Mid-term Exam-2 (2010/05/18)
Advanced Heat Transfer

1. Consider a fluid (moderate Pr, but $\Pr \neq 1$) with constant freestream conditions, u_∞ and T_∞, in parallel flow over a flat plate. Assume the flow to be steady, two-dimensional, laminar, and incompressible, and the fluid properties to be constant. The plate temperature varies in the flow direction according to

$$T_w = T_\infty + Ax^n$$

   where A is a constant, n is a positive index ($n \geq 1$) and x is the axial coordinate measured from the leading edge of the plate.

   (a) Please write the boundary layer equations and the appropriate initial and boundary conditions describing the velocity and temperature fields in the flow. (8%)

   (b) Please derive an ordinary differential equation to show that the dimensionless temperature, θ, is a function of the similarity variable η only, where

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \quad \text{and} \quad \eta = y\sqrt{u_\infty / \nu x}$$

   In the above expression, y is the vertical coordinate measured upward from the surface of the plate and ν is the momentum diffusivity of the fluid. (24%)

2. A laminar forced convection flows over a flat plate with a constant heat flux, q''_w. The Nusselt number should vary with the Reynolds number and Prandtl number. Perform an order of magnitude analysis on this problem.

   (a) $\mathrm{Nu} \sim \mathrm{Re}^a \mathrm{Pr}^b$ for Pr<<1, a, b=? (5%)

   (b) $\mathrm{Nu} \sim \mathrm{Re}^c \mathrm{Pr}^d$ for Pr>>1, c, d=? (5%)

   (c) Temperature scale, $\Delta T \sim \mathrm{A} \bullet \mathrm{Re}^e \mathrm{Pr}^f$, A, e, f=? in (a) and (b) (8%)

3. Consider a modified form of Couette flow in which there are two immiscible fluids with constant properties sandwiched between two infinitely long and wide, parallel flat plates, as shown in Fig. 1. The flow is steady, incompressible, parallel, and laminar. The top plate moves at velocity V to the right, and the bottom plate is stationary. Gravity acts in the negative z-direction. There is no forced pressure gradient pushing the fluids through the channel. You may neglect surface tension effects and assume that the interface is horizontal. The pressure at the bottom of the flow (z=0) is equal to P_0. The bottom wall is kept at constant temperature, T_1 and the top wall is maintained at constant temperature, T_2. Assume that viscous dissipation cannot be neglected.

(a) Calculate the velocity field. (20%)
(b) Calculate the pressure field. (8%)
(c) Find the temperature distribution. (16%)
(d) Find the heat flux to each of the walls. (6%)

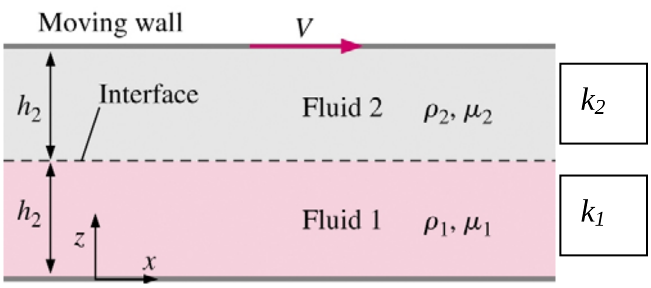

Fig. 1