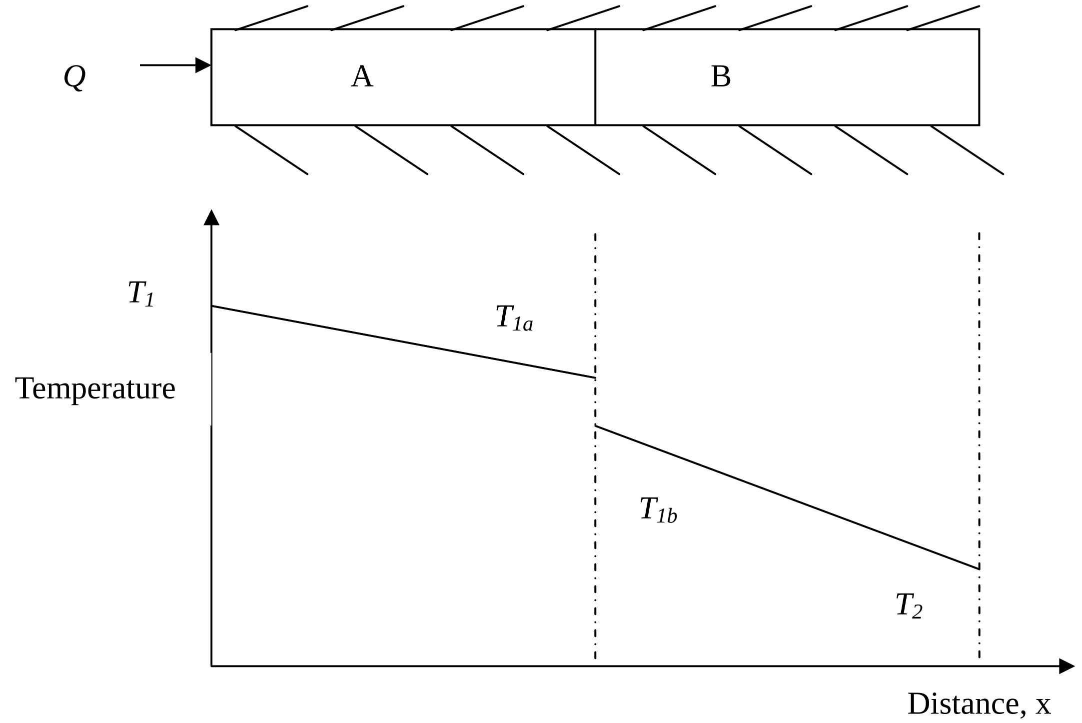

Fig. 1

2. A freezer compartment shown in Fig. 2 is covered with a layer of frost. The thickness of the frost layer is L. If the compartment is exposed to an ambient air at the temperature, T_∞ and a convective heat transfer coefficient, h, characterizes heat transfer by natural convection from the exposed surface of the layer, estimate the time required to completely melt the frost. The frost may be assumed to have a mass density, ρ_f, a latent heat of fusion, h_{sf}, and a fusion temperature, T_f. Note: You may neglect the heat transfer between compartment wall and frost layer. (10%)

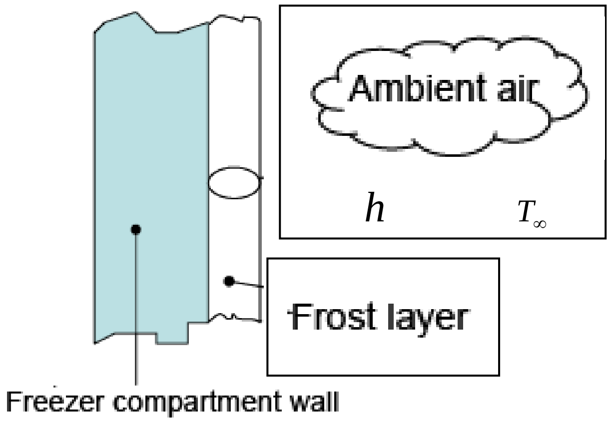

Fig. 2

3. The thermal conductivity of a plane wall varies with temperature according to the relation

$$k(T) = k_0(1+\beta T^2)$$

The surfaces at x= 0 and x= L are maintained at uniform temperatures T_1 and T_2, respectively. Find the relation for the heat flux through the slab. (15%)

4. The cylindrical system illustrated in Fig. 3 has negligible variation of temperature in the r and z directions. Assume that $\Delta r = r_o - r_i$ is small compared to r_i and denote the length in the z direction normal to the page, as L.
   (a) Beginning with a properly defined control volume and considering energy generation and storage effects, derive the differential equation that prescribes the variation in temperature with the angular coordinate ϕ. (10%)
   (b) For the steady-state conditions with no internal heat generation and with constant properties, determine the temperature distribution $T(\phi)$ in terms of the constants T_1, T_2, r_i and r_o. (10%)
   (c) For the conditions of part (b), write the expression for the heat transfer rate $\dot{Q}_\phi$. (5%)

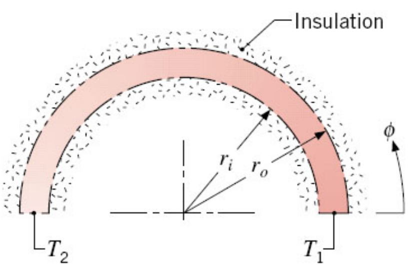

Fig. 3

5. Consider a sold sphere having an infinite radius, R. Inside the extremely large sphere, there is a small hollow space with a spherical shape. The radius of the hollow sphere is r_i (<<R). The whole solid space ($r_i \le r \le R$) is initially held at a uniform temperature, T_i. Suddenly, the interior surface at $r = r_i$ is kept at a constant temperature, T_0 ($\neq T_i$). By means of the method of similarity solution, please determine the temperature distribution, $T(r,t)$, within the solid sphere. Assume that the solid sphere has constant material properties and its thermal diffusivity is α. (15%)

Hint: (1) First, you may normalize the governing equation by the following dimensionless variables:

$$\theta = \frac{T - T_0}{T_i - T_0}, \quad \eta = \frac{r}{r_i}, \quad \tau = \frac{\alpha t}{r_i^2}.$$

(2) Second, you may apply Kirchhoff's transformation to this problem.

6. A semi-infinite solid is subjected to a periodic ambient temperature given by

$$T_a = T_\infty + T_0 \cos \omega t \qquad (1)$$

where T_∞, T_0 and ω are constant.

The differential equation governing the heat diffusion within the solid is shown as follows:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

The boundary conditions are given by

$$k\frac{\partial T}{\partial x} = h(T - T_a) \qquad \text{at } x = 0 \qquad (2a)$$

$$T \to T_\infty \qquad \text{as } x \to \infty \qquad (2b)$$

where α is the thermal diffusivity of the solid, k is the thermal conductivity of the solid and h is the convective heat transfer coefficient of the ambient fluid.

(a) Please normalize equations (2), (2a) and (2b) by using the following dimensionless variables: (5%)

$$\theta = \frac{T - T_\infty}{T_0}, \quad X = \frac{x}{L}, \quad \tau = \frac{t}{\beta}, \quad Bi = \frac{hL}{k}.$$

where $L = \sqrt{\alpha\beta}$, $\quad \beta = \dfrac{2\pi}{\omega}$.

(b) Solve the normalized temperature distribution θ of part (a). (10%)

(c) If h becomes infinite, what does the temperature distribution $T(x,t)$ become? (5%)